Two fundamental Archimedes' principle problems involve finding the buoyant force on an object, either floating or completely submersed in an incompressible fluid, and deciding if an object floats or sinks. These and many other Archimedes' law problems start with the equations $F_g = mg = (\rho g)V$ for the force of gravity and $F_b = \rho_f g V_f$ for the buoyancy, where ρ is the density of the object, ρ_f is the density of the fluid in which it is wholly or partially immersed, V is the volume of the object, and V_f is the volume of fluid displaced. If the object is floating with no other forces acting, then $\rho V = \rho_f V_f$.

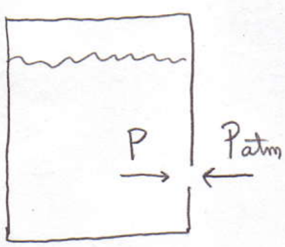
For a fluid in motion, the volume flow rate gives the volume of fluid that passes a cross section per unit time and is given by Av, where A is the cross-sectional area of the tube and v is the fluid speed.

Bernoulli's equation is used to solve some problems. It relates conditions (density, fluid speed, pressure, and height above Earth) at one point in the steady flow of a nonviscous, incompressible fluid to conditions at another point. If you are given all but one of these quantities you can use Bernoulli's equation to solve for the unknown quantity.

Questions and Example Problems from Chapter 13

Question 1

A closed tank is completely filled with water. A valve is then opened at the bottom of the tank and water begins to flow out. When the water stops flowing, will the tank be completely empty, or will there still be a noticeable amount of water in it? Explain your answer.



There will still be water in the tank.

The water will stop flowing out when the pressure P is equal to Patm.

Question 2

A steel beam is suspended completely under water by a cable that is attached to one end of the beam, so it hangs vertically. Another identical beam is also suspended completely under water, but by a cable that is attached to the beam so it hangs horizontally. Which beam, if either, experiences the greater buoyant force? Provide a reason for your answer. Neglect any change in water density with depth.

Buoyant force is the same for each.

$F_B = \rho_{fluid} V_{\substack{fluid \\ displaced}}\, g = \rho_{fluid} V_{sub}\, g$ and V_{sub} is the same for each case ($V_{sub} = V_{beam}$).

Problem 1

A pirate in a movie is carrying a chest (0.30 m × 0.30 m × 0.20 m) that is supposed to be filled with gold. To see how ridiculous this is, determine the weight (in newtons) of the gold. To judge how large this weight is, remember that 1 N = 0.225 lb.

$\rho_{gold} = 1.93 \times 10^4 \, Kg/m^3$

$\rho = m/V \rightarrow m = \rho V$

$V = 0.30m \times 0.30m \times 0.20\,m$

$= 0.018 m^3$

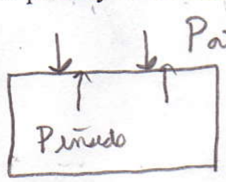
$m = (1.93 \times 10^4 \, Kg/m^3)(0.018 m^3)$

$m = 347 \, Kg$

$w = mg = (347Kg)(9.80m/s^2) \rightarrow \boxed{w = 3.40 \times 10^3 N \; (\approx 766 \, lbs)}$

Problem 2

An airtight box has a removable lid of area 1.3×10^{-2} m^2 and negligible weight. The box is taken up a mountain where the air pressure outside the box is 0.85×10^5 Pa. The inside of the box is completely evacuated. What is the magnitude of the force required to pull the lid off the box?



$P = F/A \longrightarrow F = PA$

$\Rightarrow$ the net force pushing down on the lid is

$F_{outside} - F_{inside} = P_{outside} A - P_{inside} A,$

but $P_{inside} = 0$ since inside of box is completely evacuated

$F_{outside} = P_{outside} A = (0.85 \times 10^5 Pa)(1.3 \times 10^{-2} m^2) = 1105 N$

$\Rightarrow$ to pull the lid off, we must exert a force of $\boxed{\text{at least } 1105 N}$

Problem 3

High-heeled shoes can cause tremendous pressure to be applied to a floor. Suppose that the radius of a heel is 6.00×10^{-3} m. At times during a normal walking motion, nearly the entire body weight acts perpendicular to the surface of the heel. Find the pressure that is applied to the floor under the heel because of the weight of a 50.0 kg woman.

$m = 50.0 Kg$

$w = mg = (50.0Kg)(9.80m/s^2) = 490N \rightarrow$ this is the force exerted on an area equal to the area of the heel

$r = 6.00 \times 10^{-3} m$

$P = F/A = \dfrac{F}{\pi r^2} \longrightarrow P = \dfrac{(490N)}{\pi (6.00 \times 10^{-3} m)^2} = \boxed{4.33 \times 10^6 Pa}$

Problem 4

At a depth of 10.9 km, the Challenger Deep in the Marianas Trench of the Pacific Ocean is the deepest site in any ocean. Yet, in 1960, Donald Walsh and Jacques Piccard reached the Challenger Deep in the bathyscaphe *Triete*. Assuming that seawater has a uniform density of 1024 kg/m^3, calculate the force the water would exert at a depth of 10.9 km on a round observation window of diameter 25 cm.

$P_o = P_{atm} = 1.013 \times 10^5 \, Pa$

$P = ?$

$d = 10.9 \, Km = 10.9 \times 10^3 \, m$

$\rho = 1024 \, Kg/m^3$

$P = P_o + \rho g d$

$P = (1.013 \times 10^5 \, Pa) + (1024 \, \frac{Kg}{m^3})(9.80 \, m/s^2)(10.9 \times 10^3 \, m)$

$P = 1.095 \times 10^8 \, Pa$

this is pressure at a depth of 10.9 Km

$P = F/A \rightarrow F = PA$

$\Rightarrow$ the force the water exerts on the windows is equal to

$F = PA = P(\pi r^2)$

$= (1.095 \times 10^8 \, Pa) \, \pi \, (12.5 \times 10^{-2} \, m)^2 = \boxed{5.4 \times 10^6 \, N}$

Problem 5

At a given instant, the blood pressure in the heart is 1.6×10^4 Pa. If an artery in the brain is 0.45 m above the heart, what is the pressure in the artery? Ignore any pressure changes due to blood flow.

$\rho_{blood} = 1060 \, Kg/m^3$

$d = 0.45 \, m$

$P = 1.6 \times 10^4 \, Pa$

$P_o = ?$

$P = P_o + \rho g d$

$P_o = P - \rho g d$

$= (1.6 \times 10^4 \, Pa) - (1060 \, Kg/m^3)(9.80 \, m/s^2)(0.45 \, m)$

$\boxed{P_o = 1.1 \times 10^4 \, Pa}$

Problem 6

A solid block is attached to a spring scale. When the block is suspended in air the scale reads 20.0 N; when it is completely immersed in water the scale reads 17.7 N. What is the (a) volume and (b) density of the block?

$W_{air} = 20.0\,N$

$W_{water} = 17.7\,N$

$\Big\}$ bouyant force $F_B = W_{air} - W_{water} = 2.30\,N$

(a) $F_B = \rho_{fluid} V_{sub}\, g \rightarrow$ if block is completely submerged then $V_{sub} = V_{block}$

$$F_B = \rho_{fluid} V_{block}\, g \rightarrow V_{block} = \frac{F_B}{\rho_{fluid}\, g} = \frac{2.30\,N}{(1.00\times10^3\,\frac{Kg}{m^3})(9.80\,m/s^2)}$$

$$\boxed{V_{block} = 2.35\times10^{-4}\,m^3}$$

(b) $W_{air} = mg \longrightarrow m = W_{air}/g = \frac{20.0\,N}{9.80\,m/s^2} = 2.04\,Kg$

$$\rho = \frac{m}{V} = \frac{2.04\,Kg}{2.35\times10^{-4}\,m^3} \rightarrow \boxed{\rho = 8.68\times10^3\,Kg/m^3}$$

Problem 7

Only a small part of an iceberg protrudes above the water, while the bulk lies below the surface. The density of ice is 917 kg/m^3 and that of seawater is 1025 kg/m^3. Find the percentage of the iceberg's volume that lies below the surface.

$\Rightarrow$ since the iceberg is floating, the bouyant force must equal the weight

$$F_B = w \longrightarrow \rho_{fluid} V_{sub}\, g = m g \qquad m = \rho V$$

$$\rho_{fluid} V_{sub}\, g = (\rho_{ice} V_{ice})\, g$$

$$\rho_{fluid} V_{sub} = \rho_{ice} V_{ice}$$

$$V_{sub} = \left(\frac{\rho_{ice}}{\rho_{fluid}}\right) V_{ice} \longrightarrow V_{sub} = \left(\frac{917\,Kg/m^3}{1025\,Kg/m^3}\right) V_{ice}$$

$$V_{sub} = 0.895\, V_{ice} \rightarrow \boxed{89.5\%\ \text{submerged}}$$

Problem 8

A water line with an internal radius of 6.5×10^{-3} m is connected to a shower head that has 12 holes. The speed of the water in the line is 1.2 m/s. (a) What is the volume flow rate in the line? (b) At what speed does the water leave one of the holes (effective hole radius = 4.6×10^{-4} m) in the head?

$r_1 = 6.5 \times 10^{-3}$ m

$V_1 = 1.2$ m/s

(a) volume flow rate (m^3/s) $\quad Q = Av$

$$Q = Av = (\pi r^2)v$$

$$= \pi(6.5\times10^{-3}\,m)^2(1.2\,m/s)$$

$$\boxed{Q = 1.6\times10^{-4}\ m^3/s}$$

(b) $r_2 = 4.6\times10^{-4}$ m

$V_2 = ?$

$A_2V_2 = A_1V_1$ note: $A_2 = 12\pi r_2^2$ since shower head has 12 holes

$$V_2 = A_1V_1/A_2$$

$$V_2 = \frac{\pi(6.5\times10^{-3}m)^2(1.2m/s)}{12\pi(4.6\times10^{-4}m)^2} \longrightarrow \boxed{V_2 = 2.0\times10^{1}\ m/s}$$

Problem 9

Water flows through a pipe of radius 8.0 cm with a speed of 10.0 m/s. It then enters a smaller pipe of radius 3.0 cm. What is the speed of the water as it flows through the smaller pipe? Assume that the water is incompressible.

$r_1 = 8.0$ cm

$V_1 = 10.0$ m/s

$r_2 = 3.0$ cm

$V_2 = ?$

$$A_1V_1 = A_2V_2$$

$$(\pi r_1^2)V_1 = (\pi r_2^2)V_2$$

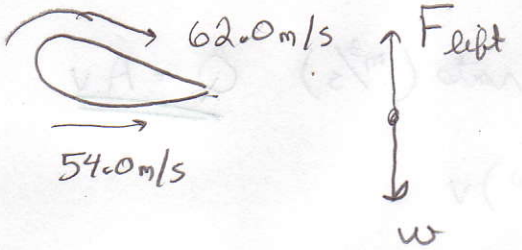
$$V_2 = \frac{r_1^2}{r_2^2}V_1 = \left(\frac{r_1}{r_2}\right)^2 V_1$$

$$V_2 = \left(\frac{r_1}{r_2}\right)^2 V_1$$

$$= \left(\frac{8.0\,cm}{3.0\,cm}\right)^2(10.0\,m/s) \longrightarrow \boxed{V_2 = 71\ m/s}$$

Problem 10

An airplane has an effective wing surface area of 16 m^2 that is generating the lift force. In level flight the air speed over the top of the wings is 62.0 m/s, while the air speed beneath the wings is 54.0 m/s. What is the weight of the plane?



level flight $\longrightarrow a_y = 0$

$$\Sigma F_y = m a_y = 0 \longrightarrow F_{lift} = w$$

$F_{lift} = (P_2 - P_1)A$ where $P_2 - P_1$ is the pressure difference

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \quad \text{assume } y_1 = y_2$$

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2 \longrightarrow P_2 - P_1 = \tfrac{1}{2}\rho(v_1^2 - v_2^2)$$

$$F_{lift} = \tfrac{1}{2}\rho(v_1^2 - v_2^2)A = \tfrac{1}{2}(1.29\,\text{kg/m}^3)\left[(62.0\,\text{m/s})^2 - (54.0\,\text{m/s})^2\right](16\,\text{m}^2)$$

$$= 9.58 \times 10^3\,\text{N} \longrightarrow \boxed{w = 9.58 \times 10^3\,\text{N}}$$

Problem 11

Suppose that a 15 m/s wind is blowing across the roof of your house. The density of air is 1.29 kg/m^3. (a) Determine the reduction in pressure (below atmospheric pressure of stationary air) that accompanies this wind. (b) Explain why some roofs are "blown outward" during high winds.

from Bernoulli's equation: $P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$

assume $y_1 = y_2 \longrightarrow P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$

$$P_1 - P_2 = \tfrac{1}{2}\rho(v_1^2 - v_2^2)$$

$$= \tfrac{1}{2}(1.29\,\text{kg/m}^3)\left[(15\,\text{m/s})^2 - (0\,\text{m/s})^2\right] = 145\,\text{Pa}$$

$$\boxed{P_1 - P_2 = 1.5 \times 10^2\,\text{Pa}}$$

(b) Since $P = F/A \rightarrow F = PA$. The pressure inside the house is greater than the pressure outside the house. This causes a net outward force on the roof. If the net force is large enough, the roof can actually be pushed off.