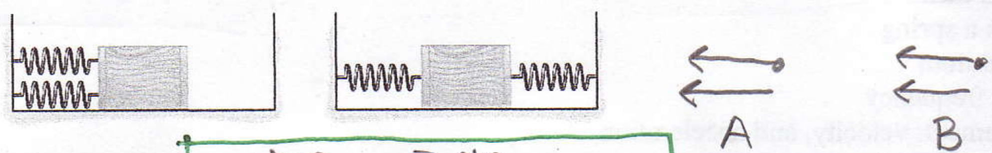
Some problems make use of the relationships among angular frequency, frequency, and period for simple harmonic motion: $\omega = 2\pi f$, and $f = 1/T$. Occasionally the period is given indirectly by describing a time interval. You must then know, for example, that the time the oscillator takes to go from maximum displacement in one direction to maximum displacement in the other direction is $T/2$ or the time it takes to go from maximum displacement to zero displacement is $T/4$. If these time intervals or others are given, you should be able to calculate the period, frequency, and angular frequency. You should also know how to find the maximum speed and maximum acceleration in terms of the angular frequency and amplitude: $v_{max} = A\omega$ and $a_{max} = A\omega^2$. Some problems require you to know the relationship between the angular frequency and the appropriate physical properties of the oscillating system: $\omega = \sqrt{k/m}$ for an undamped spring-object system.

Some problems can be solved using the principle of mechanical energy conservation. For a spring-object system, the mechanical energy E is given by: $E = ½ mv^2 + ½ kx^2 + mgh$.

Questions and Example Problems from Chapter 14

Question 1
The drawing shows identical springs that are attached to a box in two different ways. Initially, the springs are unstrained. The box is then pulled to the right and released. In each case, the initial displacement of the box is the same. At the moment of release, which box, of either, experiences the greater net force due to the spring? Provide a reason for your answer.



$\boxed{\text{net force is the same}}$

Question 2
Suppose that a grandfather clock (a simple pendulum) is running slowly. That is, the time it takes to complete each cycle is longer than it should be. Should one shorten or lengthen the pendulum to make the clock keep the correct time? Why?

$T = 2\pi\sqrt{\frac{L}{g}}$ ⇒ to decrease the period we should $\boxed{\text{decrease the length}}$ of the pendulum

Problem 1
The equilibrium length of a certain spring with a force constant of k = 250 N/m is 0.20 m. (a) What force is required to stretch this spring to twice its equilibrium length? (b) Is the force required to compress the spring to half its length the same as in part (a)? Explain.

x = displacement from unstretched or uncompressed length

(a) $\left.\begin{array}{l} x = 0.20\text{ m} \\ k = 250\text{ N/m} \end{array}\right\}$ $F = -kx = -(250\text{ N/m})(0.20\text{ m}) = \boxed{-5.0\times10^1\text{ N}}$ (− sign means force is in opposite direction of displacement)

(b) now $x = -0.10$ m

$F = -kx = -(250\text{ N/m})(-0.10\text{ m}) = \boxed{25\text{ N}}$ → no, the magnitude + direction of F are different

Problem 2

An air-track glider attached to a spring oscillates between the 10 cm mark and the 60 cm mark on the track. The glider completes 10 oscillations in 33 s. What are the (a) period, (b) frequency, (c) amplitude, and (d) maximum speed of the glider?

(a) since the glider completes 10 oscillations in 33s, the period is $T = \frac{33\text{s}}{10 \text{ oscillations}} = 3.3 \text{ s/oscillation} \longrightarrow \boxed{T = 3.3\text{s}}$

(b) $f = 1/T \rightarrow f = \frac{1}{3.3\text{s}} \longrightarrow \boxed{f = 0.30\text{Hz}}$

(c) glider oscillates between 10cm + 60cm mark; glider travel a distance of 50cm so the amplitude $\boxed{A = 25\text{cm} = 0.25\text{ m}}$

(d) $V_{max} = \omega A = 2\pi f A$

$= 2\pi(0.30\text{Hz})(0.25\text{m}) \longrightarrow \boxed{V_{max} = 0.48 \text{ m/s}}$

Problem 3

A computer to be used in a satellite must be able to withstand accelerations of up to 25 times the acceleration due to gravity, In a test to see whether it meets this specification, the computer is bolted to a frame that is vibrated back and forth in simple harmonic motion at a frequency of 9.5 Hz. What is the minimum amplitude of vibration that must be used in this test?

$a_{max} = 25g = 25(9.80\text{m/s}^2) = 245\text{ m/s}^2$

$f = 9.5\text{ Hz}$

$A = ?$

$a_{max} = (2\pi f)^2 A$

$= 4\pi^2 f^2 A$

$A = \frac{a_{max}}{4\pi^2 f^2} \rightarrow A = \frac{(245\text{m/s}^2)}{4\pi^2(9.5\text{Hz})^2}$

$A = 0.069\text{m} = \boxed{6.9\times10^{-2}\text{m}}$

Problem 4
A block of mass $m = 0.750$ kg is fastened to an unstrained horizontal spring whose spring constant is $k = 82.0$ N/m. The block is given a displacement of +0.120 m, where the + sign indicates that the displacement is along the +x axis, and then released from rest. (a) What is the force (magnitude and direction) that the spring exerts on the block just before the block is released? (b) Find the frequency of the resulting oscillatory motion. (c) What is the maximum speed of the block? (d) Determine the magnitude of the maximum acceleration of the block?

$m = 0.750\,Kg$
$K = 82.0\,N/m$
$x = 0.120\,m$

(a) $F = -Kx = -(82.0\,N/m)(0.120\,m)$

$F = -9.84\,N$ $\boxed{F = 9.84\,N \text{ to the left}}$

(b) $f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{(82.0\,N/m)}{0.750\,Kg}}$

$\boxed{f = 1.66\,Hz}$

(c) $V_{max} = 2\pi f A$ $A = 0.120\,m$ since block is released from rest

$V_{max} = 2\pi(1.66\,Hz)(0.120\,m) \longrightarrow \boxed{V_{max} = 1.25\,m/s}$

(d) $a_{max} = (2\pi f)^2 A = 4\pi^2(1.66\,Hz)^2(0.120\,m) = \boxed{13.1\,m/s^2}$

Problem 5
The shock absorbers in the suspension system of a car are in such bad shape that they have no effect on the behavior of the springs attached to the axles. Each of the identical springs attached to the front axle supports 320 kg. A person pushes down on the middle of the front end of the car and notices that it vibrates through 5 cycles in 3.0 s. Find the spring constant of either spring.

$\Rightarrow$ if the spring vibrates through 5 cycles in 3.0 s, then the frequency of vibration is $f = \frac{5\,\text{cycles}}{3.0\,s} = 1.67\,Hz$

$f = 1.67\,Hz$
$m = 320\,Kg$
$K = ?$

$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \longrightarrow 2\pi f = \sqrt{\frac{K}{m}}$

$4\pi^2 f^2 = K/m$

$K = 4\pi^2 f^2 m$

$K = 4\pi^2(1.67\,Hz)^2(320\,Kg)$

$\boxed{K = 3.5\times10^4\,N/m}$

Problem 6

A 200 g mass attached to a horizontal spring oscillates at a frequency of 2.0 Hz. At one instant, the mass is at x = 5.0 cm and has speed v = -30 cm/s. Determine: (a) The period. (b) The amplitude. (c) The maximum speed. (d) The total energy.

$m = 0.200\,Kg$

$f = 2.0\,Hz$

$x = 5.0\,cm$

$v = -30\,cm/s$

(a) $T = 1/f = \frac{1}{2.0\,Hz} \rightarrow \boxed{T = 0.50\,s}$

(b) $E = \frac{1}{2}KA^2 = \frac{1}{2}mv^2 + \frac{1}{2}Kx^2$

$\frac{1}{2}KA^2 = \frac{1}{2}mv^2 + \frac{1}{2}Kx^2$

$A^2 = m/K\,v^2 + x^2 \rightarrow A = \sqrt{\frac{m}{K}v^2 + x^2}$

$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \rightarrow K = (2\pi f)^2 m = 4\pi^2(2.0\,Hz)^2(0.200\,Kg) = 31.6\,N/m$

$A = \sqrt{\frac{(0.200\,Kg)}{(31.6\,N/m)}(0.30\,m/s)^2 + (0.050\,m)^2} = \boxed{0.055\,m = 5.5\,cm}$

(c) $V_{max} = 2\pi f A = 2\pi(2.0\,Hz)(0.055\,m) = \boxed{0.69\,m/s}$

(d) $E = \frac{1}{2}KA^2 = \frac{1}{2}mV_{max}^2 = \frac{1}{2}(0.200\,Kg)(0.69\,m/s)^2$

$= \boxed{0.048\,J}$

Problem 7

An archer pulls the bowstring back for a distance of 0.470 m before releasing the arrow. The bow and string act like a spring whose spring constant is 425 N/m. (a) What is the elastic potential energy of the drawn bow? (b) The arrow has a mass of 0.0300 kg. How fast is it traveling when it leaves the bow?

$x = 0.470\,m$

$K = 425\,N/m$

(a) $U_s = \frac{1}{2}Kx^2$

$= \frac{1}{2}(425\,N/m)(0.470\,m)^2$

$\boxed{U_s = 46.9\,J}$

(b) from conservation of mechanical energy:

$\frac{1}{2}mv_f^2 + \underbrace{\frac{1}{2}Kx_f^2}_{=0} + \underbrace{mgy_f}_{=0} = \underbrace{\frac{1}{2}mv_i^2}_{=0} + \frac{1}{2}Kx_i^2 + \underbrace{mgy_i}_{=0}$

$\frac{1}{2}mv_f^2 = \frac{1}{2}Kx_i^2 \rightarrow V_f = \sqrt{\frac{K}{m}}\,x_i$

$V_f = \sqrt{\frac{(425\,N/m)}{0.0300\,Kg}}\,(0.470\,m) \rightarrow \boxed{V_f = 55.9\,m/s}$

Problem 8

A 1.00×10^{-2} kg block is resting on a horizontal frictionless surface and is attached to a horizontal spring whose spring constant is 124 N/m. The block is shoved parallel to the spring axis and is given an initial speed of 8.00 m/s, while the spring is initially unstrained. What is the amplitude of the resulting simple harmonic motion?

$m = 1.00 \times 10^{-2}\,Kg$

$K = 124\,N/m$

when $x = 0\,m$, $v = 8.00\,m/s$

$A = ?$

$\Rightarrow$ for a horizontal spring oscillating in SHM, $E_{total} = \frac{1}{2}KA^2$

$$\frac{1}{2}KA^2 = \underbrace{\frac{1}{2}Kx^2}_{=0} + \frac{1}{2}mv^2$$

$$\frac{1}{2}KA^2 = \frac{1}{2}mv^2$$

$$A^2 = \frac{m}{K}v^2 \longrightarrow A = \sqrt{\frac{m}{K}}\,v$$

$$A = \sqrt{\frac{(1.00 \times 10^{-2}\,Kg)}{124\,N/m}}\,(8.00\,m/s) \longrightarrow \boxed{A = 7.18 \times 10^{-2}\,m}$$

Problem 9

A 0.40 kg mass is attached to a spring with a force constant of 26 N/m and released from rest a distance of 3.2 cm from the equilibrium position of the spring. What is the speed of the mass when it is halfway to the equilibrium position?

$m = 0.40\,Kg$

$K = 26\,N/m$

$x_i = 3.2 \times 10^{-2}\,m$

$v_i = 0\,m/s$

$x_f = 1.6 \times 10^{-2}\,m$

$v_f = ?$

from conservation of mechanical energy:

$$\frac{1}{2}mv_f^2 + \frac{1}{2}Kx_f^2 + \underbrace{mgy_f}_{=0} = \underbrace{\frac{1}{2}mv_i^2}_{=0} + \frac{1}{2}Kx_i^2 + \underbrace{mgy_i}_{=0}$$

$$\frac{1}{2}mv_f^2 + \frac{1}{2}Kx_f^2 = \frac{1}{2}Kx_i^2$$

$$mv_f^2 = K(x_i^2 - x_f^2)$$

$$v_f = \sqrt{\frac{K}{m}(x_i^2 - x_f^2)}$$

$$v_f = \sqrt{\frac{(26\,N/m)}{0.40\,Kg}\left[(3.2 \times 10^{-2}\,m)^2 - (1.6 \times 10^{-2}\,m)^2\right]}$$

$$\boxed{v_f = 0.22\,m/s}$$

Problem 10
Astronauts on a distant planet set up a simple pendulum of length 1.2 m. The pendulum executes simple harmonic motion and makes 100 complete vibrations in 280 s. What is the acceleration due to gravity?

$L = 1.2 m$

$f = 100 \text{ vibrations} / 280 s = 0.357 Hz$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \longrightarrow 2\pi f = \sqrt{\frac{g}{L}}$$

$$4\pi^2 f^2 = \frac{g}{L}$$

$$g = 4\pi^2 f^2 L = 4\pi^2 (0.357 Hz)^2 (1.2 m)$$

$$\boxed{g = 6.04 m/s^2}$$

Problem 11
If the period of a simple pendulum is to be 2.0 s, what should be its length?

$T = 2.0 s \Rightarrow$ for a simple pendulum: $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

$L = ?$

$$T = 1/f = 2\pi\sqrt{L/g}$$

$$T^2 = 4\pi^2 (L/g)$$

$$L = \frac{gT^2}{4\pi^2} \longrightarrow L = \frac{(9.80 m/s^2)(2.0 s)^2}{4\pi^2}$$

$$\boxed{L = 0.99 m}$$