Questions and Example Problems from Chapter 3

Question 1

(a) Can a vector have nonzero magnitude if a component is zero? If no, why not? If yes, give an example. (b) Can a vector have zero magnitude and a nonzero component? If no, why not? If yes, give an example.

(a) $\boxed{\text{Yes,}}$ as long as the other component is non-zero. ex $A_x = 0$ m, $A_y = 5$ m

(b) $\boxed{\text{No,}}$ if a vector has zero magnitude, both components must be zero.

Question 2

Vector **A** points along the +y axis and has a magnitude of 100.0 units. Vector **B** points at an angle of 60.0° above the +x axis and has a magnitude of 200.0 units. Vector **C** points along the +x axis and has a magnitude of 150.0 units. Which vector has (a) the largest x component and (b) the largest y component?

$A_x = 0$ $\quad B_x = (200.0)\cos 60° = 100.0$ $\quad C_x = 150.0$ $\quad$ (a) $\boxed{C}$

$A_y = 100.0$ $\quad B_y = (200.0)\sin 60° = 173.2$ $\quad C_y = 0$ $\quad$ (b) $\boxed{B}$

Question 3

For a projectile, which of the following quantities are constant during the flight: x, y, v_x, v_y, v, a_x, a_y? Which or the quantities are zero throughout the flight?

constant $\rightarrow$ $\boxed{V_x,\ a_x,\ +\ a_y}$

zero $\rightarrow$ $\boxed{a_x}$

Question 4

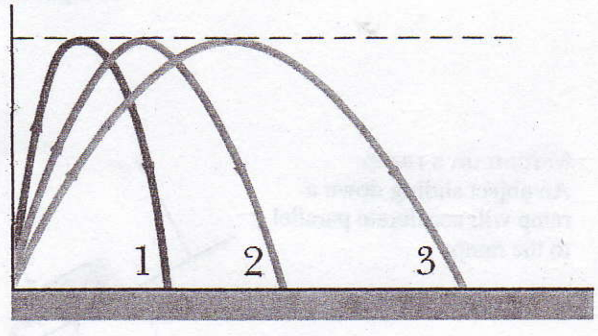
A tennis ball is hit upward into the air and moves along an arc. Neglecting air resistance, where along the arc is the speed of the ball (a) a minimum and (b) a maximum? Justify your answers.

(a) the speed of the ball is a minimum at $\boxed{\text{the highest point}}$ where $V_y = 0$ m/s. Min. speed is $V_{min} = V_x = (V_x)_i$

(b) the speed is a max $\boxed{\text{right when it leaves the racket}}$

Question 5

The figure shows three paths for a football kicked from ground level. Ignoring the effects of air resistance, rank the paths according to (a) time of flight, (b) initial vertical velocity component, (c) initial horizontal velocity component, and (d) initial speed.



(a) $\boxed{\text{all tie}}$

(b) $\boxed{\text{all tie}}$

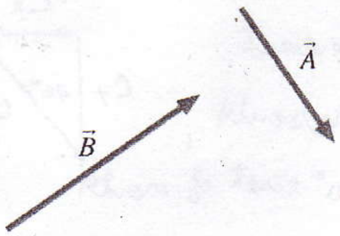
(c) $\boxed{1, 2, 3}$

(d) $\boxed{1, 2, 3}$

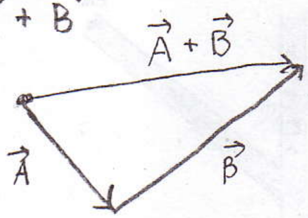
Note: since they all rise to the same height, we know that $(V_y)_i$ is the same for each so they all take the same amount of time to reach their highest point

Problem 1

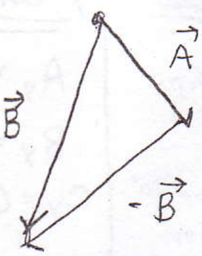
Trace the vectors in the figure below onto your paper. Then use graphical methods to draw the vectors (a) $\vec{A}+\vec{B}$ and (b) $\vec{A}-\vec{B}$.



(a) $\vec{A}+\vec{B}$



(b) $\vec{A}-\vec{B} = \vec{A}+(-\vec{B})$

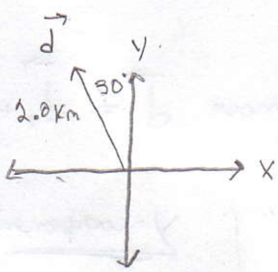


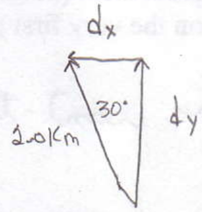
Problem 2

Draw each of the following vectors, then find its x- and y-components.

a) $\vec{d}$ = (2.0 km, 30° left of +y-axis)
b) $\vec{v}$ = (5.0 cm/s, - x-direction)
c) $\vec{a}$ = (10 m/s², 40° left of -y-axis)

a)

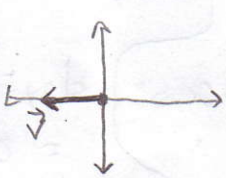




$d_x = -(2.0\text{km})\sin 30° = \boxed{-1.0\text{ m}}$

$d_y = (2.0\text{km})\cos 30° = \boxed{1.7\text{ m}}$

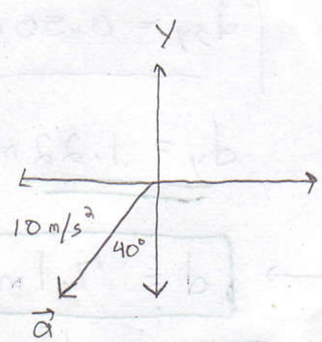
b)

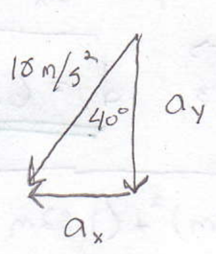


$\boxed{V_x = -5.0\text{cm/s}}$

$\boxed{V_y = 0}$

c)



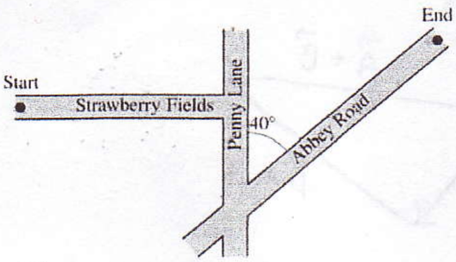


$a_x = -(10\text{ m/s}^2)\sin 40°$
$= \boxed{-6.4\text{ m/s}^2}$

$a_y = -(10\text{ m/s}^2)\cos 40°$
$= \boxed{-7.7\text{ m/s}^2}$

Problem 3

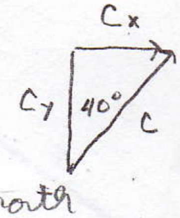
While visiting England, you decide to take a jog and find yourself in the neighborhood shown on the map in the figure below. What is your displacement after running 2.0 km on Strawberry Fields, 1.0 km on Penny Lane, and 4.0 km on Abbey Road?



$\vec{A}$ = 2.0 Km to the east

$\vec{B}$ = 1.0 Km to the south

$\vec{C}$ = 4.0 Km at 40° east of north



vector	x-component	y-component
$\vec{A}$	$A_x = 1.0\,Km$	$A_y = 0$
$\vec{B}$	$B_x = 0$	$B_y = -1.0\,Km$
$\vec{C}$	$C_x = (4.0\,Km)\sin 40° = 2.57\,Km$	$C_y = (4.0\,Km)\cos 40° = 3.06\,Km$
$\vec{d} = \vec{A} + \vec{B} + \vec{C}$	$d_x = 3.57\,Km$	$d_y = +2.06\,Km$

$d_x = 3.57\,Km$

$d_y = 2.06\,Km$

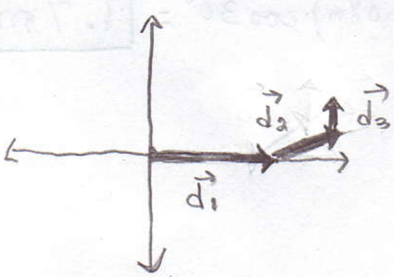
$d = \sqrt{d_x^2 + d_y^2} = 4.1\,Km$

$\theta = \tan^{-1}(d_y/d_x) = 30.°$

Problem 4

A golfer, putting on a green, requires three strokes to "hole the ball". During the first put, the ball rolls 5.0 m due east. For the second out, the ball travels 2.1 m at an angle of 20.0° north of east. The third put is 0.50 m due north. What displacement (magnitude and direction relative to east) would have been needed to "hole the ball" on the very first put?

we want to know $\vec{d} = \vec{d_1} + \vec{d_2} + \vec{d_3}$



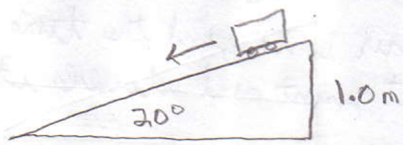
vector	x-component	y-component
$\vec{d_1}$	$d_{1x} = 5.0\,m$	$d_{1y} = 0\,m$
$\vec{d_2}$	$d_{2x} = (2.1\,m)\cos 20.0° = 1.97\,m$	$d_{2y} = (2.1\,m)\sin 20.0° = 0.718\,m$
$\vec{d_3}$	$d_{3x} = 0\,m$	$d_{3y} = 0.50\,m$
$\vec{d} = \vec{d_1} + \vec{d_2} + \vec{d_3}$	$d_x = 6.97\,m$	$d_y = 1.22\,m$

$$d = \sqrt{d_x^2 + d_y^2} = \sqrt{(6.97\,m)^2 + (1.22\,m)^2} \longrightarrow \boxed{d = 7.1\,m}$$

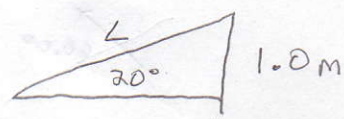
$$\theta = \tan^{-1}(d_y/d_x) = \tan^{-1}\left(\frac{1.22\,m}{6.97\,m}\right) \longrightarrow \boxed{\theta = 9.9°}$$

Problem 5

A piano has been pushed to the top of the ramp at the back of a moving van. The workers think it is safe, but as they walk away, it begins to roll down the ramp. If the back of the truck is 1.0 m above the ground and the ramp is inclined at 20°, how much time do the workers have to get to the piano before it reaches the bottom of the ramp?



define +x-axis as down ramp



$L = 1.0\,m / \sin 20° = 2.92\,m$

$$a_x = g \sin\theta = (9.80\,m/s^2)\sin 20° = 3.35\,m/s^2$$

$x_i = 0\,m$

$x_f = 2.92\,m$

$(v_i)_x = 0$

$a_x = 3.35\,m/s^2$

$\Delta t = ?$

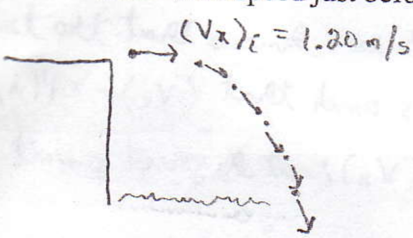
$$x_f = \overset{=0}{x_i} + \overset{=0}{(v_x)_i}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

$$x_f = \tfrac{1}{2}a_x(\Delta t)^2 \longrightarrow \Delta t = \sqrt{\frac{2x_f}{a_x}}$$

$$\Delta t = \sqrt{\frac{2(2.92\,m)}{3.35\,m/s^2}} \longrightarrow \boxed{\Delta t = 1.32\,s}$$

Problem 6

A diver runs horizontally with a speed of 1.20 m/s off a platform that is 10.0 m above the water. What is the speed just before striking the water?



$\Rightarrow$ first we need to find the time Δt to hit the water

$y_i = 10.0\,m$

$y_f = 0\,m$

$(v_y)_i = 0\,m/s$

$a_y = -9.80\,m/s^2$

$\Delta t = ?$

$$\overset{=0}{y_f} = y_i + \overset{=0}{(v_y)_i}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

$$0 = y_i + \tfrac{1}{2}a_y(\Delta t)^2$$

$$\Delta t = \sqrt{\frac{-2y_i}{a_y}} = \sqrt{\frac{-2(10.0\,m)}{-9.80\,m/s^2}}$$

$$\Delta t = 1.43\,s$$

$\Rightarrow$ to get the speed v at $t = 1.43\,s$, we find $(v_x)_f$ + $(v_y)_f$ at $t = 1.43\,s$ and use $v_f = \sqrt{(v_x)_f^2 + (v_y)_f^2}$

$(v_x)_f = (v_x)_i = 1.20\,m/s$

$(v_y)_i = 0\,m/s$

$(v_y)_f = ?$

$a_y = -9.80\,m/s^2$

$\Delta t = 1.43\,s$

$$(v_y)_f = \overset{=0}{(v_y)_i} + a_y\Delta t \longrightarrow (v_y)_f = a_y\Delta t$$

$$(v_y)_f = (-9.80\,m/s^2)(1.43\,s) = -14.0\,m/s$$

speed $v_f = \sqrt{(v_x)_f^2 + (v_y)_f^2} = \sqrt{(1.20\,m/s)^2 + (-14.0\,m/s)^2}$

$$v_f = 14.1\,m/s$$

Problem 7

The punter on a football team tries to kick a football so that it stays in the air for a long "hang time". (a) If the ball is kicked with an initial velocity of 25.0 m/s at an angle of 60.0° above the ground, what is the "hang time"? (b) How far does the ball travel before it hits the ground?

note: the easy way to find the time in the air is to find the time to the highest point and double it

a) $(V_y)_i = (25.0\,m/s)\sin 60.0°$
$= 21.7\,m/s$

$(V_y)_f = 0\,m/s$ at highest point

$a_y = -9.80\,m/s^2 \quad \Delta t = ?$

$\underbrace{(V_y)_f}_{=0} = (V_y)_i + a_y\Delta t \rightarrow 0 = (V_y)_i + a_y\Delta t$

$\Delta t = -(V_y)_i/\Delta t = -\dfrac{(21.7\,m/s)}{-9.80\,m/s^2} = 2.21\,s$

time in air $= 2(2.21\,s) = \boxed{4.42\,s}$

b) $(V_x)_i = (25.0\,m/s)\cos 60.0°$
$= 12.5\,m/s$

$X_o = 0\,m \quad x = ?$

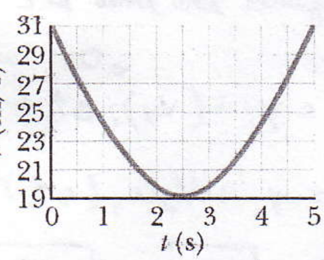
$\Delta t = 4.42\,s$

$x = \underbrace{x_o}_{=0} + (V_x)_i\Delta t \rightarrow x = (V_x)_i\Delta t$

$x = (12.5\,m/s)(4.42\,s) \rightarrow \boxed{x = 55.3\,m}$

Problem 8

A golf ball is struck at ground level. The speed of the golf ball as a function of the time is shown in the figure below, where t = 0 at the instant the ball is struck. (a) How far does the golf ball travel horizontally before returning to ground level? (b) What is the maximum height above ground level attained by the ball?



note: from the graph we know that the time in the air is $\Delta t = 5.0\,s$ and that $(V_x)_i = 19\,m/s$ (because $V_{min} = (V_x)_i$ at highest point)

(a) $(V_x)_i = 19\,m/s$

$\Delta t = 5.0\,s$

$X_i = 0 \quad x_f = ?$

$x_f = \underbrace{x_i}_{=0} + (V_x)_i\Delta t$

$x_f = (19\,m/s)(5.0\,s) = 95\,m$

(b) To get max height we need to know $(V_y)_i$

$(V_y)_f = 0\,m/s$ at highest point

$(V_y)_i = ?$

$a_y = -9.80\,m/s^2$

$\Delta t = 2.5\,s$ to highest point

$y_i = 0\,m$

$y_f = ?$

$\underbrace{(V_y)_f}_{=0} = (V_y)_i + a_y\Delta t \rightarrow 0 = (V_y)_i + a_y\Delta t$

$(V_y)_i = -a_y\Delta t = -(-9.80\,m/s^2)(2.5\,s)$

$(V_y)_i = 24.5\,m/s$

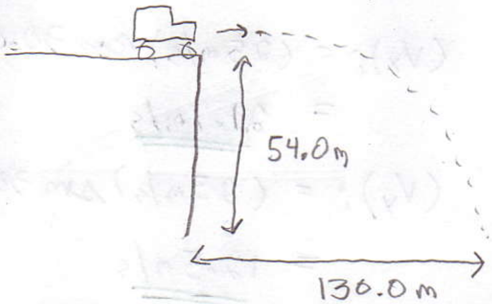
$y_f = \underbrace{y_i}_{=0} + (V_y)_i\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$

$y_f = (24.5\,m/s)(2.5\,s) + \tfrac{1}{2}(-9.80\,m/s^2)(2.5\,s)^2$

$y_f = 31\,m$

Problem 9

A car drives straight off the edge of a cliff that is 54.0 m high. The police at the scene of the accident note that the point of impact is 130.0 m from the base of the cliff. How fast was the car traveling when it went over the cliff?



⇒ first get the time in the air from the vertical motion

$y_i = 54.0\,m$

$y_f = 0\,m$

$(v_y)_i = 0\,m/s$

$a_y = -9.80\,m/s^2$

$\Delta t = ?$

$\overset{=0}{y_f} = y_i + \overset{=0}{(v_y)_i}\Delta t + \frac{1}{2}a_y(\Delta t)^2$

$0 = y_i + \frac{1}{2}a_y(\Delta t)^2$

$\Delta t = \sqrt{\frac{-2y_i}{a_y}}$

$\Delta t = \sqrt{\frac{-2(54.0\,m)}{-9.80\,m/s^2}} = \underline{\underline{3.32\,s}}$

⇒ now get the speed from the horizontal motion

$x_i = 0\,m$

$x_f = 130.0\,m$

$(v_x)_i = ?$

$\Delta t = 3.32\,s$

$x_f = \overset{=0}{x_i} + (v_x)_i\Delta t$

$(v_x)_i = x_f/\Delta t \rightarrow (v_x)_i = \frac{130.0\,m}{3.32\,s} = \underline{\underline{39.2\,m/s}}$

$\boxed{\text{car's speed was } 39.2\,m/s}$

Problem 10

A golfer, standing on a fairway, hits a shot to a green that is elevated 5.50 m above the point where she is standing. If the ball leaves her club with a velocity of 46.0 m/s at an angle of 35.0° above the ground, find the time the ball is in the air before it hits the green.

$(v_i)_y = (46.0\,m/s)\sin 35.0°$
$= 26.4\,m/s$

$y_i = 0\,m$

$y_f = 5.50\,m$

$a_y = -9.80\,m/s^2$

$\Delta t = ?$

$y_f = \overset{=0}{y_i} + (v_i)_y\Delta t + \frac{1}{2}a_y(\Delta t)^2$

$\frac{1}{2}a_y(\Delta t)^2 + (v_i)_y\Delta t - y_f = 0$

$(-4.90\,m/s^2)(\Delta t)^2 + (26.4\,m/s)\Delta t - 5.50\,m = 0$

↓ quadratic equation

$a = -4.90\,m/s^2$

$b = 26.4\,m/s$

$c = -5.50\,m$

$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

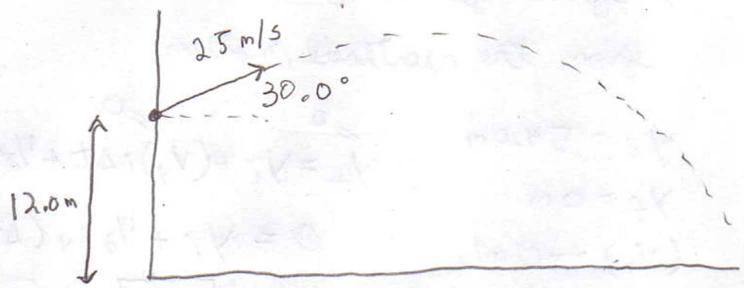
$\Delta t = 0.214\,s$ or $5.17\,s$

$\Delta t = 5.17\,s$

note: $\Delta t = 0.214\,s$ is when $y = 5.50\,m$ on the way up

Problem 11

King Arthur's knights use a catapult to launch a rock from their vantage point on top of the castle wall, 12.0 m above the moat. The rock is launched at a speed of 25 m/s and an angle of 30.0° above the horizontal. How far from the castle wall does the launched rock hit the ground?



$(V_x)_i = (25\,m/s)\cos 30.0°$

$= \underline{21.7\,m/s}$

$(V_y)_i = (25\,m/s)\sin 30.0°$

$= \underline{12.5\,m/s}$

⇒ first get the time in the air from the vertical motion:

$y_i = 12.0\,m$

$y_f = 0\,m$

$(V_y)_i = 12.5\,m/s$

$a_y = -9.80\,m/s^2$

$\Delta t = ?$

$\overset{=0}{y_f} = y_i + (V_y)_i \Delta t + \tfrac{1}{2} a_y (\Delta t)^2$

$\tfrac{1}{2} a_y (\Delta t)^2 + (V_y)_i \Delta t + y_i = 0$

$\underbrace{(-4.90\,m/s^2)(\Delta t)^2 + (12.5\,m/s)\Delta t + 12.0\,m = 0}_{\text{quadratic equation}}$

$a = -4.90\,m/s^2$

$b = 12.5\,m/s$

$c = 12.0\,m$

$\Delta t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Delta t = \dfrac{-12.5\,m/s \pm 19.8\,m/s}{2(-4.90\,m/s^2)}$

$\Delta t = -0.744\,s$ or $\underline{3.29\,s}$

$x_i = 0\,m$

$x_f = ?$

$(V_x)_i = 21.7\,m/s$

$\Delta t = 3.29\,s$

$x_f = \overset{=0}{x_i} + (V_x)_i \Delta t$

$x_f = (V_x)_i \Delta t = (21.7\,m/s)(3.29\,s)$

$x_f = 71.4\,m \longrightarrow \boxed{x_f = 71\,m}$