Answer Key

Problem 1
The earth rotates once per day about an axis passing through the north and south poles, an axis that is perpendicular to the plane of the equator. Assuming the earth is a sphere with a radius of 6.38×10^6 m, determine the speed and centripetal acceleration of a person situated at the equator.

$r = 6.38 \times 10^6 \text{ m}$

$T = 1 \text{ day} = 86{,}400 \text{ s}$

$$v = \frac{2\pi r}{T} = \frac{2\pi(6.38\times10^6 \text{ m})}{86{,}400 \text{ s}} = \boxed{464 \text{ m/s}}$$

$$a = \frac{v^2}{r} = \frac{(464 \text{ m/s})^2}{(6.38\times10^6 \text{ m})} \longrightarrow \boxed{a = 3.37\times10^{-2} \text{ m/s}^2}$$

Problem 2
Each of the space shuttle's main engines is fed liquid hydrogen by a high-pressure pump. Turbine blades inside the pump rotate at 617 rev/s. A point on one of the blades traces out a circle with a radius of 0.020 m as the blade rotates. (a) What is the magnitude of the centripetal acceleration that the blade must sustain at the point? (b) Express this acceleration as a multiple of $g = 9.80 \text{ m/s}^2$.

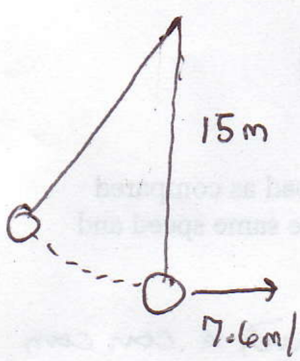
$\Rightarrow$ if the blade rotates at 617 rev/s, the period is given by:

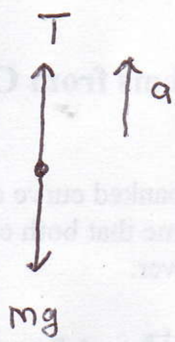
$T = 1/617 \text{ rev/s} = 1.62\times10^{-3} \text{ s}$

$$\left.\begin{array}{l} a = v^2/r \\ v = 2\pi r/T \end{array}\right\} \quad a = \frac{(2\pi r/T)^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi(0.020 \text{ m})^2}{(1.62\times10^{-3} \text{ s})^2}$$

$$a = 3.0\times10^5 \text{ m/s}^2$$

Problem 3
A 2100 kg demolition ball swings at the end of a 15 m cable on the arc of a vertical circle. At the lowest point of the swing, the ball is moving at 7.6 m/s. Determine the tension in the cable.





$\sum F_y = m a_y \qquad a = v^2/r$ upward

$\sum F_y = mv^2/r$

$T - mg = mv^2/r \longrightarrow T = mg + mv^2/r$

$$T = m(g + v^2/r)$$

$$T = (2100 \text{ kg})\left[9.80 \text{ m/s}^2 + \frac{(7.6 \text{ m/s})^2}{15 \text{ m}}\right] \longrightarrow \boxed{T = 2.9\times10^4 \text{ N}}$$

Problem 4
A child is twirling a 0.0120 kg ball on a string in a horizontal circle whose radius is 0.100 m. The ball travels once around the circle in 0.500 s. (a) Determine the centripetal force acting on the ball. (b) If the speed of the ball is doubled, does the centripetal force double? If not, by what factor does the centripetal force increase?

$m = 0.0120\,\text{Kg}$

$r = 0.100\,\text{m}$

$T = 0.500\,\text{s}$

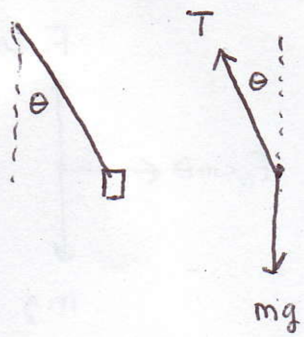
$$\sum F = ma = mv^2/r \qquad v = \frac{2\pi r}{T}$$

$$\sum F = \frac{m(2\pi r/T)^2}{r} = \frac{4\pi^2 r\, m}{T^2}$$

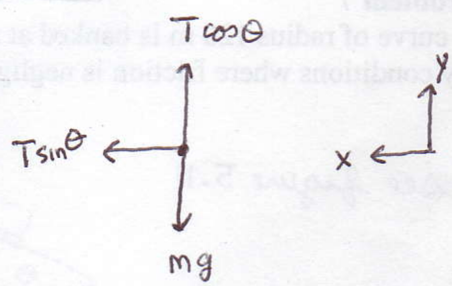
(a) $$\sum F = \frac{4\pi^2(0.100\,\text{m})(0.0120\,\text{Kg})}{(0.500\,\text{m})^2} \longrightarrow \boxed{\sum F = 0.189\,\text{N}}$$

(b) Since $\sum F = mv^2/r$ $(\sum F \propto v^2)$, if v doubles the centripetal force increase by factor of 4

Problem 5
A block is hung by a string from the inside roof of a van. When the van goes straight ahead at a speed of 28 m/s, the block hangs vertically down. But when the van maintains this same speed around an unbaked curve (radius = 150 m), the block swings toward the outside of the curve. Then the string makes an angle θ with the vertical. Find θ.



break into x + y components ⇒



$$\sum F_x = ma_x \qquad a_x = v^2/r$$

$$\sum F_x = mv^2/r$$

$$\sum F_y = ma_y = 0 \qquad T\sin\theta = mv^2/r$$

$$T\cos\theta - mg = 0$$

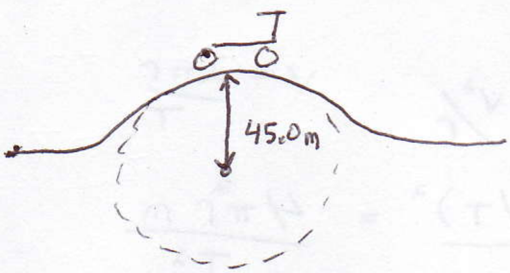
$$T = \frac{mg}{\cos\theta} \longrightarrow \left(\frac{mg}{\cos\theta}\right)\sin\theta = mv^2/r$$

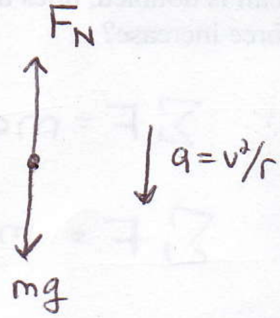
$$mg\tan\theta = mv^2/r \longrightarrow \tan\theta = \frac{v^2}{gr}$$

$$\theta = \tan^{-1}\left(v^2/gr\right) = \tan^{-1}\left[\frac{(28\,\text{m/s})^2}{(9.8\,\text{m/s}^2)(150\,\text{m})}\right]$$

$$\boxed{\theta = 28^\circ}$$

Problem 6
A motorcycle is traveling up one side of a hill and down the other side. The crest is a circular arc with a radius of 45.0 m. Determine the maximum speed that the cycle can have while moving over the crest without losing contact.





define down as +

$$\sum F_y = ma_y \qquad F_N = mg - mv^2/r$$

* when the motorcycle just loses contact with the road, the normal force is 0

$$mg - F_N = mv^2/r \qquad 0 = mg - mv^2/r$$

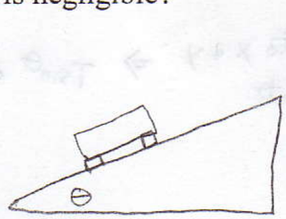
$$v^2 = gr \longrightarrow v = \sqrt{gr} = \sqrt{(9.80\,m/s^2)(45.0\,m)}$$

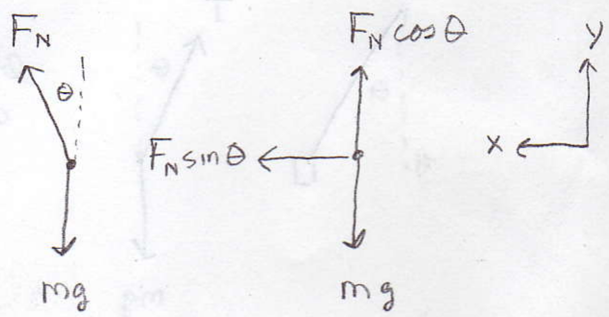
$$\boxed{v = 21.0\,m/s}$$

Problem 7 $\xrightarrow{}$ do

A curve of radius 120 m is banked at an angle of 18°. At what speed can it be negotiated under icy conditions where friction is negligible?

see figure 5.11





$$\sum F_y = ma_y \quad a_y = 0 \qquad \sum F_x = ma_x \quad a_x = v^2/r$$

$$\sum F_y = 0 \qquad \sum F_x = mv^2/r \longrightarrow F_N \sin\theta = \frac{mv^2}{r}$$

$$F_N \cos\theta = mg \qquad \underbrace{\left(\frac{mg}{\cos\theta}\right)}_{= F_N} \sin\theta = mv^2/r$$

$$F_N = \frac{mg}{\cos\theta} \qquad mg\tan\theta = mv^2/r \longrightarrow v^2 = rg\tan\theta$$

$$v = \sqrt{rg\tan\theta} = \sqrt{(120\,m)(9.80\,m/s^2)\tan 18°}$$

$$\boxed{v = 2.0 \times 10^1\,m/s}$$