# Physics 2A
# Chapter 3: Vectors and Motion in Two Dimensions

*"The only thing in life that is achieved without effort is failure."* – Source unknown

*"We are what we repeatedly do. Excellence, therefore, is not an act, but a habit."* – Aristotle

*"Act as if what you do makes a difference, because it does."* – Source unknown

**Reading:** pages 67 – 94; skip sections 3.5 and 3.8

## Outline:

⇒ using vectors (covered on PowerPoint slides)
- vector addition
- multiplication by a scalar
- vector subtraction

⇒ using vectors in motion diagrams (read on your own)

⇒ coordinate systems and vector components
- adding vectors using components

⇒ motion on a ramp

⇒ motion in two dimensions
- projectile motion
- solving problems
- lots of examples

## Problem Solving

Vectors can be given either in terms of a magnitude and direction or in terms of components; answers may be requested in either of these forms. This means you may need to convert from one form to another. If you know the components of a vector, you can easily find the magnitude and direction. For a vector $\vec{a}$ with components $a_x$ and $a_y$, the magnitude is given by

$$a = \sqrt{a_x^2 + a_y^2}$$

and the angle is given by $\theta = \arctan\left(\frac{a_y}{a_x}\right)$

After you have found $\theta$, check to be sure that $\theta$ is in the correct quadrant. If it is not, then add 180° to the value of the angle you get from the calculator.

Projectile motion problems are the same as problems from chapter 2 except that you have two sets of equations, one for the horizontal motion and one for the vertical motion. Remember that for projectile motion, $a_x = 0$ and $a_y = -9.8$ m/s$^2$.

The equations that describe the horizontal motion are:

$x_f = x_i + (v_x)_i \Delta t$
$(v_x)_f = (v_x)_i = \text{constant}$

The equations that describe the vertical motion are:

$y_f = y_i + (v_y)_i \Delta t + \frac{1}{2} a_y(\Delta t)^2$
$(v_y)_f = (v_y)_i + a_y(\Delta t)$
$(v_y)_f^2 = (v_y)_i^2 + 2a_y\Delta y$

If up is defined as positive, then $a_y = -g = -9.8\ m/s^2$. If down is defined as positive, then $a_y = 9.8\ m/s^2$. If the initial speed and launch angle of the projectile are given, then $(v_x)_i = v_i \cos(\theta)$ and $(v_y)_i = v_i \sin(\theta)$, assuming θ is given with respect to the +x axis.

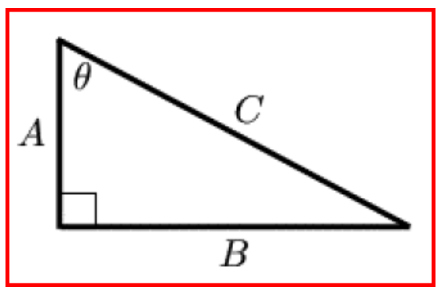
One of the key things to realize when dealing with two dimensional motion is that you can treat each dimension separately. The x part of the motion and the y part of the motion are completely independent of each other; each behaves as if the other did not exist. However, the two motions are connected by the variable Δt.

## Mathematical Skills

Trigonometry is a large portion of the mathematics used in this chapter. Here is a listing of the important elements you should know very well.

**The Pythagorean theorem**

The square of the hypotenuse of a right triangle equals the sum of squares of the other two sides. In the diagram $C^2 = A^2 + B^2$. The theorem is true only if the triangle contains a right angle (90°). The theorem is used, for example, to calculate the magnitude of a vector in the *xy* plane given its *x* and *y* components.




**Trigonometric functions.**

For the triangle shown, $A = C\cos\theta$ and $B = C\sin\theta$. The relations follow directly from the definition of the sine and cosine and are used to find the components of a vector, given the magnitude and the angle it makes with an axis. Also know that for the triangle above $\tan\theta = B/A$. This relationship, in the form $\theta = \arctan(a_y/a_x)$, is used to find the angle a vector makes with a coordinate axis.

WARNING! For any values of $a_x$ and $a_y$ the equation $\theta = \arctan(a_y/a_x)$ has two solutions for $\theta$. If you use a calculator to evaluate $\theta$, it will only give you an angle in the 1st or 4th quadrant. If you know that the angle must be in the 2nd or 3rd quadrant (because of the signs of the x- and y-coordinates), then you must add 180° to the angle given by the calculator.

# SUMMARY

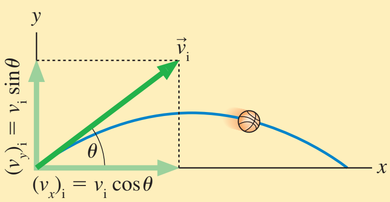
**The goals of Chapter 3 have been to learn more about vectors and to use vectors as a tool to analyze motion in two dimensions.**

## GENERAL PRINCIPLES

### Projectile Motion

A projectile is an object that moves through the air under the influence of gravity and nothing else.

The path of the motion is a parabola.




The motion consists of two pieces:

1. Vertical motion with free-fall acceleration, $a_y = -g$.
2. Horizontal motion with constant velocity.

Kinematic equations:

$$x_f = x_i + (v_x)_i\,\Delta t$$

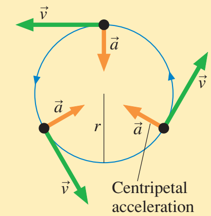
$$(v_x)_f = (v_x)_i = \text{constant}$$

$$y_f = y_i + (v_y)_i\,\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

$$(v_y)_f = (v_y)_i - g\,\Delta t$$

### Circular Motion

For an object moving in a circle at a constant speed:



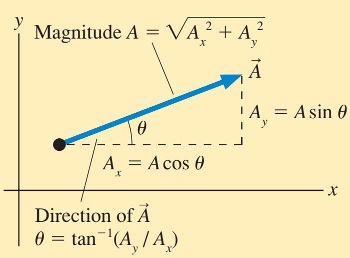

- The period $T$ is the time for one rotation.
- The frequency $f = 1/T$ is the number of revolutions per second.
- The velocity is tangent to the circular path.
- The acceleration points toward the center of the circle and has magnitude

$$a = \frac{v^2}{r}$$

## IMPORTANT CONCEPTS

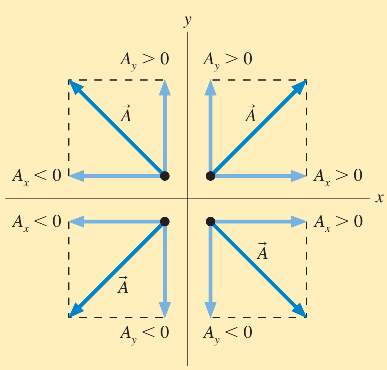
### Vectors and Components

A vector can be decomposed into $x$- and $y$-**components.**




The magnitude and direction of a vector can be expressed in terms of its components.

The sign of the components depends on the direction of the vector:

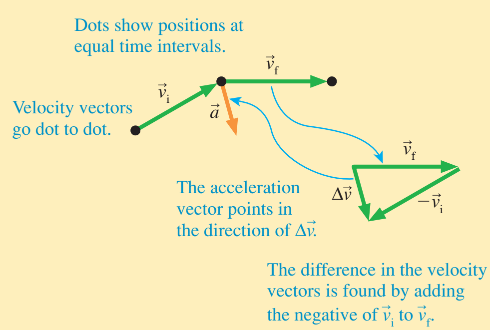



### The Acceleration Vector

We define the acceleration vector as

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\Delta\vec{v}}{\Delta t}$$

We find the acceleration vector on a motion diagram as follows:

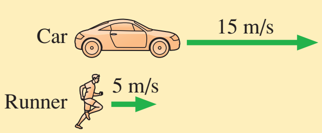



## APPLICATIONS

### Relative motion

Velocities can be expressed relative to an observer. We can add relative velocities to convert to another observer's point of view.

c = car, r = runner, g = ground




The speed of the car with respect to the runner is:

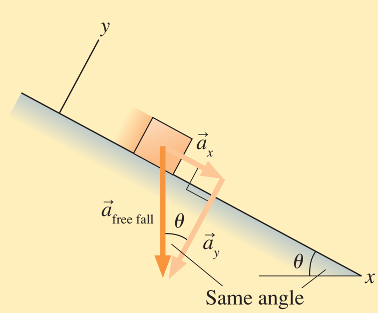
$$(v_x)_{cr} = (v_x)_{cg} + (v_x)_{gr}$$

### Motion on a ramp

An object sliding down a ramp will accelerate parallel to the ramp:

$$a_x = \pm g\sin\theta$$

The correct sign depends on the direction in which the ramp is tilted.



# Questions and Example Problems from Chapter 3

**Question 1**

(a) Can a vector have nonzero magnitude if a component is zero? If no, why not? If yes, give an example. (b) Can a vector have zero magnitude and a nonzero component? If no, why not? If yes, give an example.

**Question 2**

Vector **A** points along the +y axis and has a magnitude of 100.0 units. Vector **B** points at an angle of 60.0$^{o}$ above the +x axis and has a magnitude of 200.0 units. Vector **C** points along the +x axis and has a magnitude of 150.0 units. Which vector has (a) the largest x component and (b) the largest y component?

**Question 3**

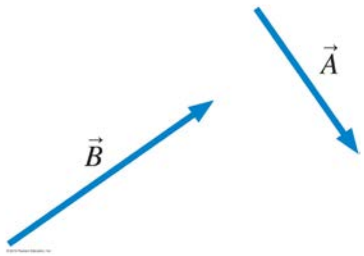
For a projectile, which of the following quantities are constant during the flight: x, y, $v_x$, $v_y$, v, $a_x$, $a_y$? Which or the quantities are zero throughout the flight?

**Question 4**

A tennis ball is hit upward into the air and moves along an arc. Neglecting air resistance, where along the arc is the speed of the ball (a) a minimum and (b) a maximum? Justify your answers.

**Problem 1**

Trace the vectors in the figure below onto your paper. Then use graphical methods to draw the vectors (a) $\vec{A}+\vec{B}$ and (b) $\vec{A}-\vec{B}$.



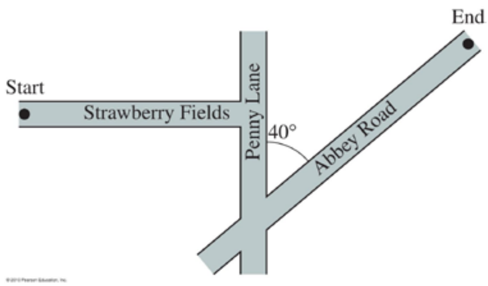

**Problem 2**

Draw each of the following vectors, then find its x- and y-components.

a) $\vec{d}$ = (2.0 km, 30° left of +y-axis)
b) $\vec{v}$ = (5.0 cm/s, - x-direction)
c) $\vec{a}$ = (10 m/s$^2$, 40° left of -y-axis)

**Problem 3**

While visiting England, you decide to take a jog and find yourself in the neighborhood shown on the map in the figure below. What is your displacement after running 2.0 km on Strawberry Fields, 1.0 km on Penny Lane, and 4.0 km on Abbey Road?




**Problem 4**

A golfer, putting on a green, requires three strokes to "hole the ball". During the first put, the ball rolls 5.0 m due east. For the second out, the ball travels 2.1 m at an angle of 20.0º north of east. The third put is 0.50 m due north. What displacement (magnitude and direction relative to east) would have been needed to "hole the ball" on the very first put?

**Problem 5**

A piano has been pushed to the top of the ramp at the back of a moving van. The workers think it is safe, but as they walk away, it begins to roll down the ramp. If the back of the truck is 1.0 m above the ground and the ramp is inclined at 20°, how much time do the workers have to get to the piano before it reaches the bottom of the ramp?

**Problem 6**

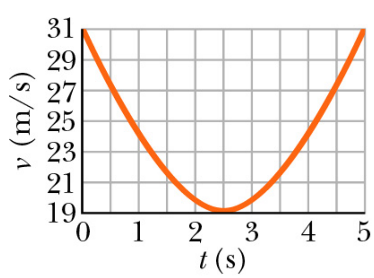
A diver runs horizontally with a speed of 1.20 m/s off a platform that is 10.0 m above the water. What is the speed just before striking the water?

**Problem 7**

The punter on a football team tries to kick a football so that it stays in the air for a long “hang time”. (a) If the ball is kicked with an initial velocity of 25.0 m/s at an angle of 60.0° above the ground, what is the “hang time”? (b) How far does the ball travel before it hits the ground?

**Problem 8**

A golf ball is struck at ground level. The speed of the golf ball as a function of the time is shown in the figure below, where t = 0 at the instant the ball is struck. (a) How far does the golf ball travel horizontally before returning to ground level? (b) What is the maximum height above ground level attained by the ball?



**Problem 9**

A car drives straight off the edge of a cliff that is 54.0 m high. The police at the scene of the accident note that the point of impact is 130.0 m from the base of the cliff. How fast was the car traveling when it went over the cliff?

**Problem 10**

A golfer, standing on a fairway, hits a shot to a green that is elevated 5.50 m above the point where she is standing. If the ball leaves her club with a velocity of 46.0 m/s at an angle of $35.0^{o}$ above the ground, find the time the ball is in the air before it hits the green.

**Problem 11**

King Arthur's knights use a catapult to launch a rock from their vantage point on top of the castle wall, l2.0 m above the moat. The rock is launched at a speed of 25 m/s and an angle of 30.0° above the horizontal. How far from the castle wall does the launched rock hit the ground?