Chapter 18: Ray Optics
Questions & Problems

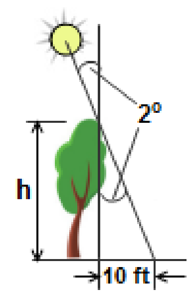
$\theta_r = \theta_i$ $\quad n_1 \sin\theta_1 = n_2 \sin\theta_2$ $\quad n = \frac{c}{v}$ $\quad \theta_c = \sin^{-1}(\frac{n_2}{n_1})$ $\quad \frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ $\quad m = \frac{h'}{h} = -\frac{s'}{s}$

Example 18.1
At high noon, the sun is almost directly above (about 2.0º from the vertical) and a tall redwood tree casts a shadow that 10.0 ft long. How tall is the redwood tree?

Solution
This problem employs the fact that rays travel in straight lines. At high noon, the sun shines from its center, 2º from the vertical axis, so that a triangle can be made from the picture on the right. Apply basic trig, the tangent of 2º gives

$$\tan 2.0° = \frac{10\text{ ft}}{h} \longrightarrow h = \frac{10\text{ ft}}{\tan 2.0°} = \boxed{290\text{ ft} = h}$$

where we have rounded to two significant figures. When the sun is less than 45º from the vertical, we expect the shadow to be shorter than the height of the object and longer than the height if the sun is more than 45º.

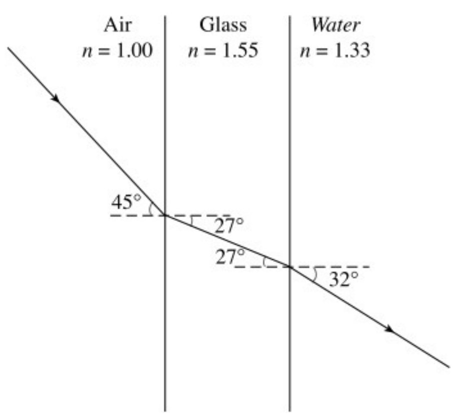


Example 18.2
You shine your laser pointer through the flat glass side of a rectangular aquarium at an angle of incidence of 45º. The index of refraction of this type of glass is 1.55.

a) At what angle from the normal does the beam from the laser pointer enter the water inside the aquarium?
b. Does your answer to part a depend on the index of refraction of the glass?

P18.45. **Prepare:** We will apply Snell's law at each interface and then come to an interesting conclusion (that we might expect).

$n_1 \sin\theta_1 = n_2 \sin\theta_2$



Solve: Use subscripts a, g, and w for air, glass, and water, respectively. Remember, all angles are measured from the normal.

(a)

$$\theta_g = \sin^{-1}\left(\frac{n_a \sin\theta_a}{n_g}\right) = \sin^{-1}\left(\frac{1.00\sin 45°}{1.55}\right) = 27°$$

$$\theta_w = \sin^{-1}\left(\frac{n_g \sin\theta_g}{n_w}\right) = \sin^{-1}\left(\frac{1.55\sin 27°}{1.33}\right) = 32°$$

So the laser enters the water at an angle of $32°$ from the normal.

(b) No, the result doesn't depend on the index of refraction of the pane of glass. Try the calculation directly from air to water to compare.

$$\theta_w = \sin^{-1}\left(\frac{n_a \sin\theta_a}{n_w}\right) = \sin^{-1}\left(\frac{1.00\sin 45°}{1.33}\right) = 32°$$

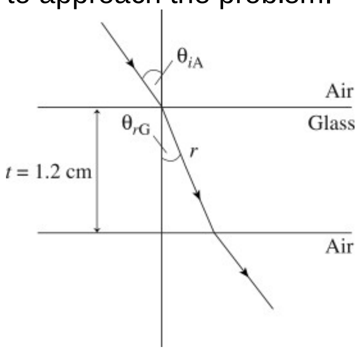
Another way to see this is that since $n_a \sin\theta_a = n_g \sin\theta_g$ and $n_g \sin\theta_g = n_w \sin\theta_w$, then it must be true (if two things are each equal to a third thing then they must be equal to each other) that $n_a \sin\theta_a = n_w \sin\theta_w$.

Assess: A flat pane of glass does nothing to change the angle of the light entering the water, but it *does* slightly displace the ray laterally. Just as the index of refraction of the glass doesn't matter, neither does the thickness of the glass, but the amount the ray is displaced is affected by the thickness.

Example 18.3

A ray of light traveling through air encounters a 1.2-cm-thick sheet of glass at a 35° angle of incidence. How far does the light ray travel in the glass before emerging on the far side?

P18.46. Prepare: Knowing Snell's law $(n_1 \sin\theta_1 = n_2 \sin\theta_2)$ and a little trigonometry we can solve this problem. The following figure shows the details and helps us decide how to approach the problem.



Solve: Knowing the index of refraction of air and glass and the angle of incidence in air, we can use Snell's law to determine the angle of refraction in glass.

From Snell's law: $n_A \sin\theta_{iA} = n_G \sin\theta_{rG}$

or

$$\theta_{rG} = \sin^{-1}[(n_A/n_G)\sin\theta_{iA}] = \sin^{-1}[(1.0/1.5)\sin 35°] = 22.5°$$

Knowing the angle of refraction in glass and the thickness of the glass, it is a matter of straightforward trigonometry to obtain the distance the light travels in glass.

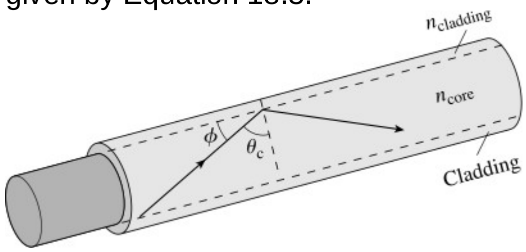
$$\cos\theta_{rG} = t/r \quad \text{or} \quad r = t/\cos\theta_{rG} = 1.2\text{ cm}/\cos 22.5° = 1.3\text{ cm}.$$

Assess: Looking at the figure, we know that the distance should be a little more than the thickness of the glass and it is.

Example 18.4

The glass core of an optical fiber has index of refraction 1.60. The index of refraction of the cladding is 1.48. What is the maximum angle between a light ray and the wall of the core if the ray is to remain inside the core?

P18.56. Prepare: Use the ray model of light. We want the ray of light to remain in the core and not refract into the cladding. For an angle of incidence greater than the critical angle, the ray of light undergoes total internal reflection. The critical angle of incidence is given by Equation 18.3.



Solve:

$$\theta_c = \sin^{-1}\left(\frac{n_{cladding}}{n_{core}}\right) = \sin^{-1}\left(\frac{1.48}{1.60}\right) = 67.7°$$

Thus, the maximum angle a light ray can make with the wall of the core to remain inside the fiber is 90° – 67.7° = 22.3°.

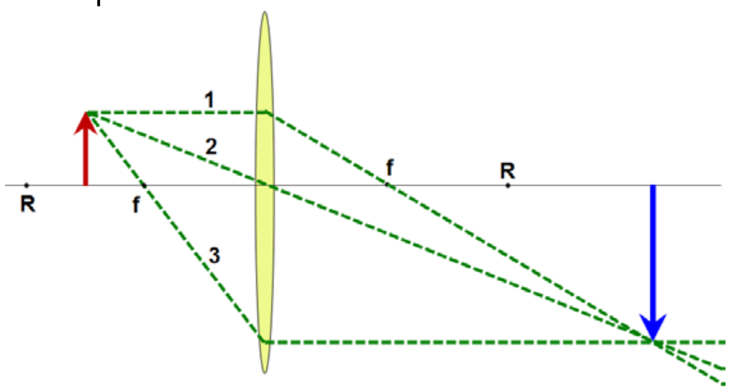
Assess: We can have total internal reflection because $n_{core} > n_{cladding}$.

Example 18.5

An object is 15 cm in front of a converging lens with a focal length of 10 cm. Use ray tracing to determine the location of the image. What are the image characteristics: (i) real or virtual?, (ii) upright or inverted?, (iii) smaller, larger or same?, and (iv) the image location?

Solution

Use ray tracing to locate the image. The following figure shows the ray-tracing diagram using the steps of Tactics Box 18.2.

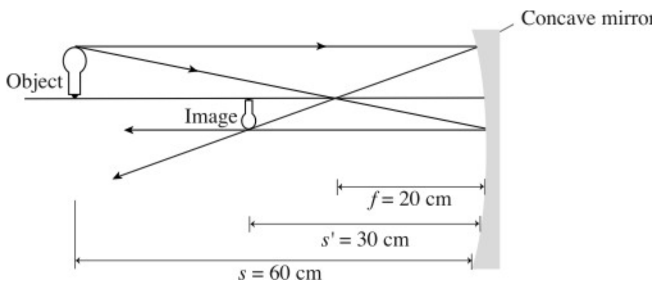


You can see from the diagram that the image is in the plane where the three special rays converge. The image is located at $s' = 30$ cm to the right of the converging lens, magnification is more than 1, and is inverted and real. Ray tracing must be done to scale to obtain useful answers.

Example 18.6

A light bulb is 60 cm from a concave mirror with a focal length of 40 cm. A 5-cm-long mascara brush is held upright 20 cm from the mirror. Use ray tracing to determine the location of the image. What are the image characteristics: (i) real or virtual?, (ii) upright or inverted?, (iii) smaller, larger or same?, and (iv) the image location?

P18.24. Prepare: Two rays are needed to determine the location of the image. A ray incident parallel to the optic axis will be reflected back through the focal point and a ray incident through the focal point will be reflected back parallel to the optic axis. Real images are formed when the reflected light actually passes through the site of the image.



Solve: The figure shows that the image is at a distance of 30 cm from the mirror. Since it has an orientation opposite that of the object, it is inverted. The image is real since the reflected rays actually pass through the image (we could see the image on a screen placed at this location). Finally, since the image is smaller than the object, the magnification is less than one.

Assess: While ray tracing is relatively simple, it can yield a lot of information.

Example 18.7

A 2.0-cm tall object is 15 cm in front of a converging lens that has a 20 cm focal length. Where is the image located and what is the height of the image? What are the characteristics of the image (real or virtual, upright or inverted, enlarged or reduced)?

P18.31. Prepare: Assume that the converging lens is a thin lens and Equation 18.11 is applicable.

Solve: Using the thin-lens formula,

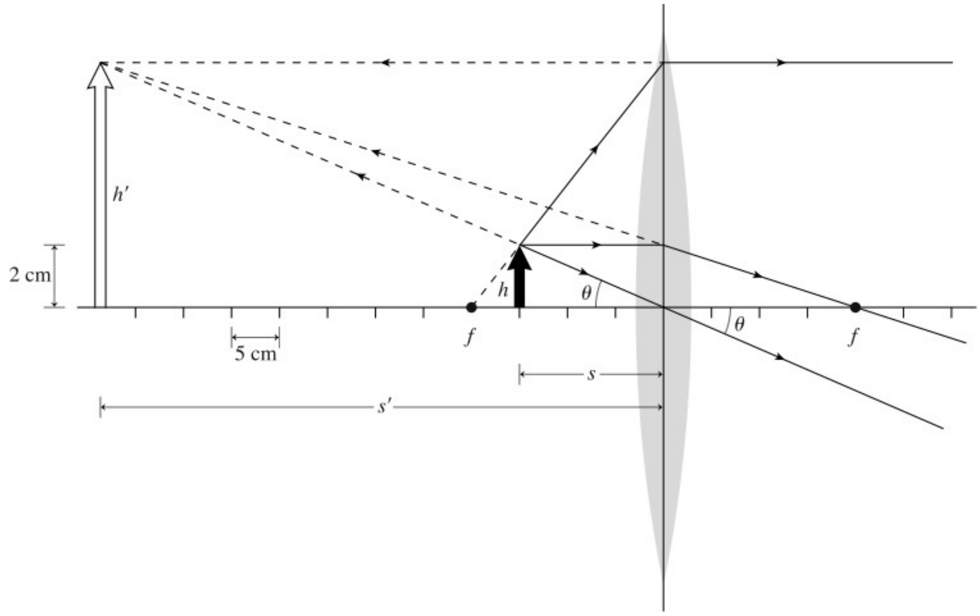
$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{15\text{ cm}}+\frac{1}{s'}=\frac{1}{20\text{ cm}} \Rightarrow \frac{1}{s'}=-\frac{1}{60\text{ cm}} \Rightarrow s'=-60\text{ cm}$$

The image height is obtained from

$$M=-\frac{s'}{s}=-\frac{-60\text{ cm}}{15\text{ cm}}=+4$$

Thus, the image is four times larger than the object or $h'=Mh=4h=4(2.0\text{ cm})=8.0\text{ cm}.$ The image is upright.

We also show later the ray-tracing diagram. The three special rays after refracting do not converge. Instead the rays appear to come from a point that is 60 cm on the same side of the lens as the object, so $s' = -60$ cm. The image is upright and has a height of 8.0 cm.



Assess: The ray-tracing diagram confirms that the position and height obtained above are correct.

Example 18.8

A 1.0-cm tall object is 60 cm in front of a diverging lens that has a -30 cm focal length. Where is the image located and what is the height of the image? What are the characteristics of the image (real or virtual, upright or inverted, enlarged or reduced)?

P18.34. Prepare: Assume that the diverging lens is a thin lens and Equation 18.11 applies.

Solve: Using the thin-lens formula, $\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{-30\text{ cm}} - \frac{1}{60\text{ cm}} = -\frac{1}{20\text{ cm}} \Rightarrow s' = -20\text{ cm}$

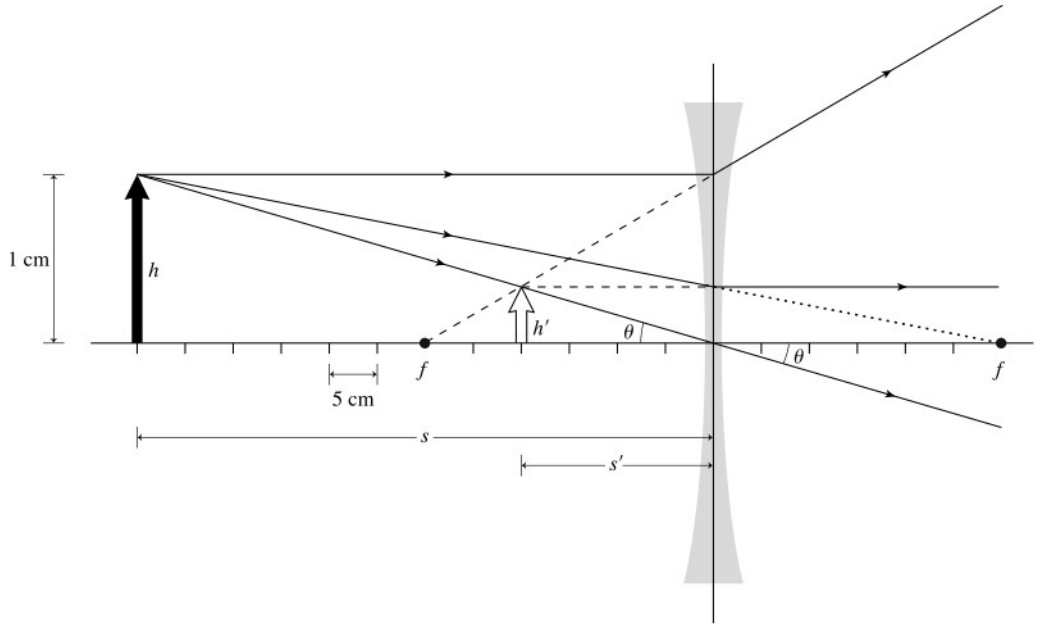
Assess: The ray-tracing diagram confirms that both the position and height obtained previously are correct.

The image height is obtained from

$$M = -\frac{s'}{s} = -\frac{-20\text{ cm}}{60\text{ cm}} = \frac{1}{3} = 0.33$$

Thus, $h' = Mh = (0.33)(1.0\text{ cm}) = 0.33\text{ cm},$ and the image is upright because *M* is positive.

We also show the ray-tracing diagram in the following figure. After refraction from the diverging lens, the three special rays do not converge. However, the rays appear to meet at a point that is 20 cm on the same side as the object. So $s' = -20\text{ cm}.$ The image is upright and has a height of 0.33 cm.



Example 18.9

A 3.0-cm tall object is 15 cm in front of a convex mirror that has a -25 cm focal length. Where is the image located and what is the height of the image? What are the characteristics of the image (real or virtual, upright or inverted, enlarged or reduced)?

P18.35. Prepare: We'll first use the thin-lens equation to find the image position and then we'll use $h'/h = -s'/h.$

We are given $s = 15\text{ cm},\ f = -25\text{ cm},$ and $h = 3.0\text{ cm}.$

Solve:

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{-25\text{ cm}} - \frac{1}{15\text{ cm}} = -0.107\text{ cm}^{-1}$$

So $s' = 1/-0.107\text{ cm}^{-1} = -9.4\text{ cm}.$ So the image is 9.4 cm behind the mirror.

$$h' = -h\frac{s'}{s} = -3.0\text{ cm}\left(\frac{-9.4\text{ cm}}{15\text{ cm}}\right) = 1.9\text{ cm}$$

Assess: A quick ray-tracing diagram confirms that both the position and height given are probably correct.

Example 18.10

A 3.0-cm tall object is 45 cm in front of a concave mirror that has a 25 cm focal length. Where is the image located and what is the height of the image? What are the characteristics of the image (real or virtual, upright or inverted, enlarged or reduced)?

P18.38. Prepare: The object distance, image distance, and focal length are related by $1/s + 1/s' = 1/f$. The magnification is related to the object and image distance and height by $m = h'/h = -s'/s$.

Solve: The image position is obtained by

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{25\text{ cm}} - \frac{1}{45\text{ cm}} = \frac{+20}{1125\text{ cm}}$$

or $s' = 56\text{ cm}$.

The image height is obtained by

$$h' = -h(s'/s) = -(3.0\text{ cm})(56\text{ cm}/45\text{ cm}) = -3.7\text{ cm}$$

Assess: First note that the object is between the focal length and the radius of curvature of the concave mirror. This tells us that the image will be inverted (opposite orientation as the object), hence a negative image height; real (positive sign on the image distance); formed beyond the radius of curvature of the mirror; and have a magnification with a magnitude greater than one (the image appears to be taller than the object). The given information regarding images formed by a concave mirror is consistent with the calculated values.