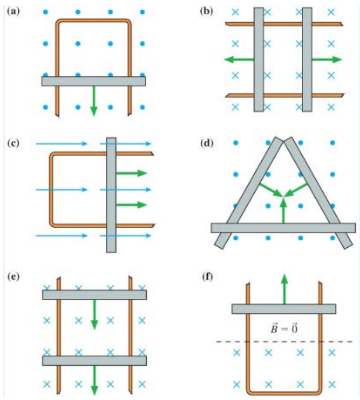
# Chapter 25: Magnetic Induction & Lenz's Law
## Questions & Problems

$$\Phi = A_{eff}B = AB\cos\theta \qquad \mathcal{E}_{induced} = N\left|\frac{\Delta\Phi}{\Delta t}\right| \qquad c=\lambda f \qquad E_{photon}=hf$$

### Example 25.1
Parts a through f of figure show one or more metal wires sliding on fixed metal rails in a magnetic field. For each, determine if the induced current is clockwise, counterclockwise, or is zero.




### Solution
In each case we apply Lenz's law: "The direction of the induced current is such that the induced magnetic field opposes the *change* in the flux."

**(a)** The magnetic field is out of the page and the flux through the rectangular loop is increasing because the loop is growing. A current will be induced that will create a magnetic field to oppose that change. The induced magnetic field must be into the page; this requires (by the right-hand rule) that the induced current be clockwise.

**(b)** The magnetic field is into the page and the flux through the rectangular loop is increasing because the loop is growing. A current will be induced that will create a magnetic field to oppose that change. The induced magnetic field must be out of the page; this requires (by the right-hand rule) that the induced current be counterclockwise.

**(c)** The magnetic field is parallel to the plane of the page and so the flux through the rectangular loop is zero and not changing. Since the flux through the loop isn't changing then the induced current is zero.

**(d)** The magnetic field is out of the page and the flux through the triangular loop is decreasing because the loop is shrinking. A current will be induced that will create a magnetic field to oppose that change. The induced magnetic field must be out of the page; this requires (by the right-hand rule) that the induced current be counterclockwise.

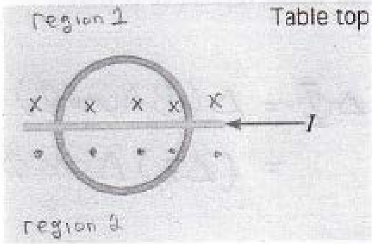
**(e)** The magnetic field is into the page but the flux through the rectangular loop is constant because the loop is not growing or shrinking as the two rails move at the same velocity. Since the flux through the loop isn't changing then the induced current is zero.

**(f)** The magnetic field is into the page but the flux through the rectangular loop is constant because the moving wire is outside the region of the magnetic field; as the loop grows it doesn't enclose any more flux. Since the flux through the loop isn't changing then the induced current is zero.

**Assess:** Flux though a loop of wire can change for various reasons, either because the area of the loop changes, or the strength of the field changes, or the orientation of the loop changes. Only if the flux through the loop changes is a current induced.

## Example 25.2

A circular loop of wire rests on a table. A long, straight wire lies on this loop, directly over its center, as the drawing illustrates. The current I in the straight wire in increasing. In what direction is the induced current, if any, in the loop? Give your reasoning.




in region I → $\vec{B}$ is into page

region II → $\vec{B}$ is out of page

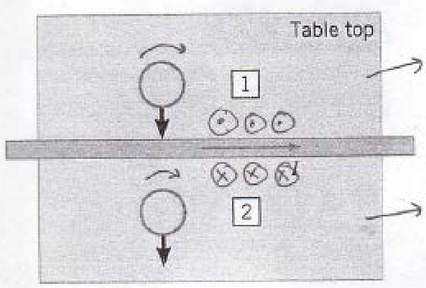
$$\Phi_{\text{region }1} = -\Phi_{\text{region }2}$$

total flux $\Phi = 0$

⇒ as I increases, the total flux is still zero so the change in flux $\Delta\Phi = 0$ → the induced emf is therefore zero so there is no induced current

## Example 25.3

A long, straight wire lies on a table and carries a current I. As shown in the drawing below, a small circular loop of wire is pushed across the top of the table from position 1 to position 2. Determine the direction of the induced current, clockwise or counterclockwise, as the loop moves past (a) position 1 and (b) position 2. Justify your answers.




→ $\vec{B}$ out of page

for a long, straight wire $B = \frac{\mu_0 I}{2\pi r}$ so

$B \propto 1/r$

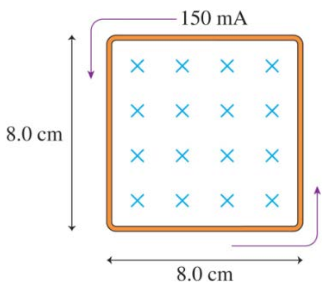
→ $\vec{B}$ into page

1 → $\vec{B}$ points out of page and is stronger closer to the wire
$\Phi$ therefore increases; induced current must therefore create a $\vec{B}$ field that points into page → induced current is $\boxed{\text{clockwise}}$

2 → $\vec{B}$ points into page and is weaker farther from the wire
$\Phi$ therefore decreases; induced current must therefore create a $\vec{B}$ field that points into page → induced current is $\boxed{\text{clockwise}}$

**Example 25.4**

The loop in figure has an induced current as shown. The loop has a resistance of 0.10Ω. Is the magnetic field strength increasing or decreasing? What is the rate of change of the field (ΔB/Δt)?




**Solution**

Please refer to Figure P25.16. Assume the field is uniform across the loop. There is a current in the loop so there must be an emf that is due to a changing flux. With the loop fixed, the area is constant so the change in flux must be due to a changing field strength.

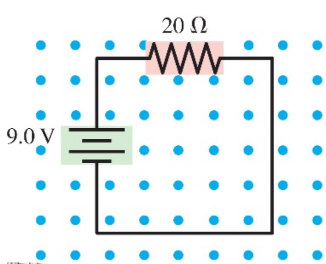
**Solve:** The induced emf is $\mathcal{E}$ = |*Δ*F/*Δt*| and the induced current is $I = \mathcal{E}/R.$ The *B* field is changing, but the area *A* is not. Take $\vec{A}$ as being into the page and parallel to $\vec{B},$ so F = *AB* and *Δ*F/*Δt* = *A*(*B*/*Δt*). We have

$$\mathcal{E} = \left|\frac{\Delta\Phi}{\Delta t}\right| = A\left|\frac{\Delta B}{\Delta t}\right| \Rightarrow \left|\frac{\Delta B}{\Delta t}\right| = \frac{IR}{A} = \frac{(150\times10^{-3}\text{ A})(0.10\ \Omega)}{(0.080\text{ m})^2} = 2.3\text{ T/s}$$

The original field and flux is into the page. The induced counterclockwise current produces an induced field and flux that is out of the page. Since the induced field opposes the change, the field must be *increasing*.

**Example 25.5**

The circuit in the figure below is a square 5.0 cm on a side. The magnetic field increases steadily from 0 T to 0.50 T in 10 ms. What is the current in the resistor during this time?




**Prepare:** The induced emf is determined by $\mathcal{E}_{\text{induced}} = \Delta\Phi/\Delta t = A(\Delta B/\Delta t).$ The induced emf is in the same direction of the applied voltage (the 9.0 volts from the battery, so the net emf is $\mathcal{E}_{\text{net}} = \mathcal{E}_{\text{applied}} + \mathcal{E}_{\text{induced}}.$ The net emf and current are related by $\mathcal{E}_{\text{net}} = IR.$ Combining these expressions and solving for the current we obtain: $I = [\mathcal{E}_{\text{applied}} + A(\Delta B/\Delta t)]/R.$

**Solve:** The current in the circuit and hence the resistor is

$$I = [\mathcal{E}_{\text{applied}} - A(\Delta B/\Delta t)]/R = [9.0\text{ V} + (25\times10^{-4}\text{ m}^2)(0.50\text{ T}/10\times10^{-3}\text{ s})]/(20\ \Omega) = 0.46\text{ A}$$

**Assess:** The current in the circuit is increased slightly due to the fact that the induced emf is in the same direction as the applied voltage.

## Example 25.6

A 300-turn rectangular loop of wire has an area per turn of $5.0\times10^{-3}\,\mathrm{m}^2$. At t = 0 s, a magnetic field is turned on, and its magnitude increases to 0.40 T when t = 0.80 s. The field is directed at an angle of $\phi$ = 30.0° with respect to the normal of the loop. (a) Find the magnitude of the average emf induced in the loop. (b) If the loop is a closed circuit whose resistance is $6.0\,\Omega$, determine the average induced current.

$N = 300$

$A = 5.0\times10^{-3}\,\mathrm{m}^2$

$\Delta B = 0.40\,\mathrm{T}$

$\Delta t = 0.80\,\mathrm{s}$

$\phi = 30.0^\circ$

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} \qquad \Delta\Phi = \Delta(BA\cos\phi) = (\Delta B)A\cos\phi$$

$$\mathcal{E} = -\frac{N(\Delta B)A\cos\phi}{\Delta t}$$

$$\mathcal{E} = -\frac{(300)(0.40\,\mathrm{T})(5.0\times10^{-3}\,\mathrm{m}^2)\cos 30.0^\circ}{0.80\,\mathrm{s}} = -0.65\,\mathrm{V}$$

$$\boxed{|\mathcal{E}| = 0.65\,\mathrm{V}}$$

(b) $I = \mathcal{E}/R = \dfrac{0.65\,\mathrm{V}}{6.0\,\Omega} \rightarrow \boxed{I = 0.11\,\mathrm{A}}$

## Example 25.7

Gamma rays with the very high energy of $2.0\times10^{13}$ eV are occasionally observed from distant astrophysical sources. What are the wavelength and frequency corresponding to this photon energy?

**Prepare:** We will use $E_{\text{photon}} = hf$ to find the frequency, then $c = \lambda f$ to arrive at the wavelength.

**Solve:**

$$f = \frac{E}{h} = \frac{2.0\times10^{13}\ \mathrm{eV}}{6.63\times10^{-34}\ \mathrm{J\cdot s}}\left(\frac{1.6\times10^{-19}\ \mathrm{J}}{1\,\mathrm{eV}}\right) = 4.8\times10^{27}\ \mathrm{Hz}$$

$$\lambda = \frac{c}{f} = \frac{3.0\times10^{8}\ \mathrm{m/s}}{4.8\times10^{27}\ \mathrm{Hz}} = 6.2\times10^{-20}\ \mathrm{m}$$

**Assess:** These are extremely high-energy photons, with correspondingly large frequencies and small wavelengths. This wavelength is much smaller than the diameter of a proton.

## Example 25.8

What is the energy of 1 mol of photons that have a wavelength of 1.0 μm?

**Prepare:** We will use Equation 25.21 and the relationship $f = c/\lambda$.

**Solve:** The energy of the single photon is

$$E_{\text{photon}} = hf = h\left(\frac{c}{\lambda}\right) = \frac{(6.63\times10^{-34}\ \mathrm{J\cdot s})(3.0\times10^{8}\ \mathrm{m/s})}{1.0\times10^{-6}\ \mathrm{m}} = 1.99\times10^{-19}\ \mathrm{J}$$

$$\Rightarrow E_{\text{mol}} = N_{\text{A}}E_{\text{photon}} = (6.023\times10^{23})(1.99\times10^{-19}\ \mathrm{J}) = 1.2\times10^{5}\ \mathrm{J}$$

**Assess:** Although the energy of a single photon is very small, a mole of photons has a significant amount of energy.