Chapters 28 and 29: Quantum Physics and Atoms
Questions & Problems

$$E_{photon} = hf = \frac{hc}{\lambda} \qquad K_{max} = E_{elec} - E_0 = hf - E_0 = eV_{stop} \qquad P = E/t \qquad E_{total} = NE_{photon}$$

$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad E_n = n^2 \frac{h^2}{8\,mL^2},\ n = 1,\ 2,\ 3,... \qquad \Delta E_{system} = hf_{photon}$$

$$\lambda = \frac{91.1\text{nm}}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)} \qquad f_{photon} = \frac{\Delta E_{atom}}{h} \qquad E_n = -\frac{E_1}{n^2} = \frac{-13.60\,\text{eV}}{n^2}, n = 1, 2, 3,...$$

$$h = 6.63 \times 10^{-34}\,\text{J} \cdot \text{s} = 4.14 \times 10^{-15}\,\text{eV} \cdot \text{s} \qquad c = 3 \times 10^8\,\text{m/s} \qquad hc = 1242 \times 10^{-9}\,\text{eV} \cdot \text{nm}$$

$$e = 1.60 \times 10^{-19}\text{C} \qquad m_e = 9.11 \times 10^{-31}\text{kg}$$

Example 28.1
One recent study has shown that x-rays with a wavelength of 0.0050 nm can produce mutations in human cells. (a) Calculate the energy in eV of a photon of radiation with this wavelength. (b) Assuming that the bond energy holding together a water molecule is typical, use Table 25.1 to estimate how many molecular bonds could be broken with this energy.

TABLE 25.1 Energies of some atomic and molecular processes

Process	Energy
Breaking a hydrogen bond between two water molecules	0.24 eV
Energy released in metabolizing one molecule of ATP	0.32 eV
Breaking the bond between atoms in a water molecule	4.7 eV
Ionizing a hydrogen atom	13.6 eV

Prepare: The energy of a photon is given by Equation 25.21: $E_{\text{photon}} = hf$.
We first compute f from Equation 25.16: $c = \lambda f$.

Solve: (a)

$$f = \frac{c}{\lambda} = \frac{3.0 \times 10^8\ \text{m/s}}{0.0050 \times 10^{-9}\ \text{m}} = 6.0 \times 10^{19}\ \text{Hz}$$

$$E_{\text{photon}} = hf = (6.63 \times 10^{-34}\ \text{J} \cdot \text{s})(6.0 \times 10^{19}\ \text{Hz}) = 4.0 \times 10^{-14}\ \text{J}$$

We want to know the answer in eV as well.

$$4.0 \times 10^{-14}\ \text{J}\left(\frac{1\,\text{eV}}{1.60 \times 10^{-19}\ \text{J}}\right) = 2.5 \times 10^5\ \text{eV} = 250\ \text{keV}$$

(b) The energy just found divided by the energy required to break a hydrogen bond between two water molecules gives the number of bonds it can break: $250\ \text{keV}/(0.24\ \text{eV}) = 1{,}000{,}000$ bonds.

Assess: This is actually a very large energy for one photon, compared to visible light photons (which have energies of just a few eV), or radio wave photons (which have even less energy than visible photons).

Example 28.2

a. Your eyes have three different types of cones with maximum absorption at 437 nm, 533 nm, and 564 nm. What photon energies correspond to these wavelengths?
b. Station KAIM in Hawaii broadcasts on the AM dial at 870 kHz, with a maximum power of 50,000 W. At maximum power, how many photons does the transmitting antenna emit each second?

Prepare: The energy and wavelength of a photon are related by $E = hc/\lambda$. Before starting to solve the problem we might want to note that the arithmetic will be easier if we first calculate the quantity *hc* ($hc = 12.42 \times 10^{-7}$ eV·m).
Solve: Photon energies corresponding to the wavelengths given are

$$E = hc/\lambda = 12.42\times10^{-7}\,\text{eV}\cdot\text{m}/(4.37\times10^{-7}\,\text{m}) = 2.84\,\text{eV}$$

In like manner for λ =533 nm obtain E = 2.33 eV, and for λ = 564 nm obtain E = 2.20 eV.
Assess: These are reasonable photon energies. Note that as the wavelength increases the energy decreases. This is consistent with the fact that the photon energy is inversely proportional to the wavelength.

Prepare: Follow Example 28.5 and use Equation 28.3. We are given $f = 870\,\text{kHz}$ and $P = 50{,}000\,\text{W}$.
Solve:

$$R = \frac{P}{hf} = \frac{50{,}000\,\text{W}}{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(870\times10^{3}\,\text{Hz})} = 8.7\times10^{31}\,\text{photons per second}$$

Assess: This result is much larger than the answer in Example 28.5 because the power of the radio station is much larger than the power output of a laser pointer and because the frequency of the radio photons is so much less than the frequency of the visible red laser photons.

Example 28.3

Electrons in a photoelectric effect experiment emerge from a copper surface with a maximum kinetic energy of 1.10 eV. What is the wavelength of the light?

Prepare: Equation 28.6 connects the maximum kinetic energy of an ejected electron with the frequency of the incident radiation and the work function of the metal. The work function for copper from Table 28.1 is 4.65 eV. Also note that we must convert eV to J using the conversion: 1 eV = 1.6×10^{-19} J.
Solve: From Equation 28.6, the maximum kinetic energy is

$$K_{\max} = hf - E_0 = h\frac{c}{\lambda} - E_0 \Rightarrow \lambda = \frac{hc}{E_0 + K_{\max}} = \frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(3.0\times10^{8}\,\text{m/s})}{4.65\,\text{eV} + 1.10\,\text{eV}} \times \frac{1\,\text{eV}}{1.6\times10^{-19}\,\text{J}} = 216\,\text{nm}$$

Assess: λ = 216 nm is the wavelength of light in the ultraviolet region of the spectrum.

Example 28.4

Light with a wavelength of 350 nm shines on a metal surface, which emits electrons. The stopping potential is measured to be 1.25 V. (a) What is the maximum speed of emitted electrons? (b) Calculate the work function and identify the metal.

Prepare: See Example 28.4. The energy of the photons is $E = hf = hc/\lambda = (6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times\ 10^{8}$ m/s)/(350 nm) = 5.68×10^{-19} J = 3.55 eV. Because $V_{stop} = K_{max}/e$ we know that the maximum energy for the photoelectrons is $K_{max} = 1.25\ \text{eV}$.

Solve: **(a)** Assume the electrons are non-relativistic so that $K = 1/2mv^2$. Solve for v.

$$v = \sqrt{\frac{2K_{max}}{m}} = \sqrt{\frac{2(1.25\ \text{eV})}{9.11\times10^{-31}\ \text{kg}}\left(\frac{1.60\times10^{-19}\ \text{J}}{1\ \text{eV}}\right)} = 6.63\times10^{5}\ \text{m/s}$$

(b) Use Equation 28.6 to find the work function.

$$E_0 = hf - K_{max} = 3.55\ \text{eV} - 1.25\ \text{eV} = 2.30\ \text{eV}$$

Looking for this value in Table 28.1 tells us the metal must be potassium.

Assess: Our results seem to correspond to values in the examples, and we did arrive spot on for the work function of one of the metals listed in Table 28.1, so we probably did the problem correctly. The units all work out nicely.

Example 28.5

Through what potential difference must an electron be accelerated from rest to have a de Broglie wavelength of 500 nm?

Prepare: Since the de Broglie wavelength is $\lambda = h/(mv)$ and is 500 nm, we can first find the speed v and then find ΔV.

Solve:

$$v = \frac{h}{m\lambda} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(500\times10^{-9}\ \text{m})} = 1456\ \text{m/s}$$

Thus,

$$e\Delta V = \frac{1}{2}mv^2 \Rightarrow \Delta V = \frac{mv^2}{2e} = \frac{(9.11\times10^{-31}\ \text{kg})(1456\ \text{m/s})^2}{2(1.6\times10^{-19}\ \text{C})} = 6.0\times10^{-6}\ \text{V}$$

Assess: A mere 6.0×10^{-6} V is able to increase an electron's speed to 1456 m/s.

Example 28.6

Estimate your de Broglie wavelength while walking at a speed of 1 m/s.

Prepare: The de Broglie wavelength for a moving particle is given by Equation 28.8, $\lambda = h/p = h/(mv)$. Your mass is, say, $m \approx 70$ kg, and your velocity is 1 m/s. Thus, your momentum is $p = mv \approx (70\ \text{kg})(1\ \text{m/s}) =\ 70\ \text{kg}\cdot\text{m/s}$.

Solve: Your de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{70\ \text{kg}\cdot\text{m/s}} \approx 9\times10^{-36}\ \text{m}$$

Assess: This is a very small wavelength—too small to be measured.

Example 28.7

The allowed energies of a quantum system are 1.0 eV, 2.0 eV, 4.0 eV, and 7.0 eV.

a. Sketch an energy-level diagram for this quantum system.
b. How many emissions lines are possible? What wavelengths appear in the system's emission spectrum? Which portion of the EM spectrum do each of the lines fall into (i.e. IR, visible, UV, …)?

Solution

The wavelengths in the emission spectrum correspond to photons whose energies are equal to the differences in energies of the allowed energy levels. We'll use $\lambda = hc/E$.

Solve: The energy differences and corresponding wavelengths are

$$2.0\text{ eV} - 1.0\text{ eV} = 1.0\text{ eV} \qquad \lambda = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{1.0\text{ eV}}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = 1200\text{ nm}$$

$$4.0\text{ eV} - 1.0\text{ eV} = 3.0\text{ eV} \qquad \lambda = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{3.0\text{ eV}}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = 410\text{ nm}$$

$$4.0\text{ eV} - 2.0\text{ eV} = 2.0\text{ eV} \qquad \lambda = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{2.0\text{ eV}}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = 620\text{ nm}$$

$$7.0\text{ eV} - 4.0\text{ eV} = 3.0\text{ eV} \qquad \lambda = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{3.0\text{ eV}}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = 410\text{ nm}$$

$$7.0\text{ eV} - 2.0\text{ eV} = 5.0\text{ eV} \qquad \lambda = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{5.0\text{ eV}}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = 250\text{ nm}$$

$$7.0\text{ eV} - 1.0\text{ eV} = 6.0\text{ eV} \qquad \lambda = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{6.0\text{ eV}}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = 210\text{ nm}$$

Notice that two of the transitions produce the same wavelength radiation—so while there are six different possible transitions, they only produce five different wavelengths.

Assess: The 620 nm and 410 nm lines will be visible; the 1200 nm line will be in the IR; and the 250 nm and 210 nm lines will be in the UV

Example 28.8

A proton confined in a one-dimensional box emits a 2.0 MeV gamma-ray photon in a quantum jump from n = 2 to n = 1. What is the length of the box?

Solve: The energy of the emitted gamma-ray photon by the proton in a one-dimensional box is exactly equal to the energy between levels 1 and 2. The energy levels of the proton are

$E_n = n^2 \dfrac{h^2}{8\,mL^2}$ n □ 1, 2, 3, …

The energy of the emitted photon is

$$E_2 - E_1 = \frac{4h^2}{8mL^2} - \frac{h^2}{8mL^2} = \frac{3h^2}{8mL^2}$$

$$\Rightarrow L = \sqrt{\frac{3h^2}{8m(E_2 - E_1)}} = \sqrt{\frac{3(6.63\times10^{-34}\text{ J}\cdot\text{s})^2}{8(1.67\times10^{-27}\text{ kg})(2.0\times10^{6}\text{ eV})}\times\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}} = 1.76\times10^{-14}\text{ m} = 18\text{ fm}$$

Assess: This is roughly the size of a typical nucleus.

Example 28.9

The wavelengths in the hydrogen spectrum with m = 1 form a series of spectral lines called the Lyman series. Calculate the wavelengths of the first four members of the series.

Prepare: We will use Equation 29.2 with *m* = 1 and *n* = 2, 3, 4, and 5. This series of spectral lines is called the Lyman series.

Solve: The formula for the Lyman series, Equation 29.2, simplifies to

$$\lambda = \frac{91.18 \text{ nm}}{1-(1/n)^2} \text{ where } n \,\square\, 2, 3, 4, \text{ and } 5$$

For *n* = 2, $\lambda = (91.18\text{ nm})\left(1-\frac{1}{4}\right)^{-1} = 121.6\text{ nm}.$ For *n* = 3, $\lambda = (91.18\text{ nm})\left(1-\frac{1}{9}\right)^{-1} = 102.6\text{ nm}.$ Likewise, for *n* = 4 and *n* = 5, $\lambda = 97.26\text{ nm}$ and $\lambda = 94.98\text{ nm},$ respectively.

Assess: These wavelengths are in the ultraviolet region.

Example 28.10

Write the symbol for an atom or ion with: (a) four electrons, four protons, and 5 neutrons. (b) six electrons, seven protons, and eight neutrons.

P29.6. Prepare: The number of protons determines the element, the sum of protons and neutrons is the atomic weight, and if the number of electrons differs from the number of protons then it is an ion.

Solve:

(a) Four protons gives beryllium. Five neutrons gives a weight of 9. So the symbol is $^{9}\text{Be}.$

(b) Seven protons gives nitrogen. Eight neutrons gives a weight of 15. So the symbol is $^{15}\text{N}^{+}.$

Assess: These can be verified on a periodic chart.

Example 28.11

The allowed energies of a simple atom are 0.0 eV, 4.0 eV, and 6.0 eV. (a) Draw the atom's energy-level diagram. An electron traveling at a speed of 1.6 × 10^6 m/s collisionally excites the atom. What are the minimum and maximum speeds the electron could have after the collision?

Prepare: We'll first compute the kinetic energy of an electron traveling at $1.6\times10^6\text{ m/s}.$

Solve:

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(9.11\times10^{-31}\text{ kg})(1.6\times10^6\text{ m/s})^2\left(\frac{1\text{ eV}}{1.6\times10^{-19}\text{ J}}\right) = 7.29\text{ eV}$$

If the electron excites the 6.0eV level than there is $7.29\text{eV} - 6.0\text{eV} = 1.29\text{eV}$ left. This equates to a speed of

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.29\text{ eV})\left(\frac{1.6\times10^{-19}\text{ J}}{1\text{ eV}}\right)}{9.11\times10^{-31}\text{ kg}}} = 6.7\times10^5\text{ m/s}$$

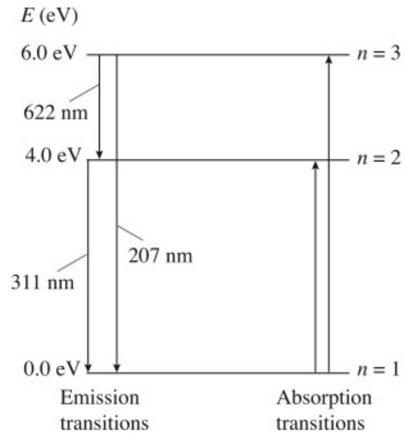
This is the minimum speed the electron could have after the collision. A similar calculation shows that when the 4.0 eV level is excited the maximum speed is $1.1\times10^6\text{ m/s}.$

Assess: These are typical electron speeds.

Example 28.12

The allowed energies of a simple atom are 0.0 eV, 4.0 eV, and 6.0 eV. (a) Draw the atom's energy-level diagram. Label each level with the energy and principle quantum number. (b) What wavelengths appear in the atom's emission spectrum? (c) What wavelengths appear in the atom's absorption spectrum?

Prepare: To conserve energy, the emission and the absorption spectra must have exactly the energy lost or gained by the atom in the appropriate quantum jumps.
Solve: (a)



(b) The energy of a light quantum is $E = hf = hc/\lambda$. We can use this equation to find the emission and absorption wavelengths. The emission energies from the previous energy-level diagram are $E_{2\to1} = 4.0$ eV, $E_{3\to1} = 6.0$ eV, and $E_{3\to2} = 2.0$ eV. The wavelength corresponding to the 2→1 transition is

$$\lambda_{2\to1} = \frac{hc}{E_{21}} = \frac{(4.14\times10^{-15}\,\text{eV}\cdot\text{s})(3.0\times10^{8}\,\text{m/s})}{4.0\,\text{eV}} = 310\text{ nm}$$

Likewise, $\lambda_{3\to1} = hc/E_{3\to1} = 210$ nm, and $\lambda_{2\to1} = 620$ nm.

(c) Absorption transitions start from the $n = 1$ ground state. The energies in the atom's absorption spectrum are $E_{1\to2} = 4.0$ eV and $E_{1\to3} = 6.0$ eV. The corresponding wavelengths are $\lambda_{1\to2} = hc/E_{1\to2} = 310$ nm and $\lambda_{1\to3} = hc/E_{1\to3} = 210$ nm.

Assess: Only those transitions whose energies match the energy intervals between the stationary states are observed as absorption or emission spectra.

Example 28.13

Determine all the possible wavelengths of photons that can be emitted from the n = 4 state of a hydrogen atom.

Prepare: We will use Equation 29.2 or 29.18 with $\lambda_0 = 91.18$ nm.
Solve: Photons emitted from the $n = 4$ state start in energy level $n = 4$ and undergo a quantum jump to a lower energy level with $m < 4$. The possibilities are $4 \to 1$, $4 \to 2$, and $4 \to 3$. According to Equation 29.18, the transition $4 \to m$ emits a photon of wavelength

$$\lambda = \frac{\lambda_0}{(1/m^2 - 1/n^2)} = \frac{91.18\text{ nm}}{(1/m^2 - 1/16)}$$

These values are given in the following table.

Transition	Wavelength
$4 \to 1$	97.3 nm
$4 \to 2$	486 nm
$4 \to 3$	1876 nm

Assess: Transitions are in the ultraviolet, visible, and infrared regions.