Chapter 28: Special Relativity

Example Problems

$$c = \lambda f \qquad n = \frac{c}{v_{mat}} = \frac{\lambda}{\lambda_{mat}} \qquad \Delta\ell = d\sin\theta_m = m\lambda, \quad m = 0,\ 1,\ 2,...$$

$$\theta \approx \sin\theta \approx \tan\theta = \frac{y}{L} \qquad y_m = m\lambda L/d \qquad y'_m = (m+\tfrac{1}{2})\lambda L/d \qquad 2t = \begin{cases} m\lambda/n \\ (m+\tfrac{1}{2})\lambda/n \end{cases}$$

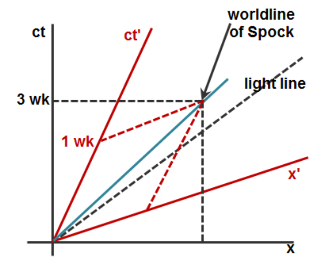
Example 28.1

a. When Spock returns his Hertz rent-a-rocket after one week's cruising in the galaxy, Spock is shocked to be billed for three weeks' rental. Assuming that he traveled on a one-way trip at the same speed. **How fast** was he traveling?

b. Antonio sets off at a steady v = 0.95c to a distant star. After exploring the star for a short time he returns at the same speed and gets home after a total absence of 80 years (as measured by earth-bound observers). **How long** do his clocks say that he was gone, and how much has he aged? (Note: there are three inertial frames).

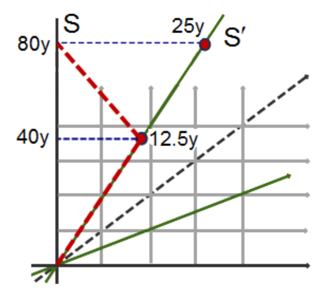
Solution

a. There is a hidden assumption in this problem: that where ever he rented his Hertz rent-a-rocket and has destination, are in the same FoR. Otherwise, this problem is meaningless in its description. Physically, what is going on? Since the rocket is moving relative to the rental place, the rental place measures a time dilated time for the rocket. Unfortunately for Spock, charges are accrued in the earth frame, not his frame. In terms of a spacetime diagram, there are two prospective. Spock's proper time (the time that Spock measures relative to him) is 1 week. The Rental place frame is that frame that observes the 3 weeks. Symbolically, we write



$$\Delta t = \gamma\Delta t'\Big|_{\substack{\Delta t=3\text{ wks}\\ \Delta t'=1\text{ wk}}} \xrightarrow[\text{for }\gamma]{\text{solving}} \gamma = \frac{\Delta t}{\Delta t'} = 3 \xrightarrow[\text{for }\beta]{\text{solving}} \boxed{\beta = 0.94}$$

b. Note that there are 3 different inertial frames: earth frame (S), rocket going out (S′) and the rocket coming back (S′′). Numerically, even though it will not make a difference (in this case), the total round trip time is given by



$$\Delta t(\text{total earth}) = \gamma\left[\Delta t'(\text{out}) + \Delta t''(\text{return})\right] + \underbrace{\Delta t(\text{turn around time})}_{\text{ignore}}$$

$$= \gamma\Delta t(\text{total rocket})$$

For β = 0.95 (γ = 3.2) we get

$$\boxed{\Delta t'(\text{total rocket}) = \frac{\Delta t(\text{total earth})}{\gamma} = \frac{80\text{y}}{3.2} = 25\text{y}}$$

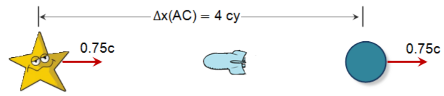
Example 28.2

A spaceship departs from Earth for the star Alpha Centauri, which are 4 light-years away. The spaceship travels at 0.75c. How long does it take to get there (a) as measured on Earth and (b) as measured by a passenger on the spaceship?

Solution

a. When the problem states that an observer on earth (S) measures the distance to Alpha Centauri as 4 light-years, it implies that they are in the same inertial FoR.

Notation: $\boxed{1\text{ light year} \equiv 1\ c\cdot y} \longrightarrow 1\ c\cdot y \approx (3\times10^8\text{m/s})\cdot(\pi\times10^7\text{s}) = 9.42\times10^{15}\text{m}$



Using the definition for the velocity, we find that

$$v = \frac{\Delta x(\text{earth})}{\Delta t(\text{earth})} \longrightarrow \Delta t = \frac{\Delta x}{v} = \frac{4\ \cancel{c}\cdot y}{0.75\cancel{c}} = \boxed{5.33y = \Delta t}$$

b. For a passenger at rest in the S′ sees the earth moving away while Alpha Centauri moving towards the spaceship is moving relative towards her and therefore, that distance is length contracted (relative to her she does not travel as far).

$$\Delta x(AC) = \frac{\Delta x'(\text{rocket})}{\gamma} \longrightarrow \Delta x' = \gamma \Delta x = \left(\frac{1}{1.513}\right)\cdot(4\ c\cdot y) = 2.65\ c\cdot y$$

To find the time, use the definition of speed again:

$$\Delta t' = \frac{\Delta x'}{v} = \frac{2.65\ \cancel{c}\cdot y}{0.75\cancel{c}} = \boxed{3.53y = \Delta t'}$$

Example 28.3

Three particles are listed in the table. The mass and speed of each particle are given as multiples of the variables m and v, which have the values m = 1.20 × 10^{-8} kg and v = 0.200c. Determine the momentum for each particle according to special relativity.

Particle	Mass	Speed
a	m	v
b	$\frac{1}{2}m$	$2v$
c	$\frac{1}{4}m$	$4v$

Solution

In special relativity the momentum of a particle is p = γmv. Because of the γ-term, doubling the particle's speed more than doubles its momentum. We expect, that particle c has the greatest momentum magnitude, followed by particle b and then by particle a. The momenta of the three particles are:

$$p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = \begin{cases} p_a = \dfrac{(1.20\times10^{-8}\ \text{kg})(0.200)(3.00\times10^{8}\ \text{m/s})}{\sqrt{1-\frac{(0.200c)^2}{c^2}}} = \boxed{0.735\ \text{kg}\cdot\text{m/s}} \\ p_b = \boxed{0.786\ \text{kg}\cdot\text{m/s}} \\ p_c = \boxed{1.20\ \text{kg}\cdot\text{m/s}} \end{cases}$$

As expected, the ranking of the momenta (largest first) is c, b, a.

Example 28.4

a. How much work must be done on an electron to accelerate it from rest to a speed of 0.990c?
b. A nuclear power reactor generates 3.0 × 10^9 W of power. In one year, what is the change in the mass of the nuclear fuel due to the energy being taken from the reactor?

Solution

a. According to the work-energy theorem, Equation 6.3, the work that must be done on the electron to accelerate it from rest to a speed of 0.990*c* is equal to the kinetic energy of the electron when it is moving at 0.990*c*. Using Equation 28.6, we find that

$$\text{KE} = mc^2\left(\frac{1}{\sqrt{1-(v^2/c^2)}}-1\right) = (9.11\times10^{-31})(3.00\times10^{8})^2\left(\frac{1}{\sqrt{1-(0.990c)^2/c^2}}-1\right) = \boxed{5.0\times10^{-13}\ \text{J}}$$

b. The energy E_0 produced in one year is the product of the power P generated and the time t, $E_0 = P\,t$. This energy is equivalent to an amount of mass m given by Equation 28.5 as $E_0 = mc^2$. The mass of nuclear fuel consumed in one year (3.15×10^7 s) is

$$mc^2 = P\,t \longrightarrow m = \frac{P\,t}{c^2} = \frac{\left(3.0\times10^9\ \text{W}\right)\left(3.15\times10^7\ \text{s}\right)}{\left(3.00\times10^8\ \text{m/s}\right)^2} = \boxed{1.1\ \text{kg}}$$