Answer Key

Some problems deal with the interference of waves reflected from the front and back surfaces of a thin film. The difference in phase comes from the difference in distances traveled and, in some cases, from the change in phase of ½ λ on reflection from a medium with a higher index of refraction. Don't forget to think about both of these sources of phase difference. Also, don't forget to use the wavelength of the wave in the film, not the wavelength in vacuum, to compute the phase difference.

When light passes through a single narrow slit and falls on a viewing screen, a pattern of bright and dark fringes is formed. The angle q that specifies the mth dark fringe on either side of the central bright fringe is given by:

$$\sin\theta = \frac{m\lambda}{W} \qquad m = 1, 2, 3, \ldots$$

where λ is the wavelength of the light and W is width of the slit.

According to the Rayleigh criterion, the minimum angle (in radians) that two point sources can subtend at a circular aperture of diameter D and still be resolved as separate objects is:

$$\theta_{\min} \approx 1.22\frac{\lambda}{D} \quad (\theta_{\min} \text{ in radians})$$

where λ is the wavelength of light.

Questions and Example Problems

Question 1
Suppose that a radio station broadcast simultaneously from two transmitting antennas at two different locations. Is it clear that your radio will have better reception with two transmitting antennas rather than one? Justify your answer.

No, with two transmitting antennas, it is possible for destructive interference to occur if:

$$\Delta L = (m + 1/2)\lambda$$

Question 2
(a) How would the pattern of bright and dark fringes produced in a Young's double slit experiment change in the light rays coming from *both slits* had their phases shifted by an amount equivalent to a half wavelength? (b) How would the pattern change if the light coming from *only one* of the slits had its phase shifted by an amount equivalent to a half wavelength?

(a) no change (both waves still in phase)

(b) the bright and dark fringes would be switched because the two waves now leave the slits out of phase

Problem 1

A rock concert is being held in an open field. Two loudspeakers are separated by 7.00 m. As an aid in arranging the seating, a test is conducted in which both speakers vibrate in phase and produce an 80.0 Hz bass tone simultaneously. The speed of sound is 343 m/s. A reference line is marked out in front of the speakers, perpendicular to the midpoint of the line between the speakers. Relative to either side of this line, what is the smallest angle that locates the places where destructive interference occurs? People seating in these places would have trouble hearing the 80.0 Hz based tone.

⇒ this is similar to double-slit interference where the two speakers are acting like two slits

for destructive interference: $\sin\theta = \frac{(m+\frac{1}{2})\lambda}{d}$ $m = 0, 1, 2, \ldots$

$\theta = \sin^{-1}\left[\frac{(m+\frac{1}{2})\lambda}{d}\right]$ → for the smallest angle we let $m = 0$

$\theta = \sin^{-1}\left[\frac{\frac{1}{2}\lambda}{d}\right] = \sin^{-1}\left[\frac{\lambda}{2d}\right]$ $\lambda f = v \longrightarrow \lambda = v/f$

$\theta = \sin^{-1}\left[\frac{v}{2df}\right] = \sin^{-1}\left[\frac{(343\,m/s)}{2(7.00\,m)(80.0\,Hz)}\right] \longrightarrow \boxed{\theta = 17.8^\circ}$

Problem 2

In a Young's double-slit experiment, the angle that locates the second-order bright fringe is 2.0°. The slit separation is 3.8×10^{-5} m. What is the wavelength of light?

second order → $m = 2$

$\theta = 2.00$

$d = 3.8 \times 10^{-5}\,m$

$\lambda = ?$

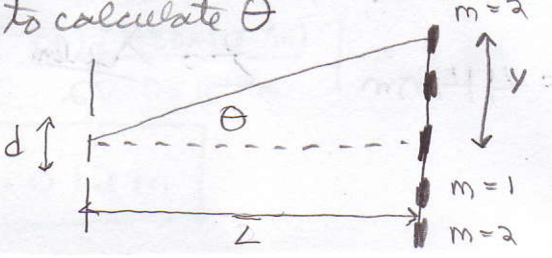
for bright fringes:

$\sin\theta = m\lambda/d$ $m = 0, 1, 2, \ldots$

$\lambda = \frac{d\sin\theta}{m} = \frac{(3.8\times10^{-5}\,m)\sin 2.0^\circ}{2}$

$\boxed{\lambda = 6.63\times10^{-7}\,m \text{ or } 663\,nm}$

note: in lab you first need to calculate θ



$\tan\theta = Y/L$

Problem 3

A Young's double slit experiment is performed using light that has a wavelength of 630 nm. The separation between the slits is 5.3×10^{-5} m. Find the angles that locate the (a) first-, (b) second-, and (c) third-order bright fringes on the screen.

$\lambda = 630\,nm = 630\times10^{-9}\,m$ ⇒ bright fringes correspond to constructive interference

$d = 5.3\times10^{-5}\,m$

$\sin\theta = m\lambda/d \longrightarrow \theta = \sin^{-1}(m\lambda/d) \quad m = 0, 1, 2, \ldots$

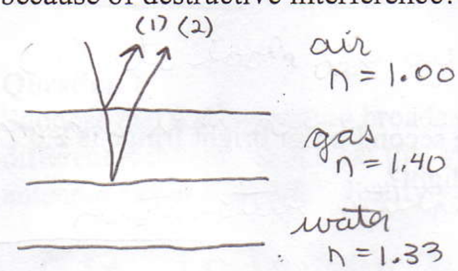
(a) $m = 1 \quad \theta = \sin^{-1}\left[\frac{(1)(630\times10^{-9}m)}{5.3\times10^{-5}m}\right] \longrightarrow \boxed{\theta = 0.68°}$

(b) $m = 2 \quad \theta = \sin^{-1}\left[\frac{(2)(630\times10^{-9}m)}{5.3\times10^{-5}m}\right] \longrightarrow \boxed{\theta = 1.4°}$

(c) $m = 3 \quad \theta = \sin^{-1}\left[\frac{(3)(630\times10^{-9}m)}{5.3\times10^{-5}m}\right] \longrightarrow \boxed{\theta = 2.0°}$

Problem 4

A mixture of yellow light (wavelength = 580 nm in vacuum) and violet light (wavelength = 410 nm in vacuum) falls perpendicularly on a film of gasoline that is floating on a puddle of water. For both wavelengths, the refractive index of gasoline is n = 1.40 and that of water is n = 1.33. What is the minimum nonzero thickness of the film in a spot that looks (a) yellow and (b) violet because of destructive interference?



wave 1 undergoes $\frac{1}{2}\lambda$ phase shift

wave 2 no phase shift

destructive interference:

$2t = m\lambda_{film} \quad m = 0, 1, 2, \ldots$

minimum non-zero thickness ⟶ $m = 1$

(a) to look yellow means violet undergoes destructive interference

$\lambda = 410\times10^{-9}\,m \qquad 2t = m\lambda_{film} = \lambda_{film}$

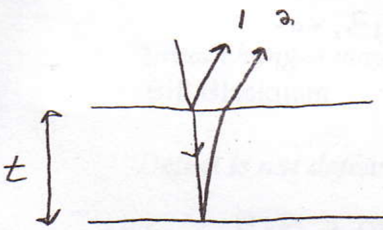
$\lambda_{film} = \frac{410\times10^{-9}m}{1.40} = 293\times10^{-9}m \qquad t = \lambda_{film}/2 = \boxed{146\,nm}$

(b) to look violet means yellow undergoes constructive interference

$\lambda_{film} = \frac{580\,nm}{1.40} = 414\,nm \qquad t = \frac{\lambda_{film}}{2} = \boxed{207\,nm}$

Problem 5

Orange light (λ_{vacuum} = 611 nm) shines on a soap film (n = 1.33) that has air on either side of it. The light strikes the film perpendicularly. What is the minimum thickness of the film for which constructive interference causes it to look bright in reflected light?



n = 1.0 $\Rightarrow$ ray 1 undergoes a $\lambda/2$ phase shift

n = 1.33 $\Rightarrow$ ray 2 undergoes no phase shift

n = 1.0

constructive interference: $2t = (m+\frac{1}{2})\lambda_{film}$

$$t = \frac{(m+\frac{1}{2})\lambda_{film}}{2} = \frac{(m+\frac{1}{2})(\lambda/n)}{2} \qquad m = 0, 1, 2, \ldots$$

for minimum thickness, m = 0 $\longrightarrow$ $t = \frac{(0+\frac{1}{2})(\lambda/n)}{2} = \frac{\lambda}{4n}$

$$t = \frac{(611\times10^{-9}\,m)}{4(1.33)} \longrightarrow \boxed{t = 115\times10^{-9}\,m \text{ or } 115\,nm}$$

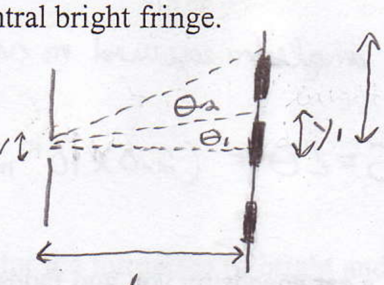
Problem 6

The width of a slit is 2.0×10^{-5} m. Light with a wavelength of 480 nm passes through this slit and falls on a screen that is located 0.50 meters away. In the diffraction pattern, find the width of the bright fringe that is next to the central bright fringe.

$W = 2.0\times10^{-5}$ m

$\lambda = 480$ nm

$L = 0.50$ m



$y = L \tan\theta$

$\Rightarrow$ for diffraction, the location of the dark fringes is given by:

$$\sin\theta = m\lambda/W \qquad m = 1, 2, 3, \ldots$$

$$\theta = \sin^{-1}(m\lambda/W)$$

$\theta_1 = \sin^{-1}\left[\frac{(1)(480\times10^{-9}\,m)}{2.0\times10^{-5}\,m}\right] = 1.4°$ $\qquad y_1 = (0.50\,m)\tan 1.4° = 0.0122\,m$

$\theta_2 = \sin^{-1}\left[\frac{(2)(480\times10^{-9}\,m)}{2.0\times10^{-5}\,m}\right] = 2.8°$ $\qquad y_2 = (0.50\,m)\tan 2.8° = 0.0245$

$y_2 - y_1 = \boxed{0.012\,m}$

Problem 7

How many dark fringes will be produced on either side of the central maximum if light ($\lambda = 651$ nm) is incident on a single slit that is 5.47×10^{-6} m wide.

location of dark fringes: $\sin\theta = m\lambda/W \quad m = 1, 2, 3, \ldots$

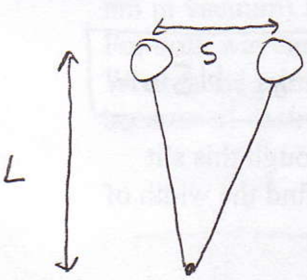
since $\sin\theta \leq 1$, we know that $m\lambda/W \leq 1$

$$m \leq W/\lambda \longrightarrow m \leq \frac{(5.47\times10^{-6}\,m)}{651\times10^{-9}\,m} \longrightarrow m \leq 8.4$$

$\Rightarrow$ since m must be an integer, $m = 8$ is the highest order fringe possible so there are $\boxed{\text{8 dark fringes on either side}}$ of the central maximum

Problem 8

Two asteroids are traveling close to each other through the solar system at a distance of 2.0×10^{10} m from the earth. With light of wavelength 550 nm, they are just resolved by the Hubble Space Telescope, whose aperture has a diameter of 2.4 m. How far apart are the asteroids?



$$\theta_{min} \approx 1.22\lambda/D = \frac{1.22(550\times10^{-9}\,m)}{2.4\,m}$$

$$\theta_{min} \approx 2.82\times10^{-7} \text{ radians}$$

$\Rightarrow$ for angles measured in radians $\longrightarrow$ $S = r\theta$

$$S = L\theta = (2.0\times10^{10}\,m)(2.82\times10^{-7}\,rad) = \boxed{5.6\times10^{3}\,m}$$

Problem 9

Late one night on a highway, a car speeds by you and fades into the distance. Under these conditions, the pupil of your eyes have diameters of about 7.0 mm. The taillights of this car are separated by a distance of 1.2 m and emit red light (wavelength = 660 nm in vacuum). How far away from you is this car when its taillights appear to merge into a single spot of light because of the effects of diffraction?

$\lambda = 660\,nm = 660\times10^{-9}\,m$

$D = 7.0\,mm = 7.0\times10^{-3}\,m$

$L = 1.2\,m$

$$\theta_{min} \approx 1.22\lambda/D \quad (\theta \text{ in radians})$$

$$= \frac{1.22(660\times10^{-9}\,m)}{7.0\times10^{-3}\,m}$$

$$\theta_{min} \approx 1.2\times10^{-4}\,rad$$

$$S = L\theta \rightarrow L = S/\theta = \frac{1.2\,m}{1.2\times10^{-4}\,m} = \boxed{1.0\times10^{4}\,m}$$