Physics 4A
Chapters 11: Impulse and Momentum

"It is good to have an end to journey toward; but it is the journey that matters, in the end."
Ursula K. Le Guin

"*Nobody made a greater mistake than he who did nothing because he could only do a little*."
Edmund Burke

Reading: pages 261 – 281; 285

Outline:

⇒ momentum and impulse
⇒ conservation of linear momentum
⇒ collisions
  inelastic collisions
  elastic collisions
⇒ explosions
⇒ momentum in two dimensions

Problem Solving

Some problems deal with the definition of linear momentum and the equation of motion $\vec{F}_{net} = d\vec{p}/dt$ for a particle. Solve this as you would any second-law problem.

Many problems of this chapter can be worked using the principle of linear momentum conservation. If you suspect the principle can be used, first check for external forces: if there are none or if they add to zero, total linear momentum is conserved. For some problems only one component of linear momentum is conserved. If the problem asks about motion in that direction, then the principle can be used. Write the momentum at the beginning of some interval in terms of the velocities and masses of the particles involved. Do the same for the momentum at the end of the interval. Equate the two expressions and solve for the unknown quantities.

Most calculations of impulse are rather straightforward. You might be given the force acting on an object as a function of time and asked to find the impulse over a given time interval. You simply evaluate the integral that defines impulse. Remember that impulse is a vector quantity and you must, for example, use the *x* component of the force to find the *x* component of the impulse. For one-dimensional motion the force might be given graphically as a function of time. You must then recognize that the impulse is the area under the curve. You might also be given sufficient information to calculate the initial and final linear momentum of a particle. Then, use $\vec{J} = \vec{p}_f - \vec{p}_i$ to calculate the impulse.

Collision problems are more complicated. First decide if the collision is one- or two-dimensional. A head-on collision is always one-dimensional. So is a completely inelastic two-body collision with one object initially at rest. If the objects move along different lines, whether initially or finally, the collision is two-dimensional.

First consider one-dimensional collisions. Since total linear momentum is always conserved in collisions, nearly every problem solution begins by writing the equation for conservation of linear momentum. There is only one such equation for a one-dimensional collision. To write it, use the component, not the magnitude, of the linear momentum of each object and write the component as the product of the appropriate mass and velocity component. Always use symbols, not numbers, even for given quantities.

Make a list of the quantities given in the problem statement and a list of the unknowns. If there is only one unknown, the linear momentum conservation equation can be solved immediately for it. Once the equation has been solved, the result can be used in other calculations. You might be asked to test for the conservation of kinetic energy, for example, and classify the collision as elastic or inelastic. You might also be asked to calculate the impulse exerted by one object on the other.

If more than one quantity is unknown, look for more information in the problem statement. If it specifies that the collision is elastic, write the equation for the conservation of kinetic energy in terms of the masses and speeds of the objects. If it specifies that the collision is completely inelastic, equate the final velocities of the objects to each other and use a single symbol to denote them.

For two-dimensional collisions there are two linear momentum conservation equations, one for each component. The amount of algebraic manipulation can usually be reduced significantly if one of the coordinate axes is placed along the direction of the total linear momentum. For some problems, however, this direction is not immediately evident. In any event, select a coordinate system and write the linear momentum conservation equations in terms of the masses, speeds, and angles between the velocities and a coordinate axis.

GENERAL PRINCIPLES

Law of Conservation of Momentum

The total momentum $\vec{P} = \vec{p}_1 + \vec{p}_2 + \cdots$ of an isolated system is a constant. Thus

$$\vec{P}_f = \vec{P}_i$$

Newton's Second Law

In terms of momentum, Newton's second law is

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Solving Momentum Conservation Problems

MODEL Choose an isolated system or a system that is isolated during at least part of the problem.

VISUALIZE Draw a pictorial representation of the system before and after the interaction.

SOLVE Write the law of conservation of momentum in terms of vector components:

$$(p_{fx})_1 + (p_{fx})_2 + \cdots = (p_{ix})_1 + (p_{ix})_2 + \cdots$$
$$(p_{fy})_1 + (p_{fy})_2 + \cdots = (p_{iy})_1 + (p_{iy})_2 + \cdots$$

ASSESS Is the result reasonable?

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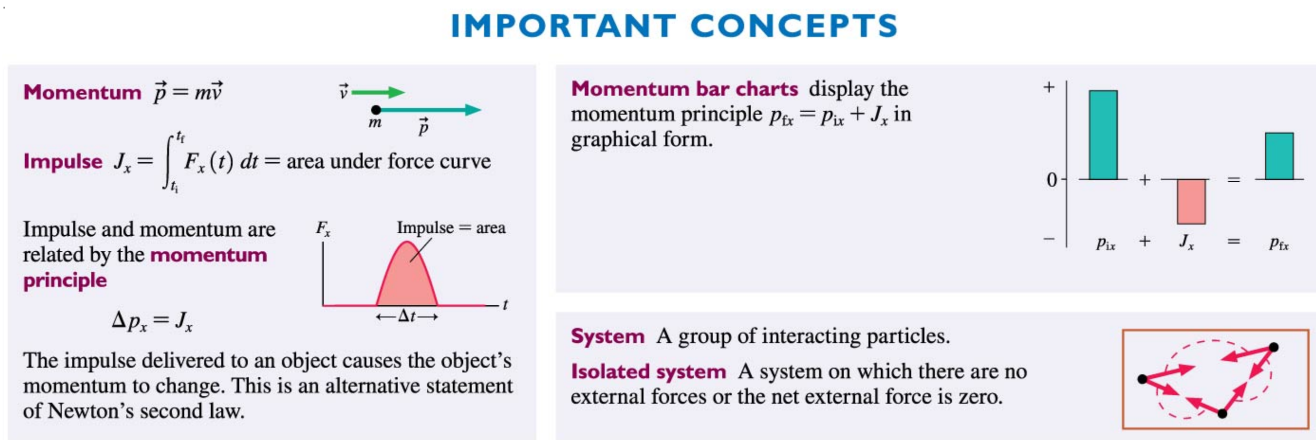

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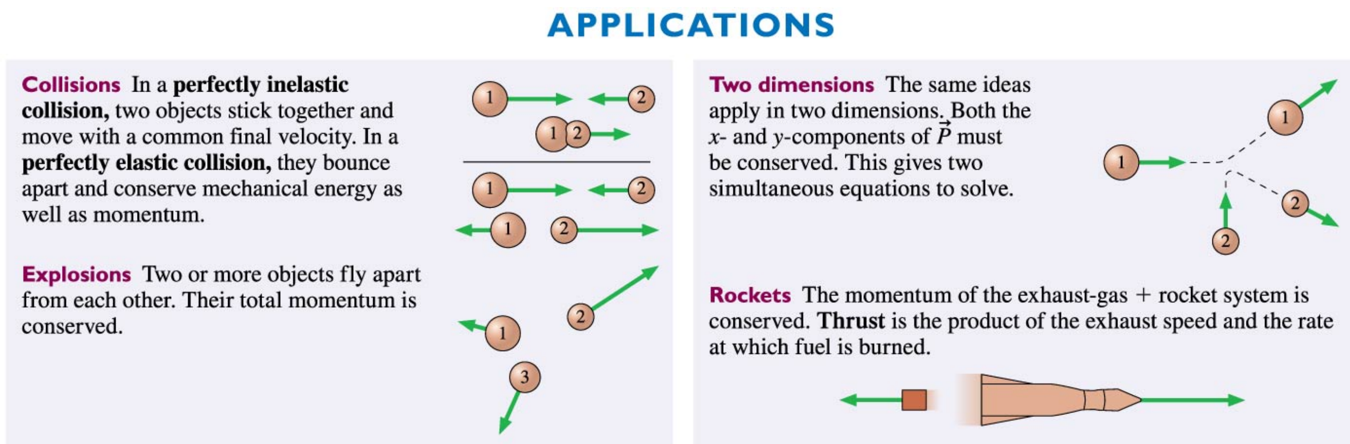

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Question and Example Problems from Chapter 11

Conceptual Question 11-4

A 0.2 kg plastic cart and a 20 kg lead cart can both roll without friction on a horizontal surface. Equal forces are used to push both carts forward for a time of 1 s, starting from rest. After the force is removed at t = 1 s, is the momentum of the plastic cart greater than, less than, or equal to the momentum of the lead cart? Explain.

Conceptual Question 11-5

A 0.2 kg plastic cart and a 20 kg lead cart can both roll without friction on a horizontal surface. Equal forces are used to push both carts forward for a distance of 1 m, starting from rest. After traveling 1 m, is the momentum of the plastic cart greater than, less than, or equal to the momentum of the lead cart? Explain.

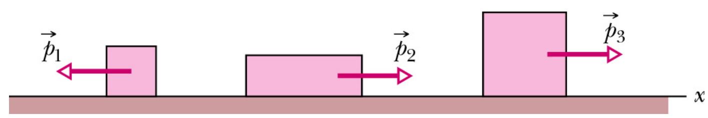
Conceptual Question 11-8

Automobiles are designed with "crumple zones" intended to collapse in a collision. Use the ideas of this chapter to explain why.

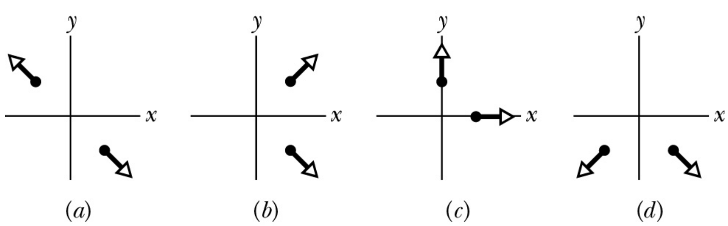
Conceptual Question 11-9
A golf club continues forward after hitting the golf ball. Is momentum conserved in the collision? Explain, making sure you are careful to identify "the system".

Conceptual Question 11-A
A container sliding along an x axis on a frictionless surface explodes into three pieces. The pieces then move along the x axis in the directions indicated in the figure below. The following table gives four sets of magnitudes (in kg·m/s) for the momenta 1, 2, and 3 of the pieces. Rank the sets according to the initial speed of the container, greatest first.

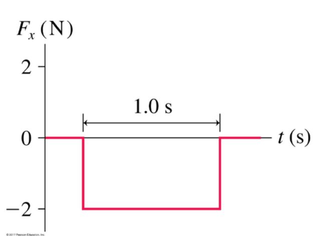
	p_1	p_2	p_3		p_1	p_2	p_3
(a)	10	2	6	(b)	10	6	2
(c)	2	10	6	(d)	6	2	10



Conceptual Question 11-B
In the four situations indicated in the figure below, an object explodes into two equal-mass fragments when the object is at the origin of the coordinate system. The velocity vectors of the fragments are indicated; they are directed either along an axis or at 45° to an axis. For each situation determine the direction of travel of the object before the explosion, or note that it was stationary.



Problem 11-8
A 2.0 kg object is moving to the right with a speed of 1.0 m/s when it experiences the force shown in the figure below. What are the object's speed and direction after the force ends?



Problem 11-10
A sled slides along a horizontal surface on which the coefficient of kinetic friction is 0.25. Its velocity at point A is 8.0 m/s and at point B is 5.0 m/s. Use the momentum principle to find how long the sled takes to travel from A to B.

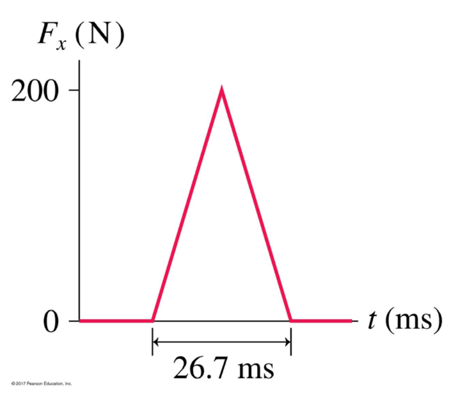
Problem 11-16
A 10-m-long glider with a mass 680 kg (including the passengers) is gliding horizontally through the air at 30 m/s when a 60 kg skydiver drops out by releasing his grip on the glider. What is the gliders's velocity just after the skydiver lets go?

Problem 11.24
A 50-g ball of clay traveling at speed v_0 hits and sticks to a 1.0 kg brick sitting at rest on a frictionless surface. **(a)** What is the speed of the brick after the collision (as a function of v_0)? **(b)** What percentage of the mechanical energy is lost in this collision?

Problem 11-28
Dan is gliding on his skateboard at 4.0 m/s. He suddenly jumps backward off the skateboard, kicking the skateboard forward at 8.0 m/s. How fast is Dan going as his feet hit the ground? Dan's mass is 50 kg and the skateboard's mass is 5.0 kg.

Problem 11-33
A 20 g ball of clay traveling east at 3.0 m/s collides with a 30 g ball of clay traveling north at 2.0 m/s. What are the speed and the direction of the resulting 50 g ball of clay? Give your answer as an angle north of east.

Problem 11-40
A 500 g cart is released from rest 1.0 m from the bottom of a frictionless, 30.0° ramp. The cart rolls down the ramp and bounces off a rubber block at the bottom. The figure shows the force during the collision. After the cart bounces, how far does it roll back up the ramp?



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Problem 11-49

(a) A bullet of mass m is fired into a block of mass M that is at rest. The block, with the bullet embedded, slides distance d across a horizontal surface. The coefficient of kinetic friction is μ_k. Find a mathematical expression for the bullet's speed v_{bullet}. **(b)** What is the speed of a 10-g bullet that, when fired into a 10-kg stationary wood block, causes the block to slide 5.0 cm across a wood table ($\mu_k = 0.20$)?

Problem 11-55

A 100 g granite cube slides down a 40° frictionless ramp. At the bottom, just as it exits onto a horizontal table, it collides with a 200 g steel cube at rest. How high above the table should the granite cube be released to give the steel cube a speed of 150 cm/s.

Problem 11.63

A 20-g ball is fired horizontally with speed v_0 toward a 100-g ball hanging motionless from a 1.0-m-long string. The balls undergo a head-on, perfectly elastic collision, after which the 100 g ball swings out to a maximum angle $\theta_{max} = 50^o$. What was v_0?

Problem 11-72

A 2100 kg truck is traveling east through an intersection at 2.0 m/s when it is hit simultaneously from the side and the rear. (Some people have all the luck!) One car is a 1200 kg compact traveling north at 5.0 m/s. The other car is a 1500 kg midsize traveling east at 10 m/s. The three vehicles become entangled and slide as one body. What are their speed and direction just after the collision?

Problem 11-A

In the figure below, block 2 (mass 1.0 kg) at rest on a frictionless surface and touching the end of an unstretched spring of spring constant 200 N/m. The other end of the spring is fixed to a wall. Block 1 (mass 2.0 kg), traveling at speed $v_1 = 4.0$ m/s, collides with block 2, and the two blocks stick together. When the blocks momentarily stop, by what distance is the spring compressed?