Physics 4A
Chapter 2: Kinematics in One Dimension

"Whether you think you can or think you can't, you're usually right." – Henry Ford

"It is our attitude at the beginning of a difficult task which, more than anything else, will affect it's successful outcome." – William James

"The first and most important step toward success is the feeling that we can succeed."
Nelson Boswell

Reading: pages 32 – 57

Outline:

⇒ uniform motion
⇒ velocity
    average and instantaneous
    finding position from velocity
⇒ acceleration
    average and instantaneous
    finding velocity from acceleration
⇒ motion with constant acceleration
    equations of constant acceleration
⇒ free fall
⇒ motion on an inclined plane

Problem Solving

All constant-acceleration problems can be solved using only the equations $x(t) = x_0 + v_0t + ½ at^2$ and $v(t) = v_0 + at$. Six quantities appear in these equations: $x(t)$, $v(t)$, x_0, v_0, a, and t. In most cases all but two are given and you are asked to solve for one or both of the others. Mathematically a typical constant-acceleration problem involves identifying the known and unknown quantities, then simultaneously solving the two kinematics equations for the unknowns. Finding solutions to simultaneous algebraic equations is discussed in the MATHEMATICAL SKILLS section for this chapter.

Other kinematics equations are derived in the text by eliminating one or another of the kinematics quantities from the expressions for $x(t)$ and $v(t)$.

Constant-acceleration problems that deal with a single object describe two events, each of which has a time, a coordinate, and a velocity associated with it. The acceleration is the additional quantity that enters the problem. It is often helpful in solving a kinematics problem to fill in a table.

For any problem write numerical values next to given quantities and question marks next to unknown quantities. When you are finished, you should have no more than two question marks. If you do, you missed some information.

You should be aware that you may assign the value 0 to the time when the particle is at *any* point along its trajectory: when the particle starts out, when it reaches a particular point, when it has a particular velocity, or any other point. A negative value for t simply means a time before the instant you chose as $t = 0$. In the kinematics equations, x_0 is the coordinate of the object and v_0 is its velocity at time $t = 0$, not necessarily when the particle starts out.

You usually have the option of selecting x_0 to be zero; this selection simply places the origin of the coordinate system at the position of the object when $t = 0$. If you do this you need deal with only the other 5 kinematics quantities.

Some problems deal with two objects. You must now write down two sets of kinematics equations, one for each object: $x_1(t) = x_{01} + v_{01}t + ½\, a_1t^2$ and $v_1(t) = v_{01} + a_1t$ for object 1 and $x_2(t) = x_{02} + v_{02}t + ½\, a_2t^2$ and $v_2(t) = v_{02} + a_2t$ for object 2. At $t = 0$, object 1 is at x_{01} and has velocity v_{01} while object 2 is at x_{02} and has velocity v_{02}. These equations can be solved for 4 unknown quantities.

Free-fall problems are exactly the same as other constant-acceleration problems for which the acceleration is given. The notation is different because we choose the y axis to be vertical, so the object moves along that axis rather than the x axis. You should realize that this is a superficial difference. The acceleration is always known (g, downward) and is usually not given explicitly in the problem statement.

Mathematical Skills

The following is a listing of mathematical knowledge you will need to understand and solve problems in this chapter. Refer to a mathematics text if you are rusty on any of it.

Functions.

You should understand the notation used for a function. The symbol $f(t)$, for example, indicates that the value of f depends on the value of t. The coordinate of a moving object, for example, is different at different times so it is a function of time. Sometimes the functional dependence is given by means of an equation, such as $x(t) = v_0t + ½\, at^2$. Sometimes it is given by means of a graph or a table of values.

If you know the dependence of x on t you can select a value for t and find the corresponding value for x. Since you can choose the value for t, t is called the independent variable. Since the value for x is determined through the functional relationship by the value of t, x is called the dependent variable.

You must carefully distinguish between a function and its value for a particular value of the independent variable. If $x(t) = 5 - 7t$, for example, then $x(2) = 5-7\times2 = -9$. If you know the coordinate of an object as a function time and wish to find its velocity at $t = 2$ s, say, you must first find the velocity as a function of time by differentiating $x(t)$ with respect to time. Only when you have done this do you substitute numerical values into the expression. An evaluation of $x(t)$ for $t = 2$ s tells you nothing about the velocity. Similarly, an evaluation of the expression $v(t)$ for a particular instant of time tells you nothing about the acceleration.

Derivatives.

You should understand the meaning of a derivative as a limit of a ratio. This is useful for a solid understanding of instantaneous velocity and acceleration. The formula $v(t) = \mathrm{d}x/\mathrm{d}t$ gives the velocity at any instant of time. To find the velocity at a given instant, you must substitute a value for t. If the object is accelerating, its velocity just before or just after the given instant is different from the velocity at the instant.

You should be able to write down derivatives of polynomials. For example, $\mathrm{d}(A + Bt + Ct^2 + Dt^3)/\mathrm{d}t = B + 2Ct + 3Dt^2$ if A, B, C, and D are constants. Your instructor may also require you to know how to evaluate the derivative with respect to t of other functions such as e^{At}, $\sin(At)$, and $\cos(At)$. The derivatives are Ae^{At}, $A\cos(At)$, and $-A\sin(At)$ respectively.

You should know the product and quotient rules for differentiation. If $f(t)$ and $g(t)$ are two functions of t, then

$$\frac{d}{dt}\left[f(t)g(t)\right] = \frac{df(t)}{dt}g(t) + f(t)\frac{dg(t)}{dt} \qquad \text{and} \qquad \frac{d}{dt}\left[\frac{f(t)}{g(t)}\right] = \frac{1}{g(t)}\frac{df(t)}{dt} - \frac{f(t)}{g^2(t)}\frac{dg(t)}{dt}$$

You should also know the chain rule. Suppose $u(t)$ is a function of t and $f(u)$ is a function of u. Then, f depends on t through u and

$$\frac{df}{dt} = \frac{df}{du}\frac{du}{dt}$$

The chain rule was used above to find the derivatives of e^{At}, $\sin(At)$, and $\cos(At)$. In each case u is taken to be At.

You should be able to interpret the slope of a line tangent to a curve as a derivative. The instantaneous velocity is the slope of the coordinate as a function of time, the instantaneous acceleration is the slope of the velocity as a function of time.

Quadratic equations.

You should be able to solve algebraic equations that are quadratic in the unknown. If $At^2 + Bt + C = 0$, then

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

When the quantity under the radical sign does not vanish, there are two solutions. Always examine both to see what physical significance they have, then decide which is required to answer the particular problem you are working. If the quantity under the radical sign is negative, the solutions are complex numbers and probably have no physical significance for problems in this course. Check to be sure you have not made a mistake.

GENERAL PRINCIPLES

Kinematics describes motion in terms of position, velocity, and acceleration.

General kinematic relationships are given **mathematically** by:

Instantaneous velocity $v_s = ds/dt =$ slope of position graph

Instantaneous acceleration $a_s = dv_s/dt =$ slope of velocity graph

Final position $s_f = s_i + \int_{t_i}^{t_f} v_s\,dt = s_i + \begin{cases}\text{area under the velocity}\\ \text{curve from } t_i \text{ to } t_f\end{cases}$

Final velocity $v_{fs} = v_{is} + \int_{t_i}^{t_f} a_s\,dt = v_{is} + \begin{cases}\text{area under the acceleration}\\ \text{curve from } t_i \text{ to } t_f\end{cases}$

Solving Kinematics Problems

MODEL Uniform motion or constant acceleration.

VISUALIZE Draw a pictorial representation.

SOLVE

- Uniform motion $s_f = s_i + v_s\Delta t$
- Constant acceleration $v_{fs} = v_{is} + a_s\Delta t$

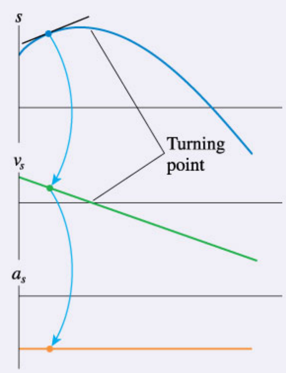
$$s_f = s_i + v_s\Delta t + \tfrac{1}{2}a_s(\Delta t)^2$$
$$v_{fs}^2 = v_{is}^2 + 2a_s\Delta s$$

ASSESS Is the result reasonable?

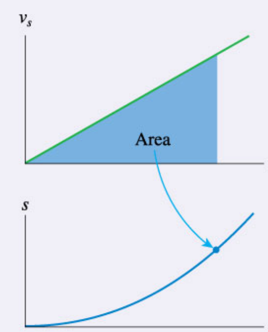
IMPORTANT CONCEPTS

Position, velocity, and acceleration are related graphically.

- The slope of the position-versus-time graph is the value on the velocity graph.
- The slope of the velocity graph is the value on the acceleration graph.
- s is a maximum or minimum at a turning point, and $v_s = 0$.



- Displacement is the area under the velocity curve.



APPLICATIONS

The **sign of v_s** indicates the direction of motion.

- $v_s > 0$ is motion to the right or up.
- $v_s < 0$ is motion to the left or down.

The **sign of a_s** indicates which way $\vec{a}$ points, *not* whether the object is speeding up or slowing down.

- $a_s > 0$ if $\vec{a}$ points to the right or up.
- $a_s < 0$ if $\vec{a}$ points to the left or down.
- The direction of $\vec{a}$ is found with a motion diagram.

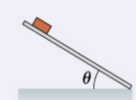
An object is **speeding up** if and only if v_s and a_s have the same sign.

An object is **slowing down** if and only if v_s and a_s have opposite signs.

Free fall is constant-acceleration motion with

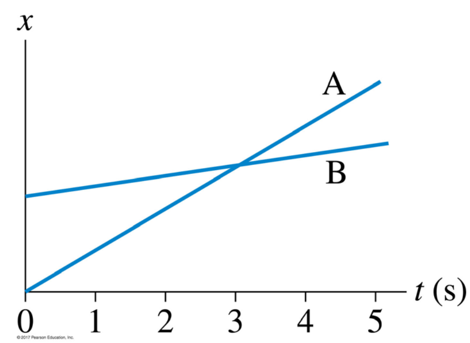
$$a_y = -g = -9.80\text{ m/s}^2$$

Motion on an inclined plane has $a_s = \pm g\sin\theta$. The sign depends on the direction of the tilt.

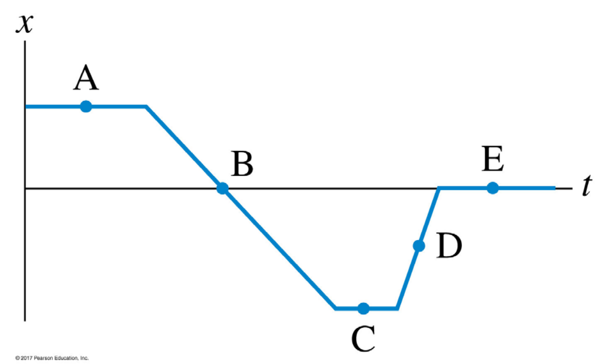


Conceptual Questions and Example Problems from Chapter 2

Conceptual Question 2.4
The figure to the right shows a position-versus-time graph for the motion of objects A and B as they move along the same axis. **(a)** At the instant t = 1 s, is the speed of A greater than, less than, or equal to the speed of B? *Explain*. **(b)** Do objects A and B ever have the same speed? If so, at what time or times? *Explain*.

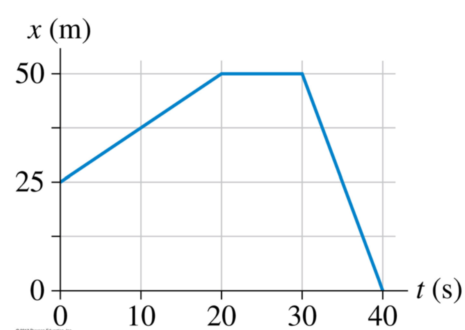


Conceptual Question 2.6
The figure below shows the position-versus-time graph for a moving object. At which letter point or points: **(a)** Is the object moving the slowest? **(b)** Is the object moving the fastest? **(c)** Is the object at rest? **(d)** Is the object moving to the left?

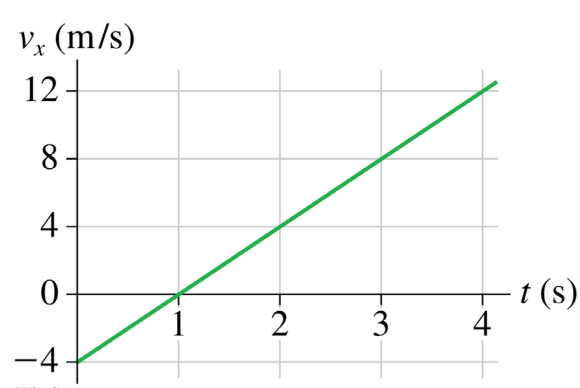


Problem 2.1
Alan leaves Los Angeles at 8:00 AM to drive to San Francisco, 400 mi away. He travels at a steady 50 mph. Beth leaves Los Angeles at 9:00 AM and drives a steady 60 mph.
(a) Who gets to San Francisco first? **(b)** How long does the first to arrive have to wait for the second?

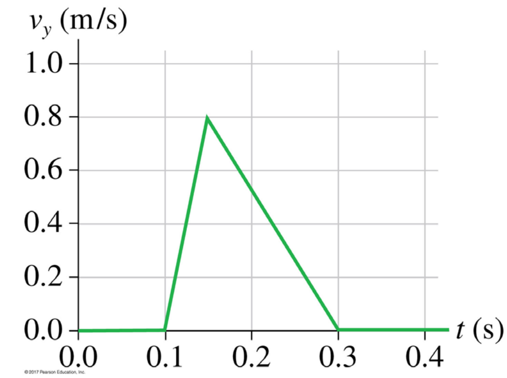
Problem 2.4
The figure to the right is the position-versus-time graph of a jogger. What is the jogger's velocity at t = 10 s, at t = 25 s, and at t = 35 s?



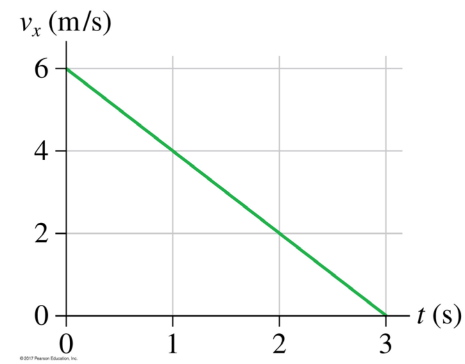
Problem 2.6
A particle starts from $x_0 = 10$ m at $t_0 = 0$ s and moves with the velocity graph shown to the right.
(a) Does this particle have a turning point? If so, at what time? **(b)** What is the object's position at t = 2s and 4 s?



Problem 2.7 and 2.10
The figure to the right is a somewhat idealized graph of the velocity of blood in the ascending aorta during one beat of the heart. Approximately how far, in cm, does the blood move during one beat? **(a)** Approximately how far, in cm, did the blood travel during this time interval? **(b)** What is the blood's acceleration during each phase of the motion, speeding up and slowing down?



Problem 2.11
The figure below shows the velocity graph of a particle moving along the x-axis. Its initial position is $x_0 = 2.0$ m at $t_0 = 0$ s. At t = 2.0 s, what are the particle's **(a)** position, **(b)** velocity, and **(c)** acceleration?

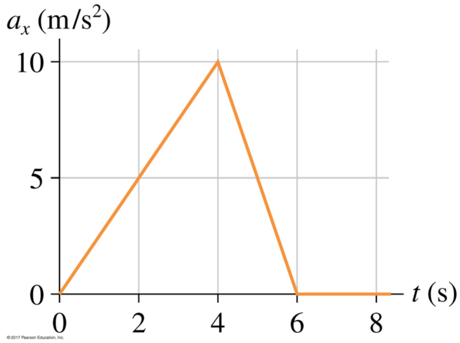


Problem 2.18
A Porsche challenges a Honda to a 400 m race. Because the Porsche's acceleration is 3.50 m/s^2 is larger than the Honda's 3.00 m/s^2, the Honda gets a 1.00 s head start. Who wins? By how many seconds?

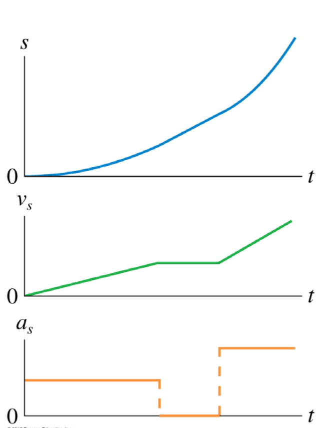
Problem 2.21
A student standing on the ground throws a ball straight up. The ball leaves the student's hand with a speed of 15 m/s when the hand is 2.0 m above the ground. How long is the ball in the air before it hits the ground. (The student moves out of the way.)

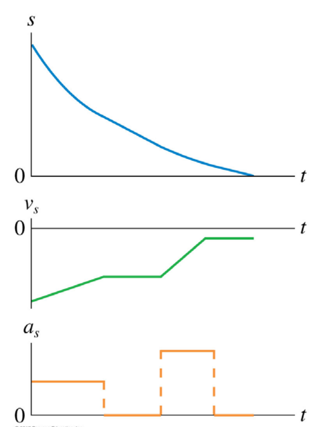
Problem 2.30
A small child gives a plastic frog a big push at the bottom of a slippery 2.0-m-long, 1.0-m-high ramp, starting it with a speed of 5.0 m/s. What is the frog's speed as it flies off the top of the ramp?

Problem 2.32
The figure below shows the acceleration graph for a particle that starts from rest at t = 0 s. What is the particle's velocity at t = 6 s?

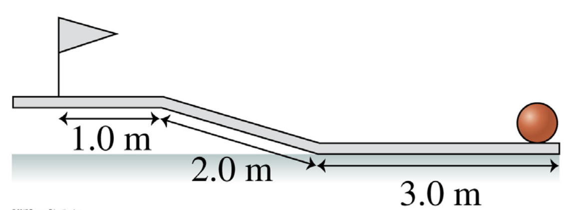


Problem 2.45 and 2.46
The figures below show a set of kinematics graphs for a ball rolling on a track. All segments of the track are straight lines. Draw a picture of the track and also indicate the ball's initial condition.





Problem 2.51
You are playing miniature golf at the golf course shown in the figure below. Due to the fake plastic grass, the ball decelerates at 1.0 m/s^2 when rolling horizontally and at 6.0 m/s^2 on the slope. What is the slowest speed with which the ball can leave your golf club if you wish to make a hole in one?



Problem 2.57
A lead ball is dropped into a lake from a diving board 5.00 m above the water. After entering the water, it sinks to the bottom with a constant velocity equal to the velocity with which it hit the water. The ball reaches the bottom 3.00 s after it is released. How deep is the lake?

Problem 2.68
Davis is driving at a steady 30 m/s when he passes Tina, who is sitting in her car at rest. Tina begins to accelerate at a steady 2.0 m/s^2 at the instant when David passes. **(a)** How far does Tina drive before passing David? **(b)** What is her speed as she passes him?

Problem 2.71
I was driving along at 20 m/s, trying to change a CD and not watching where I was going. When I looked up, I found myself 45 m from a railroad crossing. And wouldn't you know it, a train moving at 30 m/s was only 60 m from the crossing. In a split second, I realized that the train was going to beat me to the crossing and that I didn't have enough distance to stop. My only hope was to accelerate enough to cross the tracks before the train arrived. If my reaction time before starting to accelerate was 0.50 s, what minimum acceleration did my car need for me to be here today writing these words?

Problem 2.A
The position of a particle moving along the x axis is given in centimeters by $x = 9.75 + 1.50t^3$, where t is in seconds. Calculate **(a)** the average velocity during the time interval t = 2.00 s to t = 3.00 s; **(b)** the instantaneous velocity at t = 2.00 s; **(c)** the instantaneous velocity when the particle is midway between its positions at t = 2.00 s and t = 3.00 s.

Problem 2.B
The position of a particle moving along the x axis depends on the time according to the equation $x = ct^2 - bt^3$, where x is in meters and t in seconds. What are the units of **(a)** constant c and **(b)** constant b? Let their numerical values be 3.0 and 2.0, respectively. **(c)** What is the particle's average velocity between t = 1.0 s and t = 3.5 s? **(d)** At what time does the particle reach its maximum positive x position? **(e)** What is the particle's acceleration at t = 2.50 s?

Problem 2.C
A model rocket fired vertically from the ground ascends with a constant vertical acceleration of 4.00 m/s^2 for 6.00 s. Its fuel is then exhausted, so it continues upward as a free-fall particle and then falls back down. **(a)** What is the maximum altitude reached? **(b)** What is the total time elapsed from takeoff until the rocket strikes the ground?