Physics 4A
Chapter 3: Vectors and Coordinate Systems

"The only thing in life that is achieved without effort is failure." – Francis of Assisi

"We are what we repeatedly do. Excellence, therefore, is not an act, but a habit." – Aristotle

"Act as if what you do makes a difference, because it does." – William James

Reading: pages 65 – 76

Outline:

⇒ vectors and scalars (PowerPoint)
⇒ adding vectors graphically (PowerPoint)
⇒ components of vectors
SOHCAHTOA
⇒ unit vectors
⇒ adding vectors by components

Problem Solving

All of the problems at the end of the chapter deal with vector manipulations: addition, subtraction, finding components, and finding the magnitude and direction. The vectors are given either in terms of magnitude and direction or in terms of components; answers may be requested in either of these forms. This means you may need to convert from the given form to a form suitable for the vector operation, then convert again to obtain the form required for the answer.

If a is the magnitude of a vector $\vec{a}$ in the xy plane and θ is the angle that the vector makes with the positive x axis, then the components of $\vec{a}$ are $a_x = a\cos\theta$ and $a_y = a\sin\theta$. If θ is not given with respect to the positive x-axis, you will need to use trigonometry to find the components. the ang If you know the components, you can find the magnitude and the angle with the positive x axis. The magnitude is given by

$$a = \sqrt{a_x^2 + a_y^2}$$

and the angle is given by $\theta = \arctan\left(\dfrac{a_y}{a_x}\right)$

WARNING! For any values of a_x and a_y the equation $\theta = \arctan(a_y/a_x)$ has two solutions for θ. If you use a calculator to evaluate θ, it will only give you an angle in the 1st or 4th quadrant. If you know that the angle must be in the 2nd or 3rd quadrant (because of the signs of the x- and y-coordinates), then you must add 180° to the angle given by the calculator.

The basic prescription for vector addition is: if $\vec{c} = \vec{a} + \vec{b}$, then $c_x = a_x + b_x$, $c_y = a_y + b_y$, and $c_z = a_z + b_z$. The basic prescription for vector subtraction is: if $\vec{c} = \vec{a} - \vec{b}$, then $c_x = a_x - b_x$, $c_y = a_y - b_y$, and $c_z = a_z - b_z$.

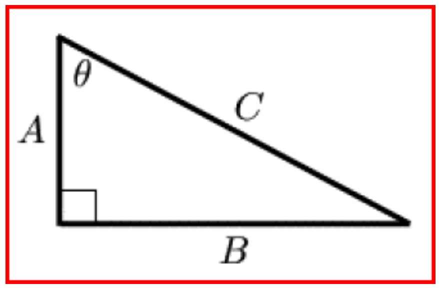
You must also know how to carry out the multiplication of a vector by a scalar.

Mathematical Skills

Trigonometry is a large portion of the mathematics used in this chapter. Here is a listing of the important elements you should know very well.

The Pythagorean theorem

The square of the hypotenuse of a right triangle equals the sum of squares of the other two sides. In the diagram $C^2 = A^2 + B^2$. The theorem is true only if the triangle contains a right angle (90°). The theorem is used, for example, to calculate the magnitude of a vector in the *xy* plane given its *x* and *y* components.



Trigonometric functions.

For the triangle shown, $A = C\cos\theta$ and $B = C\sin\theta$. The relations follow directly from the definition of the sine and cosine and are used to find the components of a vector, given the magnitude and the angle it makes with an axis. Also know that for the triangle above $\tan\theta = B/A$. This relationship, in the form $\theta = \arctan(a_y/a_x)$, is used to find the angle a vector makes with a coordinate axis.

WARNING! For any values of a_x and a_y the equation $\theta = \arctan(a_y/a_x)$ has two solutions for θ. If you use a calculator to evaluate θ, it will only give you an angle in the 1st or 4th quadrant. If you know that the angle must be in the 2nd or 3rd quadrant (because of the signs of the x- and y-coordinates), then you must add 180° to the angle given by the calculator.

Memorize these special values of the trigonometric functions:

cos(0) = 1	cos(90°) = 0	cos(180°) = -1	cos(270°) = 0
sin(0) = 0	sin(90°) = 1	sin(180°) = 0	sin(270°) = -1
tan(0) = 0	tan(90°) = ∞	tan(180°) = 0	tan(270°) = -∞

Take special care that you don't get them confused.

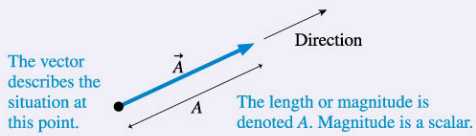
Trigonometric identities.

You should also be familiar with the following trigonometric identities:

$\sin(-A) = -\sin A$ $\quad$ $\cos(-A) = \cos A$

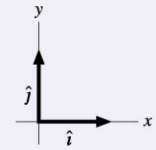
IMPORTANT CONCEPTS

A vector is a quantity described by both a magnitude and a direction.



Unit Vectors

Unit vectors have magnitude 1 and no units. Unit vectors $\hat{\imath}$ and $\hat{\jmath}$ define the directions of the x- and y-axes.



USING VECTORS

Components

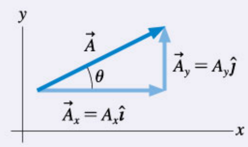
The component vectors are parallel to the x- and y-axes:

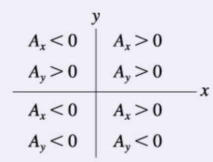
$$\vec{A} = \vec{A}_x + \vec{A}_y = A_x\hat{\imath} + A_y\hat{\jmath}$$

In the figure at the right, for example:

$$A_x = A\cos\theta \qquad A = \sqrt{A_x^2 + A_y^2}$$
$$A_y = A\sin\theta \qquad \theta = \tan^{-1}(A_y/A_x)$$

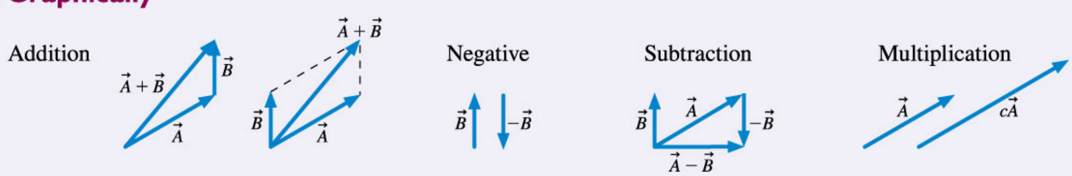
► Minus signs need to be included if the vector points down or left.





The components A_x and A_y are the magnitudes of the component vectors $\vec{A}_x$ and $\vec{A}_y$ *and* a plus or minus sign to show whether the component vector points toward the positive end or the negative end of the axis.

Working Graphically



Working Algebraically

Vector calculations are done component by component: $\vec{C} = 2\vec{A} + \vec{B}$ means $\begin{cases} C_x = 2A_x + B_x \\ C_y = 2A_y + B_y \end{cases}$

The magnitude of $\vec{C}$ is then $C = \sqrt{C_x^2 + C_y^2}$ and its direction is found using $\tan^{-1}$.

Questions and Example Problems from Chapter 3

Conceptual Question 3.8
Suppose two vectors have unequal magnitudes. Can their sum be zero? Explain.

Problem 3.4
A velocity vector 40° below the positive x-axis has a y-component of -10 m/s. What is the value of its x-component?

Problem 3.7
Draw each of the following vectors. Then find its x- and y-components.

a) $\vec{v}$ = (3.5 m/s, negative x-direction)
b) $\vec{a}$ = (1.5 m/s^2, 30° above the negative x-axis)
c) $\vec{F}$ = (50.0 N, 36.9° counterclockwise from the positive y-axis)

Problem 3.10
Draw each of the following vectors, label an angle that specifies the vector's direction, then find its magnitude and direction.

a) $\vec{B} = -4\hat{\imath} + 4\hat{\jmath}$
b) $\vec{r} = (-2.0\hat{\imath} - 1.0\hat{\jmath})$ cm
c) $\vec{v} = (-10\hat{\imath} - 100\hat{\jmath})$ m/s
d) $\vec{a} = (20\hat{\imath} + 10\hat{\jmath})$ m/s^2

Problem 3.14
Let $\vec{A} = 4\hat{i} - 2\hat{j},\ \vec{B} = -3\hat{i} + 5\hat{j},\ and\ \vec{D} = \vec{A} - \vec{B}$.

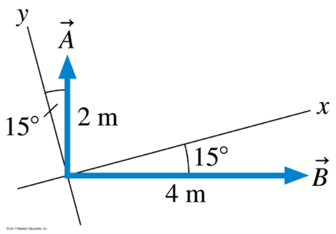
a) Write vector $\vec{D}$ in component form.
b) Draw a coordinate system and on it show vectors $\vec{A},\ \vec{B},\ and\ \vec{D}$.
c) What are the magnitude and direction of vector $\vec{D}$?

Problem 3.16
Let $\vec{A} = 4\hat{i} - 2\hat{j},\ \vec{B} = -3\hat{i} + 5\hat{j},\ and\ \vec{F} = \vec{A} - 4\vec{B}$.

a) Write vector $\vec{F}$ in component form.
b) Draw a coordinate system and on it show vectors $\vec{A},\ \vec{B},\ and\ \vec{F}$.
c) What are the magnitude and direction of vector $\vec{F}$?

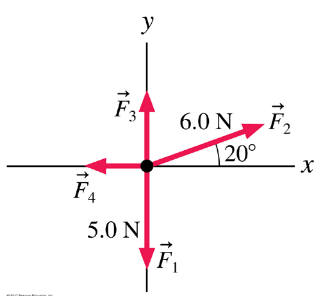
Problem 3.26

The figure below shows vectors $\vec{A}$ *and* $\vec{B}$. Find $\vec{D} = 2\vec{A} + \vec{B}$. Write your answer in component form.



Problem 3.44

Four forces are exerted on the object shown in the figure below. (Forces are measured in *Newtons*, abbreviated N.) The net force on the object is $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 4.0\hat{i}N$. What are (a) $\vec{F}_3$ and (b) $\vec{F}_4$? Give your answers in component form.



Problem A

For the vectors $\vec{a} = (3.0\,m)\hat{i} + (4.0\,m)\hat{j}$ and $\vec{b} = (5.0\,m)\hat{i} + (-2.0\,m)\hat{j}$, give $\vec{a} + \vec{b}$ in (a) unit-vector notation, and as (b) a magnitude and (c) an angle (relative to $\hat{i}$). Now give $\vec{b} - \vec{a}$ in (d) unit-vector notation, and as (e) magnitude and (f) an angle.

Problem B

Find the sum of the following four vectors in (a) unit-vector notation, and as (b) a magnitude and (c) an angle relative to +*x*.

$\vec{P}$: 10.0 m, at 25.0° counterclockwise from +x

$\vec{Q}$: 12.0 m, at 10.0° counterclockwise from +y

$\vec{R}$: 8.00 m, at 20.0° clockwise from -y

$\vec{S}$: 9.00 m, at 40.0° counterclockwise from -y