Physics 4A
Chapters 9: Work and Kinetic Energy

"Remember happiness doesn't depend upon who you are or what you have; it depends solely on what you think." – Dale Carnegie

"Most folks are about as happy as they make up their minds to be." – Abraham Lincoln

"A ship in the harbor is safe, but that is not what ships are built for." – William Shedd

"You miss 100% of the shots that you don't take." – Wayne Gretsky

Reading: pages 206 – 226

Outline:

⇒ energy overview
    basic energy model
⇒ work and kinetic energy
    kinetic energy
    work
    work done by a constant force
    scalar or dot product
    work done by a varying force
⇒ dissipative forces and thermal energy
⇒ power

Problem Solving

You should know how to calculate the work done by a force if the force is constant or if its component along the path is given as a function of the object's position. In some cases you might need to solve a Newton's second law equation to find the force. In other cases the force as a function of position might be given as a graph and you should be able to obtain the work from the area under the curve. The force in question might be the only force acting on an object or one of several.

The work-kinetic energy theorem tells us that the net work W done on a particle is equal to the change in the kinetic energy of the particle. That is, $W = \Delta K$, or since the kinetic energy is given by $\frac{1}{2} mv^2$, $W = \frac{1}{2} m(v^2_f - v^2_i)$. Here m is the mass of the particle, v_i is its speed at the beginning of the interval, and v_f is its speed at the end of the interval.

In some problems you are asked to find the net work done on an object, given its initial and final speeds (and its mass). This is a direct application of the work-kinetic energy theorem. In other problems you will use the definition of work to calculate its value, given the force and displacement, then use the work-kinetic energy theorem to find the final speed, given the initial speed.

A spring provides a good example of a variable force. If one end is fixed and the other end is moved so the spring is either extended or compressed from its equilibrium length, then the force exerted by the spring is given by $F = -kx$, where k is the spring constant. The coordinate x of the spring end is measured with the origin at the position of the movable end when the spring has its equilibrium length. If the end of the spring is moved from x_i to x_f, the work done by the spring is $W = -\frac{1}{2}k(x_f^2 - x_i^2)$.

Some problems involve calculations of the power delivered by forces. In some cases you evaluate $P = \mathrm{d}W/\mathrm{d}t$ while in others you evaluate $P = \vec{F} \bullet \vec{v}$.

Mathematical Skills

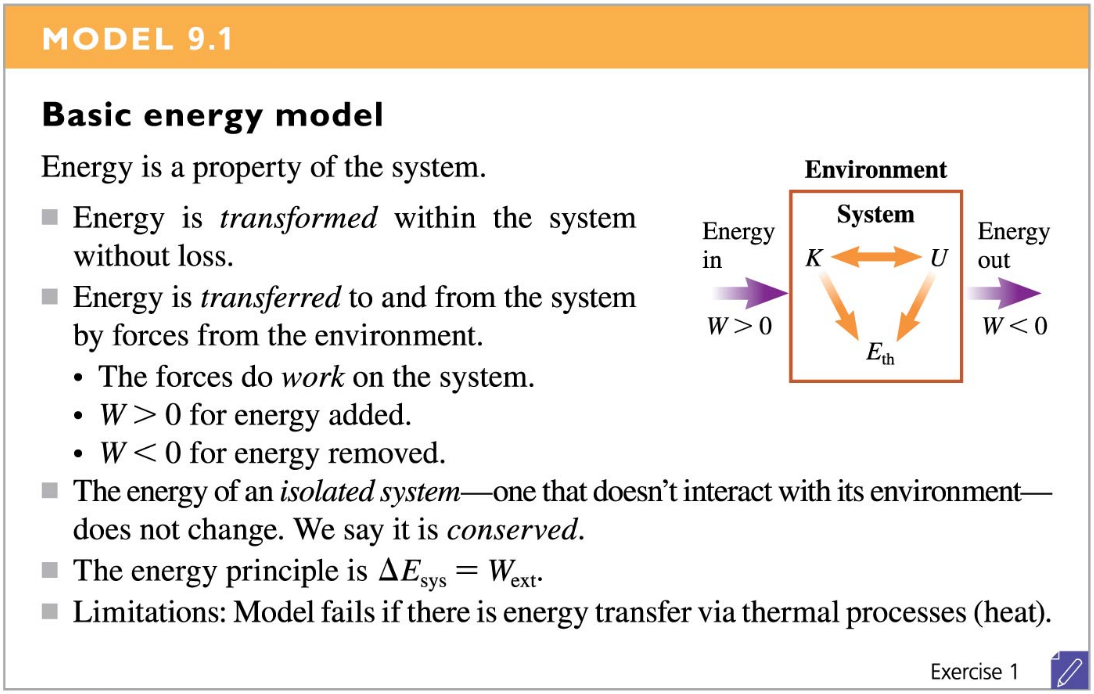
These are used extensively in this chapter to calculate the work done and the power delivered by a force: $W = \vec{F} \bullet \vec{d}$ for a constant force and $P = \vec{F} \bullet \vec{v}$ for any force. Be sure you know how to evaluate them. In particular know that

$$\vec{F} \bullet \vec{d} = Fd\cos\phi,$$

where ϕ is the angle between $\vec{F}$ and $\vec{d}$ when they are drawn with their tails at the same point. Also know how to evaluate a scalar product in terms of components:

$$\vec{F} \bullet \vec{d} = F_x d_x + F_y d_y + F_z d_z.$$

Review the appropriate sections of Chapter 3 of the text.

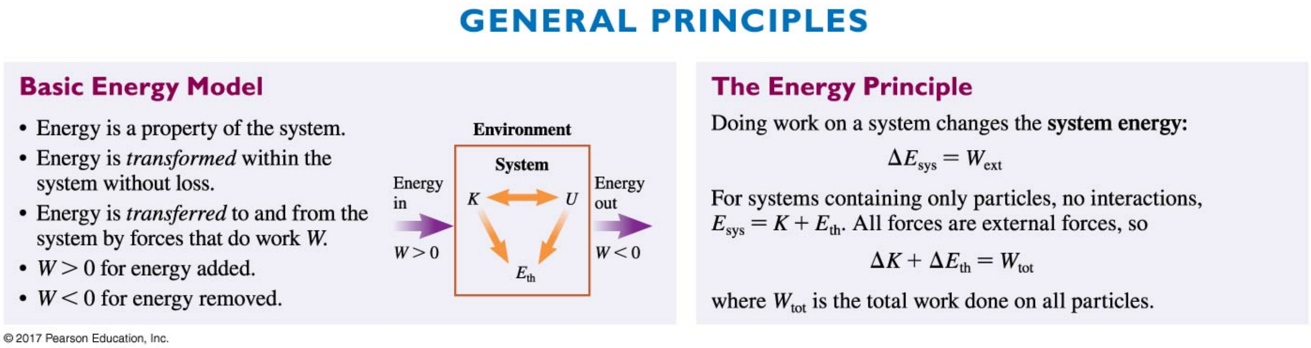


MODEL 9.1

Basic energy model

Energy is a property of the system.

- Energy is *transformed* within the system without loss.
- Energy is *transferred* to and from the system by forces from the environment.
  - The forces do *work* on the system.
  - $W > 0$ for energy added.
  - $W < 0$ for energy removed.
- The energy of an *isolated system*—one that doesn't interact with its environment—does not change. We say it is *conserved*.
- The energy principle is $\Delta E_{\text{sys}} = W_{\text{ext}}$.
- Limitations: Model fails if there is energy transfer via thermal processes (heat).



GENERAL PRINCIPLES

Basic Energy Model

- Energy is a property of the system.
- Energy is *transformed* within the system without loss.
- Energy is *transferred* to and from the system by forces that do work W.
- $W > 0$ for energy added.
- $W < 0$ for energy removed.

The Energy Principle

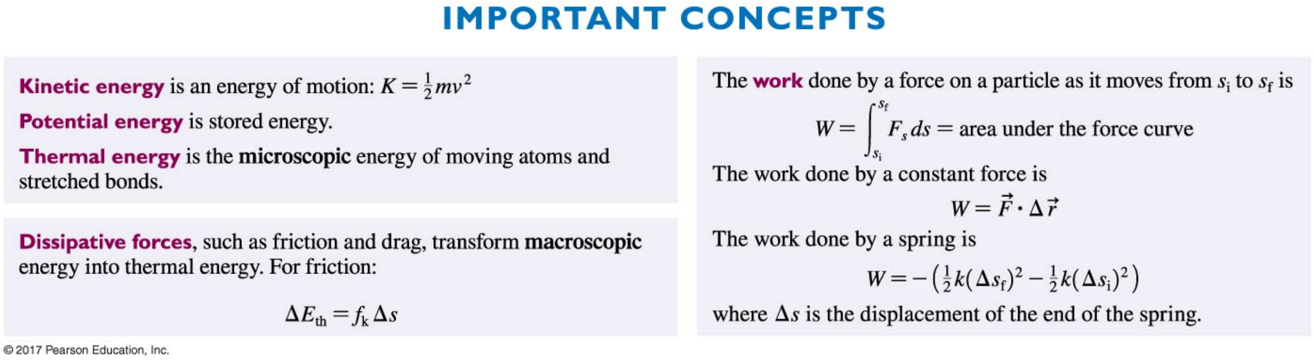
Doing work on a system changes the **system energy:**

$$\Delta E_{\text{sys}} = W_{\text{ext}}$$

For systems containing only particles, no interactions, $E_{\text{sys}} = K + E_{\text{th}}$. All forces are external forces, so

$$\Delta K + \Delta E_{\text{th}} = W_{\text{tot}}$$

where W_{tot} is the total work done on all particles.



IMPORTANT CONCEPTS

Kinetic energy is an energy of motion: $K = \frac{1}{2}mv^2$

Potential energy is stored energy.

Thermal energy is the **microscopic** energy of moving atoms and stretched bonds.

Dissipative forces, such as friction and drag, transform **macroscopic** energy into thermal energy. For friction:

$$\Delta E_{\text{th}} = f_k \Delta s$$

The **work** done by a force on a particle as it moves from s_i to s_f is

$$W = \int_{s_i}^{s_f} F_s\, ds = \text{area under the force curve}$$

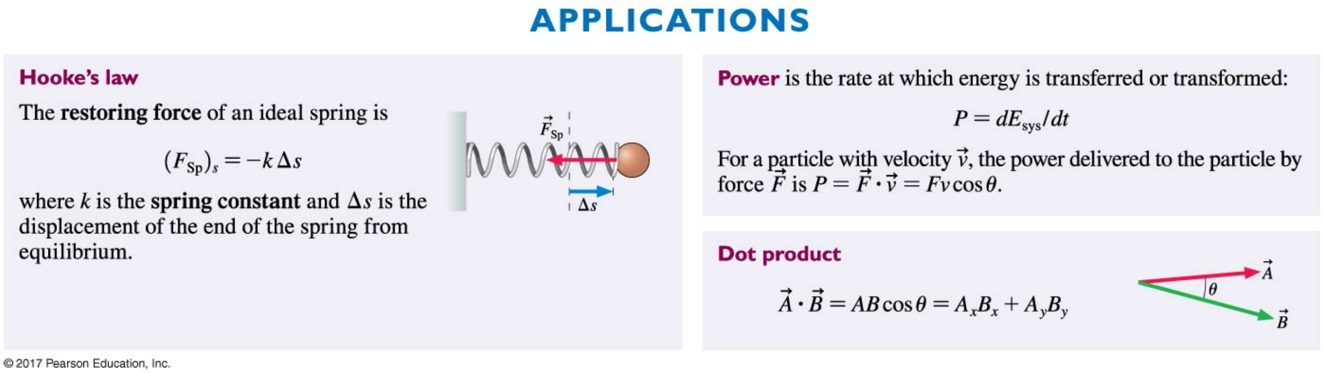
The work done by a constant force is

$$W = \vec{F} \cdot \Delta\vec{r}$$

The work done by a spring is

$$W = -\left(\tfrac{1}{2}k(\Delta s_f)^2 - \tfrac{1}{2}k(\Delta s_i)^2\right)$$

where Δs is the displacement of the end of the spring.



APPLICATIONS

Hooke's law

The **restoring force** of an ideal spring is

$$(F_{\text{Sp}})_s = -k\,\Delta s$$

where k is the **spring constant** and Δs is the displacement of the end of the spring from equilibrium.

Power is the rate at which energy is transferred or transformed:

$$P = dE_{\text{sys}}/dt$$

For a particle with velocity $\vec{v}$, the power delivered to the particle by force $\vec{F}$ is $P = \vec{F} \cdot \vec{v} = Fv\cos\theta$.

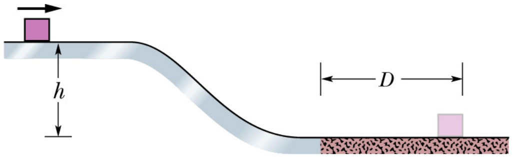
Dot product

$$\vec{A} \cdot \vec{B} = AB\cos\theta = A_xB_x + A_yB_y$$

Question and Example Problems from Chapter 9

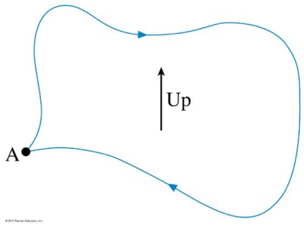
Conceptual Question 9.A

In the figure below, a block slides along a track that descends through distance h. The track is frictionless except for the lower section. There the block slides to a stop in a certain distance D because of friction. **(a)** If we decrease h, will the block now slide to a stop in a distance that is greater than, less than, or equal to D? **(b)** If instead, we increase the mass of the block, will the stopping distance now be greater than, less than, or equal to D?



Conceptual Question 9.3
An elevator held by a single cable is descending but slowing down. Is the work done by tension positive, negative, or zero? What about the work done by gravity? *Explain.*

Conceptual Question 9.7
A particle moves in a vertical plane along the closed path seen in the figure, starting at A and eventually returning to its starting point. Is the work done by gravity positive, negative, or zero.



Conceptual Question 9.8
A need to raise a heavy block by pulling it with a massless rope. You can either **(a)** pull the block straight up height h, or **(b)** pull it up a long, frictionless plane inclined at a 15 angle until its height has increased by h. Assume you will move the block at constant speed either way. Will you do more work in case a or case b? Or is the work the same in both cases? Explain.

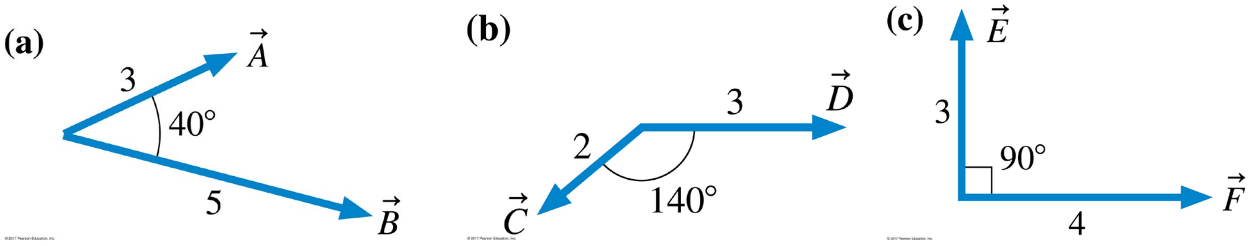
Problem 9.7
A 20 g particle is moving to the left at 30 m/s. A force on the particle causes it to move to the right at 30 m/s. How much work is done by the force?

Problem 9.10
The cable of a crane is lifting a 750 kg girder. The girder increases its speed from 0.25 m/s to 0.75 m/s in a distance of 3.5 m. **(a)** How much work is done by gravity? **(b)** How much work is done by tension?

Problem 9.11
Evaluate the dot product $\vec{A}\bullet\vec{B}$ if
(a) $\vec{A}=4\hat{i}-2\hat{j}\ \textit{and}\ \vec{B}=-2\hat{i}-3\hat{j}$
(b) $\vec{A}=4\hat{i}+2\hat{j}\ \textit{and}\ \vec{B}=2\hat{i}+3\hat{j}$

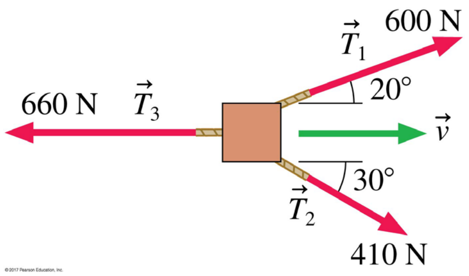
Problem 9.15

Evaluate the dot product of the three pairs of vectors in the figure below.



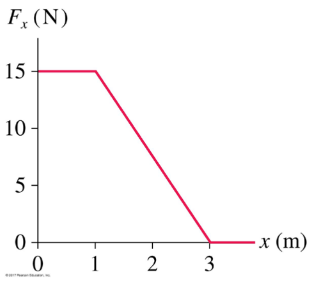
Problem 9.19

The three ropes shown in the bird's-eye view of the figure below are used to drag a crate 3.0 m across the floor. How much work is done by each of the three forces?



Problem 9.21

A 500 g particle moving along the x-axis experiences the force shown in the figure below. The particle's velocity is 2.0 m/s at x = 0 m. What is its velocity at x = 3 m?



Problem 9.28

A 60 kg student is standing atop a spring in an elevator as it accelerates upwards at 3.0 m/s^2. The spring constant is 2500 N/m. By how much is the spring compressed?

Problem 9.33
A baggage handler throws a 15 kg suitcase along the floor of an airplane luggage compartment with a speed of 1.2 m/s. The suitcase slides 2.0 m before stopping. Use work and energy to find the suitcase's coefficient of kinetic friction on the floor.

Problem 9.40
A 50 kg sprinter, starting from rest, runs 50 m in 7.0 s at constant acceleration. **(a)** What is the magnitude of the horizontal force acting on the sprinter? **(b)** What is the sprinter's power output at 2.0 s, 4.0 s, and 6.0 s?

Problem 9.49
A 50-kg ice skater is gliding along the ice, heading due north at 4.0 m/s. The ice has a small coefficient of static friction, to prevent the skater from slipping sideways, but $\mu_k = 0$. Suddenly, a wind from the northeast exerts a force of 4.0 N on the skater. **(a)** Use work and energy to find the skater's speed after gliding 100 m in this wind. **(b)** What is the minimum value of μ_s that allows her to continue moving straight north?

Problem 9.60
A 90 kg firefighter needs to climb the stairs of a 20-m-tall building while carrying a 40 kg backpack filled with gear. How much power does he need to reach the top in 55 s?

Problem 9.A
In the figure below, a horizontal force $\vec{F}_a$ of magnitude 20.0 N is applied to a 3.00 kg psychology book as the book slides a distance d = 0.500 m up a frictionless ramp at angle $\theta = 30.0°$. (a) During the displacement, what is the net work done on the book by $\vec{F}_a$, the gravitational force on the book, and the normal force on the book? (b) If the book has zero kinetic energy at the start of the displacement, what is its speed at the end of the displacement?