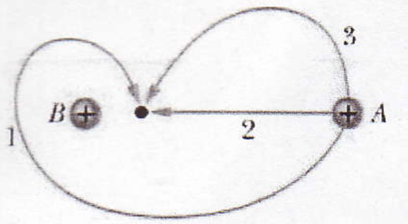
Answer Key

Questions and Example Problems from Chapter 24

Question 1

The figure below shows three paths along which we can move positively charged sphere A closer to positively charged sphere B, which is fixed in place. (a) Would sphere A be moved to a higher or lower electric potential? Is the work done (b) by our force and (c) by the electric field (due to the second sphere) positive, negative, or zero? (d) Rank the paths according to the work our force does, greatest first.



(a) $\boxed{\text{higher potential}}$

(b) $\boxed{\text{positive}}$

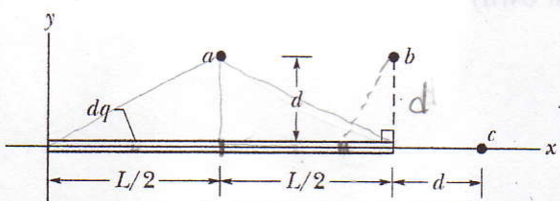
(c) $\boxed{\text{negative}}$

(d) $\boxed{\text{all equal}}$ → $\vec{E}$ force is conservative

Question 2

The figure below shows a thin, uniformly charged rod and three points at the same distance d from the rod. Without calculation, rank the magnitude of the electric potential the rod produces at these three points, greatest first.

$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$



$\boxed{A, B, C}$

$V_A > V_B$ since charge is closer to point A than point B

$V_B > V_C$ since charge is closer to point B than point C

Problem 1

A particle of charge q is fixed at point P, and a second particle of mass m and the same charge q is initially held a distance r_1 from P. The second particle is then released. Determine its speed when it is a distance r_2 from P. Let q = 3.1 μC, m = 20 mg, r_1 = 0.90 mm, and r_2 = 2.5 mm.

P $\underset{q}{\bullet}\xrightarrow{r_i}\underset{q}{\bullet}$ $\underset{q}{\bullet}\xleftarrow{r_2}\underset{q}{\bullet}\xrightarrow{v_f}$

$v_i = 0$

$r_i = r_1 = 0.90\times10^{-3}\,m$

$v_f = ?$

$r_f = r_2 = 2.5\times10^{-3}\,m$

$q = 3.1\times10^{-6}\,C$

$m = 20\times10^{-6}\,kg$

from conservation of energy:

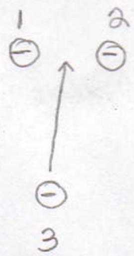
$$\underbrace{K_i}_{=0} + U_i = K_f + U_f \rightarrow K_f = U_i - U_f$$

$$\frac{1}{2}mv_f^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_i} - \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_f}$$

$$\frac{1}{2}mv_f^2 = \frac{q^2}{4\pi\varepsilon_0}\left(\frac{1}{r_i} - \frac{1}{r_f}\right) \rightarrow v_f^2 = \frac{2q^2}{4\pi\varepsilon_0 m}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$v_f = \sqrt{\frac{(8.99\times10^9\,Nm^2/C^2)\,2\,(3.1\times10^{-6}C)^2}{(20\times10^{-6}\,kg)}\left(\frac{1}{0.90\times10^{-3}m} - \frac{1}{2.5\times10^{-3}\,kg}\right)} = \boxed{2.5\times10^3\,m/s}$$

Problem 2
Two electrons are fixed 2.0 cm apart. Another electron is shot from infinity and stops midway between the two. What is its initial speed?



initial $\longrightarrow$ $V_{1i} = V_{2i} = 0 \quad V_{3i} = ?$

final $\longrightarrow$ $V_{1f} = V_{2f} = V_{3f} = 0$

from conservation of energy $\longrightarrow$ $K_f + U_f = K_i + U_i$; $K_f = 0$, $K_i = \frac{1}{2}mV_3^2$

$U_f = \frac{1}{2}mV_3^2 + U_i \qquad U_i = U_{12}$

$U_f = U_{12} + U_{13} + U_{23}$

$U_{13} + U_{23} = \frac{1}{2}mV_3^2$

$\frac{1}{2}mV_3^2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right) \qquad q_1 = q_2 = q_3 = -e = -1.602\times10^{-19}C$

$r_{12} = r_{23} = 1.0\,cm$

$m = 9.11\times10^{-31}\,Kg$

$V_3 = \left[\frac{2}{4\pi\varepsilon_0 m}\,2\left(\frac{e^2}{r}\right)\right]^{1/2} \longrightarrow \boxed{V_i = V_3 = 3.2\times10^2\ m/s}$

Problem 3
The electric potential difference between the ground and a cloud in a particular thunderstorm is 1.2×10^9 V. What is the magnitude of the change in the electric potential energy (in multiples of the electron-volt) of an electron that moves between the ground and the cloud?

$\Rightarrow$ electrons accelerate from lower to higher potential

$\Delta V = 1.2\times10^9\,V$ * if the charge is expressed in e and the potential in V, then energy is in eV

$q_0 = -1.602\times10^{-19}C = -e$

$\Delta U = ?$

$\Delta V = \Delta U/q_0 \rightarrow \Delta U = q_0\,\Delta V$

$= (-e)(1.2\times10^9 V)$

$= -1.2\times10^9\,eV$

$|\Delta U| = \boxed{1.2\times10^9\,eV = 1.2\,GeV}$

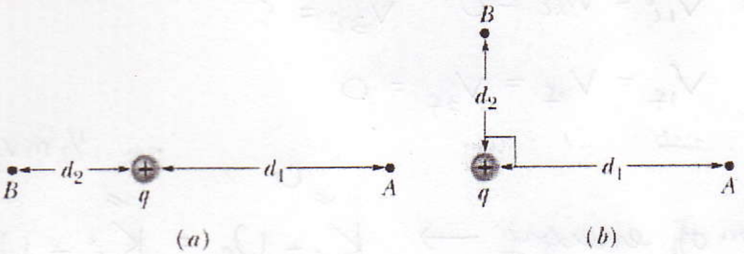
or

$\Delta U = (-1.602\times10^{-19}C)(1.2\times10^9\,V) = -1.92\times10^{-11}\,J$

$|\Delta U| = 1.92\times10^{-11}\,J\left(\frac{1\,eV}{1.602\times10^{-19}J}\right) = \boxed{1.2\times10^9\,eV = 1.2\,GeV}$

Problem 4

Consider a point charge q = 1.0 μC, point A at distance d_1 = 2.0 m from q, and point B at distance d_2 = 1.0 m. (a) If A and B are diametrically opposite each other, as in Fig. a below, what is the electric potential difference $V_A - V_B$? (b) What is that electric potential difference if points A and B are located as in Fig. b?



for a point charge:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

(a) $$V_A - V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_A} - \frac{q}{r_B}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

$$V_A - V_B = (8.99\times10^9\,\mathrm{Nm^2/C^2})(1.0\times10^{-6}\,\mathrm{C})\left(\frac{1}{2.0\,\mathrm{m}} - \frac{1}{1.0\,\mathrm{m}}\right)$$

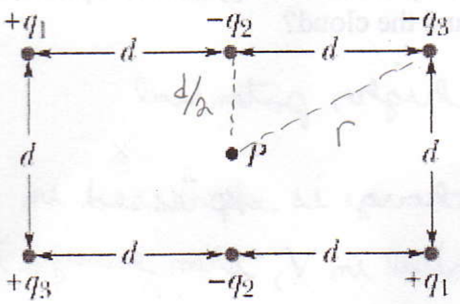
$$= \boxed{-4.5\times10^3\,\mathrm{V}}$$

(b) $V_A - V_B = \boxed{-4.5\times10^3\,\mathrm{V}}$ note: since potential is a scalar, $V_A - V_B$ is the same in each case

Problem 5

In the figure below, point P is at the center of the rectangle. With V = 0 at infinity, q_1 = 5.00 fC, q_2 = 2.00 fC, q_3 = 3.00 fC, and d = 2.54 cm, what is the net electric potential at P due to the six charged particles?

$$r = \sqrt{d^2 + (d/2)^2} \rightarrow r = \sqrt{5/4}\,d$$



for a collection of point charges:

$$V = \sum_i \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i} = \frac{1}{4\pi\varepsilon_0}\sum q_i/r_i$$

starting from left corner & going clockwise:

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{+q_1}{r} - \frac{q_2}{(d/2)} - \cancel{\frac{q_3}{r}} + \frac{q_1}{r} - \frac{q_2}{(d/2)} + \cancel{\frac{q_3}{r}}\right]$$

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q_1}{r} - \frac{2q_2}{(d/2)}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q_1}{r} - \frac{4q_2}{d}\right]$$

$$V = (8.99\times10^9\,\mathrm{Nm^2/C^2})\left[\frac{2(5.00\times10^{-15}\,\mathrm{C})}{\sqrt{5/4}(0.0254\,\mathrm{m})} - \frac{4(2.00\times10^{-15}\,\mathrm{C})}{(0.0254\,\mathrm{m})}\right]$$

$$\boxed{V = 3.34\times10^{-4}\,\mathrm{V}}$$

Problem 6

A long wire has a uniform linear charge density λ. What is the potential difference V_{ab} between two points a and b located radial distances r_a and $r_b = 2r_a$ from the wire?

for a long wire $\rightarrow E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

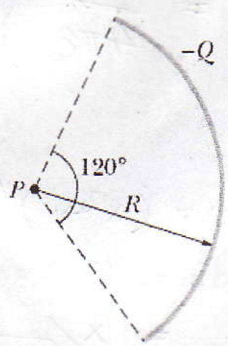
$$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s} \longrightarrow V_a - V_b = -\int_{r_b}^{r_a} \left(\frac{\lambda}{2\pi\varepsilon_0 r}\right) dr$$

$$V_a - V_b = \frac{-\lambda}{2\pi\varepsilon_0}\int_{r_a}^{r_b} \frac{dr}{r} = \frac{-\lambda}{2\pi\varepsilon_0} \ln r \Big|_{r_a}^{r_b}$$

$$= \frac{-\lambda}{2\pi\varepsilon_0}\underbrace{(\ln r_b - \ln r_a)}_{= \ln(r_b/r_a) = \ln 2} \Rightarrow \boxed{V_a - V_b = \frac{-\lambda}{2\pi\varepsilon_0}\ln 2}$$

Problem 7

The figure below, a plastic rod having a uniformly distributed charge Q = -25.6 pC has been bent into a circular arc of radius R = 3.71 cm and central angle 120°. With V = 0 at infinity, what is the electric potential at P, the center of curvature of the rod?



$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$$

note: r = R = 3.71 cm for all points on the rod so we can pull r out of the integral

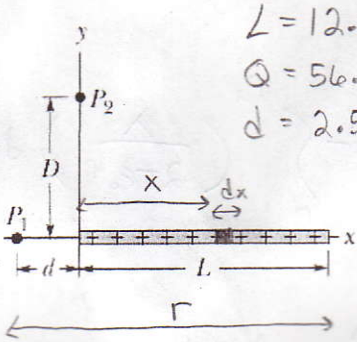
$$V = \frac{1}{4\pi\varepsilon_0 R}\int dq$$

$$\int dq = Q = -25.6\,pC$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = \frac{(8.99\times10^{9}\,Nm^2/C^2)(-25.6\times10^{-12}\,C)}{(0.0371\,m)}$$

$$\boxed{V = -6.20\,V}$$

Problem 8
The figure below shows a thin plastic rod of length L = 12.0 cm and uniform positive charge Q = 56.1 pC lying on an x axis. With V = 0 at infinity, find the electric potential at point P_1 on the axis, at distance d = 2.50 cm from one end of the rod.



$L = 12.0\times10^{-2}\,m$
$Q = 56.1\times10^{-12}\,C$
$d = 2.50\times10^{-2}\,m$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} \qquad dq = \lambda\,dx$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dx}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dx}{x+d}$$

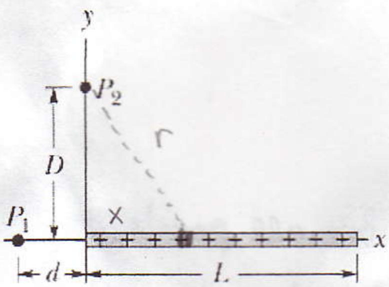
$$V = \int dV = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{x+d} = \frac{\lambda}{4\pi\varepsilon_0}\ln(x+d)\Big/_0^L$$

$$V = \frac{\lambda}{4\pi\varepsilon_0}\left[\ln(L+d)-\ln(d)\right] = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{L+d}{d}\right) \qquad \lambda = Q/L$$

$$V = \frac{(Q/L)}{4\pi\varepsilon_0}\ln\left(\frac{L+d}{d}\right) = (8.99\times10^9\,Nm^2/C^2)\left(\frac{56.1\times10^{-12}\,C}{12.0\times10^{-2}\,m}\right)\ln\left[\frac{14.5\times10^{-2}\,m}{2.5\times10^{-2}\,m}\right]$$

$$\boxed{V = 7.39\,V}$$

Problem 9
The thin plastic rod of length L = 10.0 cm in the figure below has a nonuniform linear charge density λ = cx, where c = 49.9 pC/m. (a) With V = 0 at infinity, find the electric potential at point P_2 on the y axis, a distance y = D = 3.56 cm. (b) Find the electric field component E_y at P_2. (c) Why cannot the field component E_x at P_2 be found using the result of (a)?



$$V = \int dV = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda\,dx}{r}$$

$$\lambda = cx \qquad r = \sqrt{x^2+y^2}$$

$$V = \frac{c}{4\pi\varepsilon_0}\int\frac{x\,dx}{\sqrt{x^2+y^2}} \qquad u = (x^2+y^2)^{1/2}$$
$$du = \tfrac{1}{2}(x^2+y^2)^{-1/2}(2x) = x/\sqrt{x^2+y^2}$$

$$V = \frac{c}{4\pi\varepsilon_0}\int du = \frac{c}{4\pi\varepsilon_0}(x^2+y^2)^{1/2}\Big/_0^L = \frac{c}{4\pi\varepsilon_0}\left(\sqrt{L^2+y^2}-y\right)$$

$$V = (49.9\times10^{-12}\,C/m)(8.99\times10^9\,Nm^2/C^2)\left[\sqrt{(0.10m)^2+(0.0356m)^2}-(0.0356m)\right] = \boxed{3.16\times10^{-2}\,V}$$

(b) $$E_y = -\frac{\partial V}{\partial y} = \frac{-c}{4\pi\varepsilon_0}\frac{\partial}{\partial y}\left(\sqrt{L^2+y^2}-y\right) = \frac{c}{4\pi\varepsilon_0}\left(1-\frac{y}{\sqrt{L^2+y^2}}\right) = \boxed{0.298\,N/C}$$

(c) We found V on the y axis.
To get E_x, we would need to find V(x,y) & then use $V_x = -\frac{\partial V}{\partial x}$

Problem 10

The electric potential V in the space between two flat parallel plates 1 and 2 is given (in volts) by $V = 1500x^2$, where x (in meters) is the perpendicular distance from plate 1. At $x = 1.3$ cm, (a) what is the magnitude of the electric field and (b) is the field directed toward or away from plate 1?

$V = 1500x^2 \rightarrow$ should be written as $V = (1500\ V/m^2)\, x^2$

$E_x = -\partial V/\partial x = -\partial/\partial x\,(1500\ V/m^2)\, x^2$

$E_x = -(3000\ V/m^2)\, x$

(a) at $x = 1.3$ cm, $|E_x| = (3000\ V/m^2)(1.3 \times 10^{-2}\ m)$

$|E_x| = \boxed{3.9 \times 10^1\ V/m}$

(b) V increases from plate 1 to plate 2; since $\vec{E}$ points from higher V to lower V, the field is directed downward towards plate 1 $(-\hat{i})$

Problem 11

What is the magnitude of the electric field at the point $(3.00\hat{i} - 2.00\hat{j} + 4.00\hat{k})m$ if the electric potential is given by $V = 2.00xyz^2$, where V is in volts and x, y, and z are in meters?

$E_x = -\partial V/\partial x \qquad E_y = -\partial V/\partial y \qquad E_z = -\partial V/\partial z$

$E_x = -\partial/\partial x\,(2.00xyz^2) = -2.00yz^2$

$E_y = -\partial/\partial y\,(2.00xyz^2) = -2.00xz^2$

$E_z = -\partial/\partial z\,(2.00xyz^2) = -4.00xyz$

for $x = 3.00$, $y = -2.00$, $z = 4.00$:

$E_x = -2.00(-2.00)(4.00)^2 = 64.0\ V/m$

$E_y = -2.00(3.00)(-4.00)^2 = -96.0\ V/m$

$E_z = -4.00(3.00)(-2.00)(4.00) = 96.0\ V/m$

$|\vec{E}| = \sqrt{(64.0\ V/m)^2 + (-96.0\ V/m)^2 + (96.0\ V/m)^2} = \boxed{150\ V/m}$