magnetic flux but there may be others, produced by batteries or generators, for example. Add them with their correct signs and divide the total by the resistance in the loop. Obviously, you must be very careful about the signs of the various emfs here.

Some problems ask for the electric field associated with a changing magnetic field. Use Faraday's law in the form $\oint \vec{E} \cdot d\vec{s} = -d\Phi/dt$. All of the problems have cylindrical symmetry, with the electric field lines forming circles around the cylinder axis. For them, you integrate the tangential component of the electric field around one of the field lines, with result $\oint \vec{E} \cdot d\vec{s} = 2\pi r E$, where $r$ is the radius of the circle. Find the rate of change of the magnetic flux through the circle and solve the Faraday's law equation for $E$.

Some problems ask you to use the definition of self-inductance ($L = N\Phi/i$) to compute one of the quantities that appear in it. To carry out this task, you may need to compute the magnetic flux $\Phi$ of the magnetic field, perhaps by carrying out an integration. Other problems deal with the emf generated by an inductor when the current changes. Use $\mathcal{E} = -L\, di/dt$.

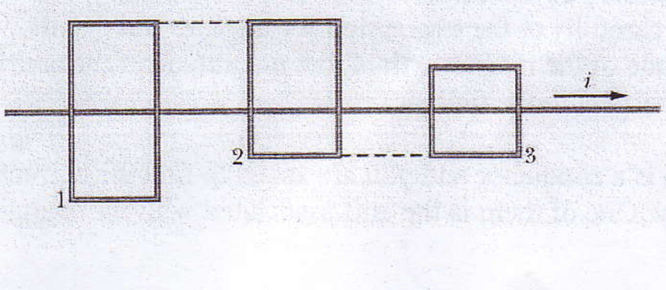
Some problems deal with $RL$ series circuits. If a source of emf, such as a battery, is in the circuit and the current is 0 at time $t = 0$ (because, for example, a switch is closed then), the current is given by $i(t) = (\mathcal{E}/R)\left(1 - e^{-t/\tau_L}\right)$, where the inductive time constant is $\tau_L = L/R$. If the current is $i_0$ at $t = 0$ and no source of emf is in the circuit, calculate $i_0 e^{-t/\tau_L}$.

You may be asked to compute the potential difference across the resistor ($iR$) or across the inductor ($L\, di/dt$). Be sure you can tell which end is at the higher potential. You may also be asked for the rate with which the source of emf is supplying energy ($i\mathcal{E}$), the rate at which the resistor is dissipating energy ($i^2R$), or the rate at which the inductor is storing energy ($Li\, di/dt$). If you are asked for the energy densities use $u_E = \frac{1}{2}\varepsilon_0 E^2$ and $u_B = B^2/2\mu_0$.

Some problems deal with the calculation of mutual inductance. Here you assume a current in one circuit and compute the magnetic flux it produces in another circuit.

## Questions and Example Problems from Chapter 30

**Question 1**
In the figure below, a long straight wire with current i passes (without touching) three rectangular wire loops with edge lengths L, 1.5L, and 2L. The loops are widely spaced (so as to not affect one another). Loops 1 and 3 are symmetric about the long wire. Rank the loops according to the size of the current induced in them if current i is (a) constant and (b) increasing, greatest first.

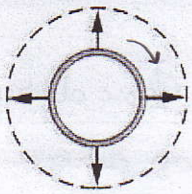



(a) $\boxed{\text{all tie}}$

(b) $\boxed{\text{2, 1+3 tie (zero)}}$

**Question 2**

If the circular conductor in the figure below undergoes thermal expansion while it is in a uniform magnetic field, a current will be induced clockwise around it. Is the magnetic field directed into the page or out of it?

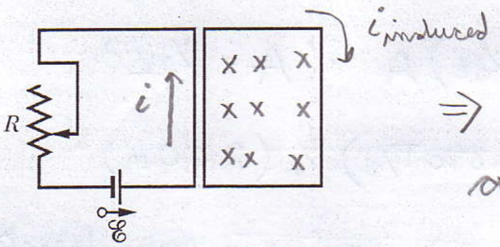


$\Rightarrow$ $\vec{B}$ induced is into the page

since area is increasing, $\Phi_B$ is increasing; since the induced current acts to oppose the change in flux, so $\boxed{\vec{B} \text{ must point out of the page}}$

**Question 3**

If the variable resistance R in the left-hand circuit of the figure below is increased at a steady rate, is the current induced in the right-hand loop clockwise or counterclockwise?



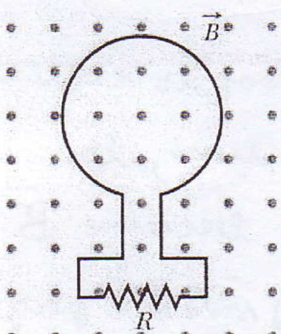

$\Rightarrow$ $i$ is decreasing so $\vec{B}$ is decreasing and therefore $\Phi_B$ is decreasing

induced current must reinforce $\vec{B}_{ext}$

$\boxed{\text{clockwise}}$

**Problem 1**

In the figure below, the magnetic flux through the loop increases according to the relation $\Phi_B = 6.0t^2 + 7.0t$, where $\Phi_B$ is in milliwebers and t is in seconds. (a) What is the magnitude of the emf induced in the loop when t = 2.0 s? (b) Is the direction of the current through R to the right or left?




$\Phi_B = 6.0t^2 + 7.0t$

should be written as $\Phi_B = (6.0\ \frac{mW}{s^2})t^2 + (7.0\ mW/s)\,t$

$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}(6.0t^2 + 7.0t) = -12t - 7.0$

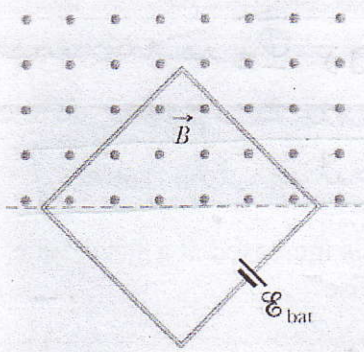
$|\mathcal{E}| = 12t + 7.0 \longrightarrow |\mathcal{E}(t = 2.0s)| = 12(2.0) + 7.0 = 31$

$$\boxed{|\mathcal{E}| = 31\,mV = 31 \times 10^{-3}\,V}$$

(b) Since $\vec{B}$ is out of the page and increasing (because $\Phi$ is increasing) the induced current must produce a $\vec{B}$ field into the page.)

current must be clockwise $\longrightarrow$ $\boxed{\text{right to left through } R}$

**Problem 2**
A square wire loop with 2.00 m sides is perpendicular to a uniform magnetic field, with half the area of the loop in the field as shown in the figure below. The loop contains an ideal battery with emf $\varepsilon = 20.0$ V. If the magnitude of the field varies with time according to $B = 0.0420 - 0.870t$, with B in teslas and t in seconds, what are (a) the net emf in the circuit and (b) the direction of the current through the battery?




⇒ since $\vec{B}$ is decreasing, $\Phi$ is decreasing and $\vec{B}_{induced}$ must be out of page

$\boxed{i \text{ induced is counterclockwise}}$

$\mathcal{E} = -N\, d\Phi_B/dt \qquad N = 1 \qquad \mathcal{E} = -\frac{d}{dt}(BA)$

$\mathcal{E} = -(dB/dt)A \qquad A = \tfrac{1}{2}L^2$

$B = 0.0420T - (0.870\,T/s)\,t$

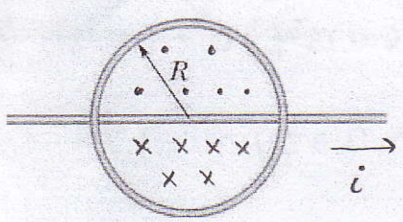
$dB/dt = -0.870\,T/s$

$\mathcal{E} = -(-0.870\,T/s)\,\tfrac{1}{2}(2.00m)^2$

$\mathcal{E} = 1.74$ → emf is in same direction as battery

$\boxed{\text{net } \mathcal{E} = 21.7V}$

**Problem 3**
In the figure below, a wire forms a closed circular loop, with radius R = 2.0 m and resistance 4.0 Ω. The circle is centered on a long straight wire; at time t = 0, the current in the long straight wire is 5.0 A rightward. Thereafter, the current changes according to $i = 5.0\text{ A} - (2.0\text{ A/s}^2)t^2$. (The straight wire is insulated, so there is no electrical contact between it and the wire of the loop.) What are the magnitude and direction of the current induced in the loop at times t > 0?



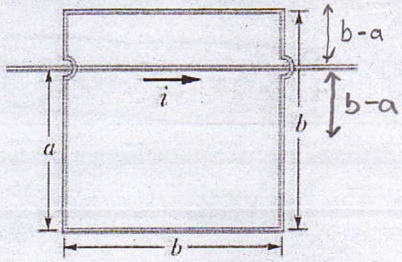

⇒ since the circular loop is centered on the long straight wire, the magnetic flux $\Phi_B = 0$ because $\vec{B}$ points out of the page in the upper half and into the page in the lower half. This will be true no matter what the current i in the straight wire is.

since $\Phi_B = 0 \longrightarrow \mathcal{E} = -\frac{d\Phi_B}{dt} = 0 \qquad \boxed{i = \mathcal{E}/R = 0}$

$$i = (4.50\ A/s^2)t^2 - (10.0\ A/s)t$$

**Problem 4**

For the situation shown in the figure below, a = 12.0 cm and b = 16.0 cm. The current in the long straight wire is given by $i = 4.50t^2 - 10.0t$, where i is in amperes and t is in seconds. (a) Find the emf in the square loop at t = 3.00 s. (b) What is the direction of the induced current in the loop?




$\Rightarrow$ flux from portion of loop above wire is canceled by flux from portion of loop below wire to radius b-a

$$\mathcal{E} = -d\Phi_B/dt$$

$$\Phi = \int \vec{B}\cdot d\vec{A} = \int_{b-a}^{a} B\,dA\cos 0° = \int_{b-a}^{a}\left(\frac{\mu_0 i}{2\pi r}\right)(b\,dr) = \frac{\mu_0 i b}{2\pi}\int_{b-a}^{a}\frac{dr}{r}$$

$$\Phi = \frac{\mu_0 i b}{2\pi}\ln r\Big|_{b-a}^{a} \rightarrow \Phi = \frac{\mu_0 i b}{2\pi}\ln\left(\frac{a}{b-a}\right)$$

$$\mathcal{E} = -d\Phi/dt = -\frac{\mu_0 b}{2\pi}\ln\left(\frac{a}{a-b}\right)di/dt = -\frac{\mu_0 b}{2\pi}\ln\left(\frac{a}{b-a}\right)(9.00t-10)$$

$$|\mathcal{E}| = \frac{(4\pi\times10^{-7}\,Tm/A)(0.16m)}{2\pi}\ln\left(\frac{0.12m}{0.04m}\right)\left[(9.00\,A/s^2)(3.00s) - 10.0\,A/s\right]$$

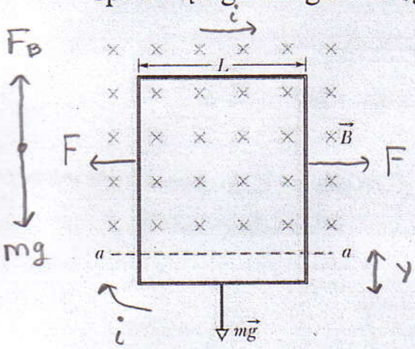
$$\boxed{|\mathcal{E}| = 5.98\times10^{-7}V}$$

(b) at t = 3.0s, $di/dt > 0 \rightarrow$ B increasing, $\Phi$ increasing

$\boxed{\text{counterclockwise}}$

**Problem 5**

In the figure below, a long rectangular conducting loop, of width L, resistance R, and mass m, is hung in a horizontal, uniform magnetic field that is directed into the page and that exists only above line *aa*. The loop is then dropped; during its fall, it accelerates until it reaches a certain terminal speed $v_t$. Ignoring air drag, find that terminal speed.




at terminal speed $\rightarrow \vec{a} = 0 \quad \sum F_y = 0$

$$F_B - mg = 0 \rightarrow F_B = mg$$

$$iLB\sin 90° = mg \rightarrow i = \frac{mg}{LB}$$

$$i = |\mathcal{E}|/R = \frac{1}{R}\left|d\Phi/dt\right| = \frac{1}{R}\left|d/dt(BA)\right|$$

$$= \frac{1}{R}B(dA/dt) = \frac{1}{R}B(L\,dy/dt) = \frac{BLv}{R}$$

$$\frac{BLv}{R} = \frac{mg}{LB} \rightarrow \boxed{v = \frac{mgR}{B^2L^2}}$$

**Problem 6**

The inductance of a close-packed coil of 400 turns is 8.0 mH. Calculate the magnetic flux through the coil when the current is 5.0 mA.

$N = 400$

$L = 8.0 \times 10^{-3}\,H$

$i = 5.0 \times 10^{-3} A$

$\Phi_B = ?$

$L = N\Phi/i \rightarrow \Phi = Li/N$

$\Phi = \dfrac{(8.0\times10^{-3} H)(5.0\times10^{-3}A)}{400} = \boxed{1.0\times10^{-7}\,Wb}$

units $\rightarrow$ $HA = (Tm^2/A)(A) = Tm^2 = \underline{Wb}$

That was easy!

**Problem 7**

A battery is connected to a series RL circuit at time t = 0. At what multiple of $\tau_L$ will the current be 0.100% less than its equilibrium value?

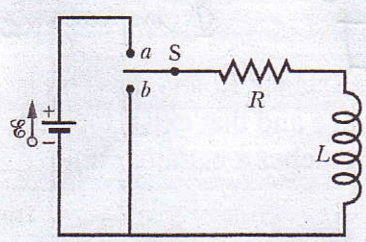
$i = \mathcal{E}/R\,(1 - e^{-t/\tau_L})$ equilibrium value $\rightarrow i = \mathcal{E}/R$

$0.999\,\mathcal{E}/R = \mathcal{E}/R\,(1 - e^{-t/\tau_L}) \rightarrow 0.999 = 1 - e^{-t/\tau_L} \rightarrow e^{-t/\tau_L} = 0.001$

$-t/\tau_L = \ln(0.001) \longrightarrow \boxed{t = 6.91\,\tau_L}$

**Problem 8**

Consider the RL circuit of the figure below. In terms of the battery emf ε, (a) what is the self-induced emf $\varepsilon_L$ when the switch has just been closed on a, and (b) what is $\varepsilon_L$ when t = 2.0$\varepsilon_L$? (c) In terms of $\varepsilon_L$, when will $\varepsilon_L$ be just one-half the battery emf ε?




$i = \mathcal{E}/R\,(1 - e^{-t/\tau_L}) \qquad \tau_L = L/R$

$\mathcal{E}_L = -L\,di/dt = -L\,d/dt\,[\mathcal{E}/R\,(1 - e^{-tL/R})]$

$\mathcal{E}_L = -\mathcal{E} e^{-t/\tau_L} \rightarrow |\mathcal{E}_L| = \mathcal{E} e^{-t/\tau_L}$

(a) $V_R + V_L = \mathcal{E} \rightarrow iR + \mathcal{E}_L = \mathcal{E}$ at $t=0$, $i=0$ so $\mathcal{E}_L = \mathcal{E}$

or $|\mathcal{E}_L| = \mathcal{E} e^{-t/\tau_L} = \mathcal{E} e^{-0} \Rightarrow \boxed{|\mathcal{E}_L| = \mathcal{E}}$ (note: $\mathcal{E}_L$ points in opposite direction as $\mathcal{E}$)

(b) $|\mathcal{E}_L| = \mathcal{E} e^{-2.0\tau_L/\tau_L} = \mathcal{E} e^{-2.0} = \boxed{0.135\,\mathcal{E}}$

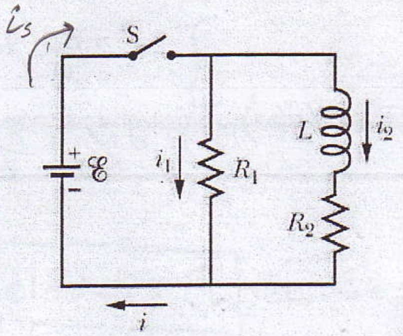
(c) $|\mathcal{E}_L| = \mathcal{E} e^{-t/\tau_L} \rightarrow \mathcal{E}/2 = \mathcal{E} e^{-t/\tau_L} \rightarrow 1/2 = e^{-t/\tau_L}$

$\ln(1/2) = -t/\tau_L \rightarrow t = -(\ln 1/2)\,\tau_L = (\ln 2)\,\tau_L$

$\boxed{t = 0.693\,\tau_L}$

**Problem 9**

In the figure below, the battery is ideal, $\varepsilon = 10$ V, $R_1 = 5.0\ \Omega$, $R_2 = 10\ \Omega$, and $L = 5.0$ H. Switch S is closed at time $t = 0$. Just afterwards, what are (a) $i_1$, (b) $i_2$, (c) the current $i_s$ through the switch, (d) the potential difference $V_2$ across resistor $R_2$, (e) the potential difference $V_L$ across the inductor L, and (f) the rate of change $di_2/dt$? A long time later, what are (g) $i_1$, (h) $i_2$, (i) $i_s$, (j) $V_2$, (k) $V_L$, and (l) $di_2/dt$?




for an inductor (current rising):

$$\left.\begin{aligned} i &= \mathcal{E}/R\,(1-e^{-t/\tau_L}) \\ \mathcal{E}_L &= -\mathcal{E}e^{-t/\tau_L} \end{aligned}\right\} \begin{aligned} &\text{at } t=0,\ i=0 \\ &t=\infty,\ i=\mathcal{E}/R \end{aligned}$$

(a) from loop rule, $\mathcal{E} - i_1R_1 = 0 \rightarrow i_1 = \mathcal{E}/R_1 = \frac{10\text{V}}{5.0\Omega} = \boxed{2.0\text{A}}$

(b) $i_2 = \mathcal{E}/R_2\,(1-e^{-t/\tau_L})$ at $t=0$s, $\boxed{i_2 = 0}$

(c) $i_s = i_1 + i_2 = \boxed{2.0\text{A}}$

(d) $V_2 = i_2R_2 = \boxed{0\text{ V}}$ (since $i_2 = 0$)

(e) $\mathcal{E}_L = -\mathcal{E}e^{-t/\tau_L} \longrightarrow \mathcal{E}_L = -\mathcal{E}$ at $t=0$s, $|\mathcal{E}_L| = \boxed{10.0\text{V}}$

(f) $di_2/dt = \mathcal{E}/L\, e^{-t/\tau_L}$

$= \frac{10.0\text{V}}{5.0\text{H}} e^0 = \boxed{2.0\text{ A/s}}$

(g) $\boxed{i_1 = 2.0\text{A}}$ ($i_1$ does not change)

(h) $i_2 = \mathcal{E}/R_2\,(1-e^{-t/\tau_L})$ at $t=\infty$, $i_2 = \mathcal{E}/R_2 = \frac{10.0\text{V}}{10\Omega} = \boxed{1.0\text{A}}$

(i) $i_s = i_1 + i_2 = \boxed{3.0\text{A}}$

(j) $V_2 = i_2R_2 = (1.0\text{A})(10.0\Omega) = \boxed{10.0\text{V}}$

(K) $\mathcal{E}_L = -\mathcal{E}e^{-t/\tau_L} \longrightarrow \boxed{\mathcal{E}_L = 0}$ at $t=\infty$

(l) $di_2/dt = d/dt\left[\mathcal{E}/R\,(1-e^{-t/\tau_L})\right]$

$= -\mathcal{E}/R\,(-1/\tau_L)\,e^{-t/\tau_L} \rightarrow \frac{di_2}{dt} = \mathcal{E}/L\, e^{-t/\tau_L}$

$\boxed{di_2/dt = 0}$ at $t=\infty$

**Problem 10**

A coil with an inductance of 2.0 H and a resistance of 10 Ω is suddenly connected to an ideal battery with ε = 100 V. At t = 0.10 s after the connection is made, what is the rate at which (a) energy is being stored in the magnetic field, (b) thermal energy is appearing in the resistance, and (c) energy is being delivered by the battery?

$L = 2.0\,H$, $R = 10\,\Omega$, $\mathcal{E} = 100\,V$

(a) $P = i\mathcal{E}_L = i\,L\,di/dt$

(or $P = dU_B/dt = d/dt(\frac{1}{2}Li^2) = L\,i\,di/dt$)

$i = \frac{\mathcal{E}}{R}(1-e^{-t/\tau_L})$, $\quad di/dt = \frac{\mathcal{E}}{L}e^{-t/\tau_L}$

$$P = L\left[\mathcal{E}/R\,(1-e^{-t/\tau_L})\right](\mathcal{E}/L\,e^{-t/\tau_L})$$

$$= \mathcal{E}^2/R\,(1-e^{-t/\tau_L})(e^{-t/\tau_L}) \longrightarrow \text{at } t = 0.10\,s \quad \boxed{P = 2.4\times10^2\,W}$$

(b) for a resistor: $P = i^2R = \left[\mathcal{E}/R\,(1-e^{-t/\tau_L})\right]^2 R$

$$P = \mathcal{E}^2/R\,(1-e^{-t/\tau_L})^2 \longrightarrow \text{at } t = 0.10\,s \quad \boxed{P = 1.5\times10^2\,W}$$

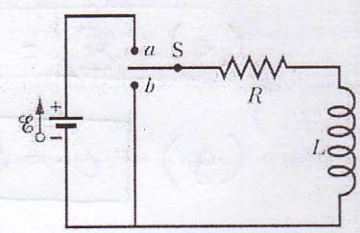
(c) for a battery: $P = i\mathcal{E} = \left[\mathcal{E}/R\,(1-e^{-t/\tau_L})\right]\mathcal{E}$

$$P = \mathcal{E}^2/R\,(1-e^{-t/\tau_L}) \longrightarrow \text{at } t = 0.10\,s \quad \boxed{P = 3.9\times10^2\,W}$$

<u>note</u>: $P_{battery} = P_{resistor} + P_{inductor}$

**Problem 11**

Prove that, after switch S in the figure to the right has been thrown from *a* to *b*, all the energy stored in the inductor will ultimately appear as thermal energy in the resistor.




for inductor: $P = i\mathcal{E}_L = i(L\,di/dt)$

$dE/dt = L\,i\,di/dt \longrightarrow E = \frac{1}{2}Li^2$ → energy stored in inductor

⇒ assume current through circuit is initially $i_0$ when switch is at a ($E = \frac{1}{2}Li_0^2$)

& $t = 0$, switch is thrown to b $\quad i = i_0 e^{-t/\tau_L} \quad \tau_L = L/R$

$$E = \int_0^\infty P\,dt = \int_0^\infty i^2R\,dt = \int_0^\infty [i_0 e^{-t/\tau_L}]^2 R\,dt = i_0^2 R\int_0^\infty e^{-2t/\tau_L}$$

$$E = i_0^2 R\left[-\tau_L/2\,e^{-2t/\tau_L}\right]\Big|_0^\infty = i_0^2 R(-\tau_L/2)(0-1)$$

$$E = \frac{1}{2}i_0^2 R\tau_L = \frac{1}{2}i_0^2 R\,[L/R] = \boxed{\tfrac{1}{2}Li_0^2}$$ → energy originally stored in inductor