Some problems deal with the analogy between an LC electrical circuit and a block-spring mechanical system. Remember that $q \Leftrightarrow x$, $L \Leftrightarrow m$, $C \Leftrightarrow 1/k$, and $i \Leftrightarrow v$.

You should know the relationship between the maximum charge on the capacitor and the maximum current: $I = \omega_d Q$. You should also know that the total energy of the circuit can be written in terms of either of these: $U = Q^2/2C = LI^2/2$. Some problems ask you to compute the maximum charge on the capacitor, given the maximum potential difference across the plates: $Q = CV_{\max}$.

Some problems deal with damped circuits. You should know that the frequency of oscillation changes with the addition of resistance and that in succeeding cycles, the maximum charge on the capacitor decreases exponentially, the relevant factor being $e^{-Rt/2L}$.

You should know how to calculate the reactances and the impedance for an RLC series circuit: $X_L = \omega_d L$, $X_C = 1/\omega_d C$, and $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where ω_d is the angular frequency. Some problems give the frequency f_d and you must use $\omega_d = 2\pi f_d$.

You should know how to compute the current given the generator emf. If $\mathcal{E} = \mathcal{E}_m \sin \omega_d t$, then $i = I \sin(\omega_d t - \phi)$, where $I = \mathcal{E}_m/Z$ and $\tan \phi = (X_L - X_C)/R$. Think of ϕ as a phase *difference*. That is, if $\mathcal{E} = \mathcal{E}_m \sin(\omega_d t + \alpha)$, then $i = I \sin(\omega_d t + \alpha - \phi)$.

Given the current, you should know how to compute the potential differences across the individual circuit elements: $v_R = iR$, $v_C = IX_C \sin(\omega_d t - \pi/2)$, and $v_L = IX_L \sin(\omega_d t + \pi/2)$.

Finally, you should know that the generator supplies energy at the rate $P_{\mathcal{E}} = i\mathcal{E}$, the inductor stores energy at the rate $P_L = iv_L$, the capacitor stores energy at the rate $P_C = iv_C$, and the resistor generates thermal energy at the rate $P_R = i^2R$. These are all time dependent quantities.

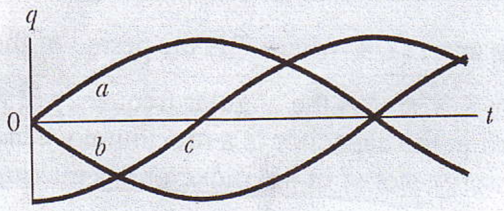
Sometimes the rms values of current and potential differences are given or requested. You should remember that if $i = I \sin(\omega_d t - \phi)$, then $i_{\text{rms}} = I/\sqrt{2}$. Similar expressions hold for the potential differences. You should also recognize that equations such as $v_L = IX_L \sin(\omega_d t + \pi/2)$ lead to $V_{L\,\text{rms}} = i_{\text{rms}} X_L$.

Questions and Example Problems from Chapter 31

Question 1

For each of the curves of q(t) in the figure below for an LC circuit, determine the least positive phase constant Φ in the equation $q = Q\cos(\omega t + \Phi)$ required to produce the curve.

$\omega t + \phi = 0 \longrightarrow t = -\phi/\omega$



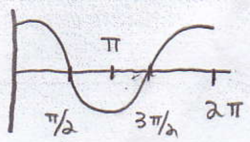
$\phi > 0$ curve shifted left

$\phi < 0$ curve shifted right

a) $\boxed{3\pi/2}$

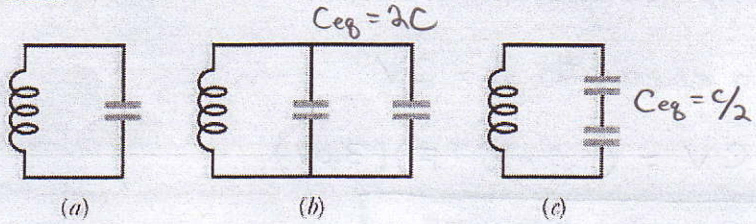
b) $\boxed{\pi/2}$

c) $\boxed{\pi}$



Question 2

The figure below shows three oscillating LC circuits with identical inductors and capacitors. Rank the circuits according to the time taken to fully discharge the capacitors during the oscillations, greatest first.



can rank according to $T = 1/f$

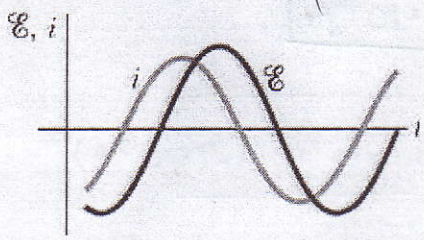
$\omega = \frac{1}{\sqrt{LC}} = 2\pi f \qquad f = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$

$T = 2\pi\sqrt{LC}$

$\boxed{b, a, c}$

Question 3

The figure below shows the current i and driving emf $\mathcal{E}$ for a series RLC circuit. (a) Does the current lead or lag the emf? (b) Is the circuit's load mainly capacitive or mainly inductive? (c) Is the angular frequency ω_d of the emf greater than or less than the natural angular frequency ω?



a) $\boxed{\text{current leads}}$

b) $\boxed{\text{mainly capacitive}}$ (ELI the ICE man)

c) $\boxed{\text{less than}}$ $\omega_d < \omega \quad X_C > X_L$

mainly capacitive

Question 4

The figure above (Question 3) shows the current i and driving emf $\mathcal{E}$ for a series RLC circuit. Relative to the emf curve, does the current curve shift leftward or rightward and does the amplitude of that curve increase or decrease if we slightly increase (a) L, (b) C, and (c) ω_d?

$X_C > X_L$

$\omega_d < \omega$

a) $\boxed{\text{rightward, increase}}$ (X_L increases, closer to resonance)

b) $\boxed{\text{rightward, increase}}$ (X_C decreases, closer to resonance)

c) $\boxed{\text{rightward, increase}}$ (ω_d/ω increases, closer to resonance)

Problem 1

In an oscillating LC circuit, L = 1.10 mH and C = 4.00 μF. The maximum charge on the capacitor is 3.00 μC. Find the maximum current.

electrical energy stored $\underline{U_E = q^2/2C} = Q^2/2C \cos^2(\omega t + \phi)$

magnetic energy stored $\underline{U_B = \frac{1}{2}Li^2} = \frac{1}{2}LI^2 \sin^2(\omega t+\phi) = \frac{Q^2}{2C}\sin^2(\omega t + \phi)$

total energy stored $U = U_E + U_B = Q^2/2C = \frac{1}{2}LI^2$ where Q = max charge on capacitor, I = max current through inductor

$\frac{1}{2}Q^2/C = \frac{1}{2}LI^2$

$I = Q/\sqrt{LC} = \frac{3.00\times10^{-6}\,C}{\sqrt{(1.10\times10^{-3}\,H)(4.00\times10^{-6}\,F)}} \rightarrow \boxed{I = 4.52\times10^{-2}\,A}$

Problem 2)
An oscillating LC circuit consisting of a 1.0 nF capacitor and a 3.0 mH coil has a maximum voltage of 3.0 V. What are (a) the maximum charge on the capacitor, (b) the maximum current through the circuit, and (c) the maximum energy stored in the magnetic field of the coil?

$C = 1.0\times10^{-9}F$

$L = 3.0\times10^{-3}H$

$V_{max} = 3.0V$

a) for a capacitor $q = CV$

$Q = CV = (1.0\times10^{-9}F)(3.0V)$

$\boxed{Q = 3.0nC = 3.0\times10^{-9}C}$

b) total energy $U = \frac{1}{2}Q^2/C = \frac{1}{2}LI^2$

$I^2 = Q^2/LC \rightarrow I = Q/\sqrt{LC} \quad (I = \omega Q)$

$$I = \frac{(3.0\times10^{-9}C)}{\sqrt{(3.0\times10^{-3}H)(1.0\times10^{-9}F)}} \rightarrow \boxed{I = 1.7\times10^{-3}A}$$

c) $U_B = \frac{1}{2}Li^2 \longrightarrow U_{B,max} = \frac{1}{2}LI^2 = \frac{1}{2}(3.0\times10^{-3}H)(1.7\times10^{-3}A)^2$

$\boxed{U_{B,max} = 4.5\times10^{-9}J}$

Problem 3
In an oscillating LC circuit with C = 64.0 μF, the current as a function of time is given by $i = (1.60)\sin(2500t + 0.680)$, where t is in seconds, i in amperes, and the phase constant in radians. (a) How soon after t = 0 will the current reach its maximum value? What are (b) the inductance L and (c) the total energy?

$i = -I\sin(\omega t+\phi) = (-1.60A)\sin\left[(2500s^{-1})t + 0.680rad\right]$

(a) current will have maximum value when $\sin(\omega t+\phi) = 1$ or $\omega t+\phi = \pi/2$

(note: when $\sin(\omega t+\phi) = 1$, $i = -1.60A$; the current has a maximum magnitude of 1.60 A $\rightarrow$ the − sign indicates the direction.)

$$\omega t+\phi = \pi/2 \rightarrow t = \frac{\pi/2-\phi}{\omega} = \frac{\pi/2 - 0.680rad}{(2500s^{-1})} \rightarrow \boxed{t = 3.56\times10^{-4}s}$$

(b) for an LC circuit:

$$\omega = 1/\sqrt{LC} \rightarrow L = \frac{1}{\omega^2C} = \frac{1}{(2500s)^2(64.0\times10^{-6}F)} \rightarrow \boxed{L = 2.50\times10^{-3}H}$$

(c) total energy $U = Q^2/2C = \frac{1}{2}LI^2$

$U = \frac{1}{2}LI^2 = \frac{1}{2}(2.50\times10^{-3}H)(1.60A)^2 \longrightarrow \boxed{U = 3.20\times10^{-3}J}$

Problem 4

What resistance R should be connected in series with an inductance L = 220 mH and capacitance C = 12.0 μF for the maximum charge on the capacitor to decay to 99.0% of its initial value in 50.0 cycles? (Assume ω′ ≈ ω.)

$$q = Qe^{-Rt/2L}\cos(\omega' t + \phi) \approx Qe^{-Rt/2L}\cos(\omega t + \phi)$$ assuming $\omega' \approx \omega$

$q = q_{max}\cos(\omega t + \phi)$ where $q_{max} = Qe^{-Rt/2L}$ Q = initial charge at $t=0$ (assuming $\phi = 0$)

$$q_{max} = Qe^{-Rt/2L} \longrightarrow q_{max}/Q = e^{-Rt/2L}$$

$$\ln(q_{max}/Q) = -Rt/2L \xrightarrow[q_{max} = 0.99Q]{} \ln(0.99) = -Rt/2L$$

$$R = -2L/t \ln(0.99)$$

* we need time for 50 cycles

$$t = 50T$$
$$= 50(2\pi\sqrt{LC})$$
$$t = 50(2\pi)\sqrt{(220\times10^{-3}H)(12.0\times10^{-6}F)} = 0.510s$$

$$R = \frac{-2(220\times10^{-3}H)}{(0.510s)}\ln(0.99)$$

$$\boxed{R = 8.66\times10^{-3}\,\Omega}$$

Problem 5

(a) At what frequency would a 6.0 mH inductor and a 10 μF capacitor have the same reactance?
(b) What would the reactance be? (c) Show that this frequency would be the natural frequency of an oscillating circuit with the same L and C.

capacitive reactance $X_C = \frac{1}{\omega_d C}$ inductive reactance $X_L = \omega_d L$

(a) we want f when $X_C = X_L \longrightarrow \frac{1}{\omega_d C} = \omega_d L$

$$\omega_d = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(6.0\times10^{-3}H)(10\times10^{-6}F)}} = 4082\text{ rad/s}$$

$$\omega = 2\pi f \longrightarrow f = \omega/2\pi = \frac{4082\text{ rad/s}}{2\pi\text{ rad}} \longrightarrow \boxed{f = 650\text{ Hz}}$$

(b) $X_L = \omega_d L = (4082\text{ rad/s})(6.0\times10^{-3}H) \longrightarrow \boxed{X_L = 24.5\,\Omega}$

$X_C = 1/\omega_d C = \frac{1}{(4082\text{ rad/s})(10\times10^{-6}F)} \longrightarrow \boxed{X_C = 24.5\,\Omega}$

(c) natural frequency of an LC circuit $= f = \omega/2\pi = \frac{1}{2\pi\sqrt{LC}}$

$$\boxed{f = 650\text{ Hz}}$$

Problem 6

An ac generator has emf $\mathcal{E} = \mathcal{E}_m \sin(\omega_d t - \pi/4)$, where $\mathcal{E}_m = 30.0$ V and $\omega_d = 350$ rad/s. The current produced in a connected circuit is $i(t) = I \sin(\omega_d t - 3\pi/4)$, where $I = 620$ mA. At what time after $t = 0$ does (a) the generator emf first reach a maximum and (b) the current first reach a maximum? (c) The circuit contains a single element other than the generator. Is it a capacitor, an inductor, or a resistor? Justify your answer. (d) What is the value of the capacitance, inductance, or resistance, as the case may be?

$$\mathcal{E} = \mathcal{E}_m \sin(\omega_d t - \pi/4)$$
$$i = I \sin(\omega_d t - 3\pi/4)$$

$\omega_d = 350 \text{ rad/s}$
$\mathcal{E}_m = 30.0\text{V}$
$I = 620\text{mA}$

(a) $\mathcal{E} = \mathcal{E}_m$ when $\sin(\omega_d t - \pi/4) = 1$ or $\omega_d t - \pi/4 = \pi/2$

$$\omega_d t = 3\pi/4$$

$$t = 3\pi/4\omega_d = \frac{3\pi}{4(350\text{rad/s})} \longrightarrow \boxed{t = 6.73\times10^{-3}\text{s}}$$

(b) $i = I$ when $\omega_d t - 3\pi/4 = \pi/2$

$$\omega_d t = 5\pi/4$$

$$t = 5\pi/4\omega_d = \frac{5\pi}{4(350\text{rad/s})} \longrightarrow \boxed{t = 1.12\times10^{-2}\text{s}}$$

(c) current peaks after the emf (current lags voltage by $\pi/2$)

$\boxed{\text{inductor}}$ ELI the ICE man

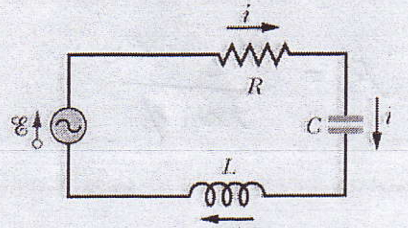
(d) $V_L = I_L X_L \qquad X_L = \omega_d L$

$$V_L = I_L(\omega_d L) \longrightarrow L = \frac{V_L}{I\omega_d} = \frac{\mathcal{E}_m}{I\omega_d}$$

$$L = \frac{(30.0\text{V})}{(620\times10^{-3}\text{A})(350\text{rad/s})} \longrightarrow \boxed{L = 0.138\text{H}}$$

Problem 7

Remove the inductor from the circuit in the figure below and set $R = 200\ \Omega$, $C = 15.0\ \mu F$, $f_d = 60.0$ Hz, and $\mathcal{E}_m = 36.0$ V. What are (a) Z, (b) ϕ and (c) I? ~~(d) Draw a phasor diagram.~~



$$X_c = \frac{1}{\omega_d C} = \frac{1}{2\pi f_d C} = \frac{1}{2\pi (60.0\,Hz)(15.0\times10^{-6}F)}$$

$$X_c = 177\,\Omega$$

$$X_L = \omega_d L = 0$$

(a) $Z = \sqrt{R^2 + (X_L - X_c)^2} = \sqrt{(200\,\Omega)^2 + (0 - 177\,\Omega)^2} \longrightarrow \boxed{Z = 267\,\Omega}$

(b) $\phi = \tan^{-1}\left(\frac{X_L - X_c}{R}\right) = \tan^{-1}\left(\frac{0 - 177\,\Omega}{200\,\Omega}\right) \longrightarrow \boxed{\phi = -41.5^\circ}$

(c) $I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (X_L - X_c)^2}} = \frac{\mathcal{E}_m}{Z} = \frac{36.0\,V}{267\,\Omega} \longrightarrow \boxed{I = 0.135\,A}$

Problem 8

(a) In an RLC circuit, can the amplitude of the voltage across an inductor be greater than the amplitude of the generator emf? (b) Consider an RLC circuit with $\mathcal{E}_m = 10$ V, R = 10 Ω, L = 1.0 H, and C = 1.0 μF. Find the amplitude of the voltage across the inductor at resonance.

(a) $\boxed{\text{yes}}$

(b) for an RLC circuit, $I = \frac{\mathcal{E}_m}{Z}$ $\quad Z = \sqrt{R^2 + (X_L - X_c)^2}$

at resonance, $X_L = X_c$ so $Z = R = 10\,\Omega$

this happens when $\omega_d = \omega = \frac{1}{\sqrt{LC}}$

$$\omega_d = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.0H)(1.0\times10^{-6}F)}} = 1000\,rad/s$$

$$V_c = I X_c = \frac{I}{\omega_d C} = \frac{(1.0A)}{(1000\,rad/s)(1.0\,\mu F)} = 1000V$$

$$V_L = I X_L = I(\omega_d L) = \left(\frac{\mathcal{E}_m}{Z}\right)(\omega_d L)$$

$$= \left(\frac{10V}{10\,\Omega}\right)(1000\,rad/s)(1.0H) \longrightarrow \boxed{V_L = 1000\,V}$$

Problem 9

A coil of inductance 88 mH and unknown resistance and a 0.94 μF capacitor are connected in series with an alternating emf of frequency 930 Hz. If the phase constant between the applied voltage and the current is 75°, what is the resistance of the coil?

$L = 88\times10^{-3}\,H$

$C = 0.94\times10^{-6}\,C$

$R = ?$

$f = 930\,Hz$

$\phi = 75°$

$$\tan\phi = \frac{X_L - X_C}{R} \longrightarrow R = \frac{X_L - X_C}{\tan\phi}$$

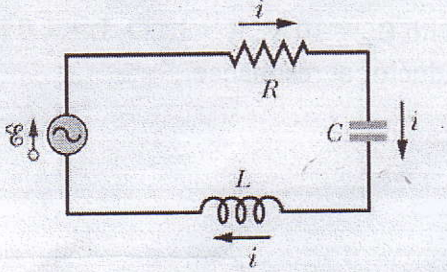
$$R = \frac{(2\pi f_d L) - \frac{1}{(2\pi f_d C)}}{\tan\phi}$$

$$R = \frac{2\pi(930\,Hz)(88\times10^{-3}\,H) - \frac{1}{(2\pi)(930\,Hz)(0.94\times10^{-6}\,C)}}{\tan 75°}$$

$$\boxed{R = 89\,\Omega}$$

Problem 10

(a) Show that the average rate at which energy is supplied to the circuit of Fig. 33-7 can also be written as $P_{avg} = \mathcal{E}^2_{rms}R/Z^2$. Then show that this expression for average power gives reasonable results for (b) a purely resistive circuit, (c) an RLC circuit at resonance, (d) a purely capacitive circuit, and (e) a purely inductive circuit.



$P_{ave} = \mathcal{E}_{rms} I_{rms} \cos\phi$

$I_{rms} = \mathcal{E}_{rms}/Z$

$\cos\phi = R/Z$

(a) $P_{ave} = \mathcal{E}_{rms}(\mathcal{E}_{rms}/Z)(R/Z) \longrightarrow \boxed{P_{ave} = \mathcal{E}^2_{rms} R/Z^2}$

(b) for a purely resistive circuit, $Z = R \rightarrow P_{ave} = \mathcal{E}^2_{rms} R/R^2 = \boxed{\mathcal{E}^2_{rms}/R}$

(c) for an RLC circuit at resonance, $Z = R$ $(X_L = X_C)$ so $P_{ave} = \boxed{\mathcal{E}^2_{rms}/R}$

(d) for a purely capacitive circuit, $R = 0$ so $\boxed{P_{ave} = 0}$

(e) for a purely inductive circuit, $R = 0$ so $\boxed{P_{ave} = 0}$

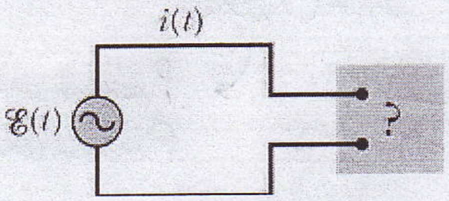
no energy is dissipated if there is no resistance

Problem 11

The figure below shows an ac generator connected to a "black box" through a pair of terminals. The box contains an RLC circuit, possibly even a multiloop circuit, whose elements and connections we do not know. Measurements outside the box reveal that

$$\mathcal{E}(t) = (75.0\text{ V})\sin\omega_d t \quad \text{and} \quad i(t) = (1.20\text{ A})\sin(\omega_d t + 42.0°).$$

(a) What is the power factor? (b) Does the current lead or lag the emf? (c) Is the circuit in the box largely inductive or largely capacitive? (d) Is the circuit in the box in resonance? (e) Must there be a capacitor in the box? (f) An inductor? (g) A resistor? (h) At what average rate is energy delivered to the box by the generator? (i) Why don't you need to know the angular frequency w_d to answer all these questions?



$i = I\sin(\omega_d t - \phi)$ $\quad I = 1.20\text{ A}$

$\phi = -42.0°$

(a) power factor $= \cos\phi = \cos(-42.0°)$

$= \boxed{0.743}$

(b) $\phi < 0 \rightarrow$ $\boxed{\text{current leads the voltage}}$ (current shifted to left)

(c) $\boxed{\text{largely capacitive}}$ (ICE)

(d) no, at resonance $\phi = 0°$

$\tan\phi = \dfrac{X_L - X_C}{R}$ since $\phi \neq \pm \pi/2$ $\quad R \neq 0$

since $\phi < 0$, must be capacitive

$\Rightarrow$ there must be a resistor and a capacitor; there doesn't have to be an inductor (if there is, $X_L < X_C$)

(e) $\boxed{\text{yes}}$ (f) $\boxed{\text{no}}$ (g) $\boxed{\text{yes}}$

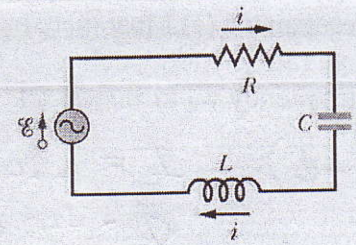
(h) $P_{ave} = \mathcal{E}_{rms} I_{rms} \cos\phi = (\mathcal{E}_m/\sqrt{2})(I/\sqrt{2})\cos\phi$

$= \frac{1}{2}\mathcal{E}_m I \cos\phi = \frac{1}{2}(75.0\text{V})(1.20\text{A})(0.743) = \boxed{33.4\text{ W}}$

(g) the answers depend on frequency only through the phase constant which is given

Problem 12

In an RLC circuit such as that of the figure below, assume that R = 5.00 Ω, L = 60.0 mH, f_d = 60.0 Hz, and $\mathcal{E}m$ = 30.0 V. For what values of the capacitance would the average rate at which energy is dissipated in the resistance be (a) a maximum and (b) a minimum? What are (c) the maximum dissipation rate and the corresponding (d) phase angle and (e) power factor? What are (f) the ~~maximum~~ minimum dissipation rate and the corresponding (g) phase angle and (h) power factor?



$R = 5.00\,\Omega$

$L = 60.0\times10^{-3}\,H$

$f_d = 60.0\,Hz$

$\mathcal{E}_m = 30.0\,V$

$P_{ave} = I_{rms}^2 R = \frac{\mathcal{E}_{rms}^2 R}{Z^2}$

$P_{ave} = \frac{\mathcal{E}_{rms}^2 R}{R^2+(X_L-X_C)^2}$

P is max when Z is min

P is min when Z is max

(a) Z is a min when $X_L = X_C$, or $\omega_d L = \frac{1}{\omega_d C}$

$\omega_d = \frac{1}{\sqrt{LC}} \rightarrow C = \frac{1}{\omega_d^2 L} = \frac{1}{(2\pi f_d)^2 L}$

$C = \frac{1}{(2\pi)^2(60.0\,Hz)^2(60.0\times10^{-3}H)} \longrightarrow \boxed{C = 1.17\times10^{-4}\,F}$

(b) Z is a max when $R^2+(X_L-X_C)^2$ is a max which occurs when $\boxed{C=0}$ $(X_C=\infty)$

(c) $P_{ave} = \frac{\mathcal{E}_{rms}^2 R}{Z^2} \rightarrow P_{ave} = \frac{(\mathcal{E}_m/\sqrt{2})^2 R}{R^2} = \frac{\mathcal{E}_m^2}{2R} = \frac{(30.0\,V)^2}{2(5.00\,\Omega)} = \boxed{90.0\,W}$

(d) $\tan\phi = \frac{X_L-X_C}{R} = 0 \rightarrow \boxed{\phi = 0°}$

(e) power factor $= \cos\phi = \cos 0° = \boxed{1}$

(f) when $C=0$, $X_C=\infty$, $Z=\infty$ so $\boxed{P_{ave}=0}$

(g) $\tan\phi = \frac{X_L-X_C}{R} = -\infty \longrightarrow \boxed{\phi = -90.0°}$

(h) power factor $=\cos\phi$

$= \cos(-90°)$

$= \boxed{0}$

Problem 13

A generator supplies 100 V to the primary coil of a transformer of 50 turns. If the secondary coil has 500 turns, what is the secondary voltage?

$V_p = 100\,V$

$V_s = ?$

$N_p = 50$

$N_s = 500$

$V_s = V_p(N_s/N_p)$

$V_s = (100\,V)(500/50)$

$\boxed{V_s = 1000\,V}$