

The integral is the same as the first one taken from the tables, with $a = 2$. So

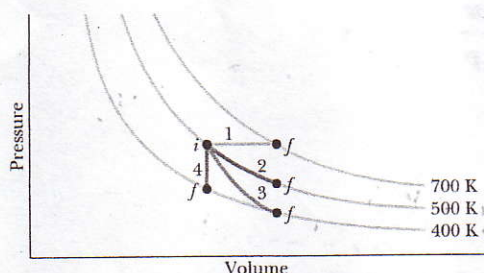
$$v_{\text{avg}}^2 = 4\pi \left[\frac{M}{2\pi RT} \right]^{3/2} \frac{3}{2(2\pi)^2} \sqrt{\frac{\pi}{p}} = 4\pi \left[\frac{M}{2\pi RT} \right]^{3/2} \frac{3R^2 T^2}{2M^2} \sqrt{\frac{2\pi RT}{M}} = \frac{3RT}{M}$$

Thus, the root-mean-square speed is $\sqrt{3RT/M}$.

Questions and Example Problems from Chapter 19

Question 1

- (a) Rank the four paths in the figure below according to the work done by the gas, greatest first.
 (b) Rank paths 1, 2, and 3 according to the change in the internal energy of the gas, most positive first and most negative last.



(a) $W = \int p dV = \text{area under curve}$

1, 2, 3, 4 (=0)

(b) $\Delta E_{\text{int}} = n C_v \Delta T$

1, 2, 3

Question 2

An ideal diatomic gas, with molecular rotation but not oscillation, loses energy as heat Q . Is the resulting decrease in the internal energy of the gas greater if the loss occurs in a constant-volume process or in a constant-pressure process?

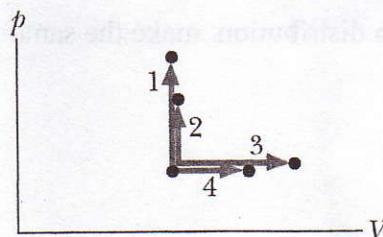
diatomic gas $\rightarrow C_v = 5/2 R$
 $\rightarrow C_p = 7/2 R$

$Q = n C_v \Delta T$ or $Q = n C_p \Delta T$
 $\Delta T = Q/n C_v$ $\Delta T = Q/n C_p$

greater if loss occurs at constant-volume

Question 3

A certain amount of energy is to be transferred as heat to 1 mol of a monatomic gas (a) at constant pressure and (b) at constant volume, and to 1 mol of a diatomic gas (c) at constant pressure and (d) at constant volume. The figure below shows four paths from an initial point to four final points on a p-V diagram. Which path goes with which process? (e) Are the molecules of the diatomic gas rotating?



monatomic gas $\rightarrow C_v = 3/2 R$ (b)
 $\rightarrow C_p = 5/2 R$ (a)

diatomic gas $\rightarrow C_v = 5/2 R$ (d)
 $\rightarrow C_p = 7/2 R$ (c)

(a) path 3

(b) path 1

(c) path 4

(d) path 2

(e) yes, because paths 2 + 4 have smaller ΔT 's than paths 1 + 3

Problem 1

A quantity of ideal gas at 10.0°C and 100 kPa occupies a volume of 2.50 m^3 . (a) How many moles of the gas are present? (b) If the pressure is now raised to 300 kPa and the temperature is raised to 30.0°C , how much volume does the gas occupy? Assume no leaks.

$$T = 10.0^\circ\text{C} = 283.15\text{ K}$$

$$p = 100 \times 10^3\text{ Pa}$$

$$V = 2.50\text{ m}^3$$

$$n = ?$$

$$pV = nRT$$

$$n = \frac{pV}{RT}$$

$$n = \frac{(100 \times 10^3\text{ Pa})(2.50\text{ m}^3)}{(8.31\text{ J/mol}\cdot\text{K})(283.15\text{ K})}$$

$$\boxed{n = 106\text{ mol}}$$

$$(b) \quad n = \frac{pV}{RT} = \text{constant} \rightarrow \frac{pV}{T} = \text{constant} \rightarrow \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$V_2 = \frac{T_2 p_1}{T_1 p_2} V_1 = \left(\frac{303.15\text{ K}}{283.15\text{ K}} \right) \left(\frac{100 \times 10^3\text{ Pa}}{300 \times 10^3\text{ Pa}} \right) (2.50\text{ m}^3) = \boxed{0.892\text{ m}^3}$$

Problem 2

The best laboratory vacuum has a pressure of about $1.00 \times 10^{-18}\text{ atm}$, or $1.01 \times 10^{-13}\text{ Pa}$. How many gas molecules are there per cubic centimeter in such a vacuum at 293 K ?

$$p = 1.01 \times 10^{-13}\text{ Pa}$$

$$T = 293\text{ K}$$

$$V = 1.0\text{ cm}^3 = 1.0 \times 10^{-6}\text{ m}^3$$

$$n = ?$$

\Rightarrow first get the number of moles n ;
we can get the number of molecules
from $N = n N_A$

$$pV = nRT \rightarrow n = \frac{pV}{RT}$$

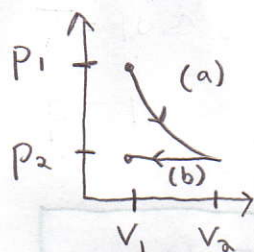
$$n = \frac{(1.01 \times 10^{-13}\text{ Pa})(1.0 \times 10^{-6}\text{ m}^3)}{(8.31\text{ J/mol}\cdot\text{K})(293\text{ K})} = \underline{\underline{4.1 \times 10^{-23}\text{ mol}}}$$

$$N = n N_A = (4.1 \times 10^{-23}\text{ mol})(6.022 \times 10^{23}\text{ mol}^{-1}) = 25$$

$$\boxed{N/V = 25\text{ molecules/cm}^3}$$

Problem 3

Air that initially occupies 0.140 m^3 at a gauge pressure of 103.0 kPa is expanded isothermally to a pressure of 101.3 kPa and then cooled at constant pressure until it reaches its initial volume. Compute the work done by the air. (Gauge pressure is the difference between the actual pressure and atmospheric pressure.)



$$V_1 = 0.140 \text{ m}^3 \quad P_1 = 103.0 \text{ kPa} + 101.3 \text{ kPa} = 204.3 \text{ kPa}$$

$$V_2 = ? \quad P_2 = 101.3 \text{ kPa}$$

path a \rightarrow for isotherm $W = nRT \ln\left(\frac{V_2}{V_1}\right)$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \rightarrow \frac{V_2}{V_1} = \frac{P_1}{P_2} \quad nRT_1 = P_1 V_1$$

$$W = nRT \ln\left(\frac{V_2}{V_1}\right) = P_1 V_1 \ln\left(\frac{P_1}{P_2}\right)$$

$$W = (204.3 \times 10^3 \text{ Pa})(0.140 \text{ m}^3) \ln\left(\frac{204.3 \text{ kPa}}{101.3 \text{ kPa}}\right) = \underline{2.00 \times 10^4 \text{ J}}$$

path b $\rightarrow W = \int p dV = p(V_f - V_i) \rightarrow W_b = (101.3 \text{ kPa})(0.140 - 0.282 \text{ m}^3)$

$$\frac{V_2}{V_1} = \frac{P_1}{P_2} \rightarrow V_2 = \left(\frac{P_1}{P_2}\right)V_1$$

$$= \left(\frac{204.3 \text{ kPa}}{101.3 \text{ kPa}}\right)(0.140 \text{ m}^3) = \underline{0.282 \text{ m}^3}$$

$$W = W_a + W_b = \underline{5.6 \times 10^3 \text{ J}}$$

Problem 4

At what temperature do atoms of helium gas have the same rms speed as molecules of hydrogen gas at 20.0°C ? (The molar masses are given in Table 19-1).

$$V_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$M_{\text{H}_2} = 2.02 \text{ g/mol}$$

$$20.0^\circ\text{C} = 293 \text{ K}$$

$$M_{\text{He}} = 4.0 \text{ g/mol}$$

$$\sqrt{\frac{3RT}{M_{\text{He}}}} = \sqrt{\frac{3R(293 \text{ K})}{M_{\text{H}_2}}} \rightarrow \frac{T}{M_{\text{He}}} = \frac{293 \text{ K}}{M_{\text{H}_2}}$$

$$T = \frac{M_{\text{He}}(293 \text{ K})}{M_{\text{H}_2}} = \frac{(4.0)(293 \text{ K})}{2.02} \rightarrow$$

$$\boxed{T = 580 \text{ K} = 307^\circ\text{C}}$$

Problem 5

At 20°C and 750 torr pressure, the mean free paths for argon gas (Ar) and nitrogen gas (N₂) are $\lambda_{\text{Ar}} = 9.9 \times 10^{-6}$ cm and $\lambda_{\text{N}_2} = 27.5 \times 10^{-6}$ cm. (a) Find the ratio of the effective diameter of argon to that of nitrogen. What is the mean free path of argon at (b) 20°C and 150 torr, and (c) -40°C and 750 torr?

$$\lambda = \frac{1}{\sqrt{2} \pi d^2 (N/V)} \quad pV = NKT \rightarrow \frac{N}{V} = \frac{p}{KT} \rightarrow \lambda = \frac{1}{\sqrt{2} \pi d^2 (p/KT)}$$

$$\frac{\lambda_{\text{N}_2}}{\lambda_{\text{Ar}}} = \frac{\frac{1}{\sqrt{2} \pi d_{\text{N}_2}^2 (p/KT)}}{\frac{1}{\sqrt{2} \pi d_{\text{Ar}}^2 (p/KT)}} = \frac{(1/d_{\text{N}_2}^2)}{(1/d_{\text{Ar}}^2)} = \left(\frac{d_{\text{Ar}}}{d_{\text{N}_2}} \right)^2 \rightarrow \frac{d_{\text{Ar}}}{d_{\text{N}_2}} = \sqrt{\frac{\lambda_{\text{N}_2}}{\lambda_{\text{Ar}}}} = \boxed{1.7}$$

$$(b) \quad \lambda = \frac{1}{\sqrt{2} \pi d^2 (p/KT)} \rightarrow \lambda = \frac{KT}{\sqrt{2} \pi d^2 p} \rightarrow \frac{p\lambda}{T} = \frac{K}{\sqrt{2} \pi d^2} = \text{constant}$$

$$\frac{p_i \lambda_i}{T_i} = \frac{p_f \lambda_f}{T_f} \rightarrow \lambda_f = \left(\frac{T_f}{T_i} \right) \left(\frac{p_i}{p_f} \right) \lambda_i = \left(\frac{293 \text{ K}}{293 \text{ K}} \right) \left(\frac{750 \text{ torr}}{750 \text{ torr}} \right) (9.9 \times 10^{-6} \text{ cm})$$

$$\boxed{\lambda_f = 5.0 \times 10^{-5} \text{ cm}}$$

$$(c) \quad \boxed{\lambda_f = 7.9 \times 10^{-6} \text{ cm}}$$

Problem 6

Twenty-two particles have speeds as follows (N_i represents the number of particles that have speed v_i):

N_i	2	4	6	8	2
v_i (cm/s)	1.0	2.0	3.0	4.0	5.0

(a) Compute their average speed v_{avg} . (b) Compute their root-mean-square speed v_{rms} . (c) Of the five speeds shown, which is the most probable speed v_p ?

$$(a) \quad v_{\text{ave}} = \frac{2(1.0 \text{ cm/s}) + 4(2.0 \text{ cm/s}) + 6(3.0 \text{ cm/s}) + 8(4.0 \text{ cm/s}) + 2(5.0 \text{ cm/s})}{22}$$

$$\boxed{v_{\text{ave}} = 3.2 \text{ cm/s}}$$

$$(b) \quad v_{\text{rms}} = \sqrt{\frac{2(1.0 \text{ cm/s})^2 + 4(2.0 \text{ cm/s})^2 + 6(3.0 \text{ cm/s})^2 + 8(4.0 \text{ cm/s})^2 + 2(5.0 \text{ cm/s})^2}{22}}$$

$$\boxed{v_{\text{rms}} = 3.4 \text{ cm/s}}$$

(c) $\boxed{v_p = 4.0 \text{ cm/s}} \Rightarrow$ this is the most probable speed because more particles (8) have this speed than any other.

Problem 7

One mole of an ideal gas undergoes an isothermal expansion. Find the energy added to the gas as heat in terms of the initial and final volumes and the temperature. (Hint: Use the first law of thermodynamics.)

⇒ for an isothermal expansion, $\Delta T = 0$ so $\Delta E_{\text{int}} = n C_v \Delta T = 0$

from 1st law $\rightarrow \Delta E = Q - W \rightarrow Q = \Delta E + W = W$

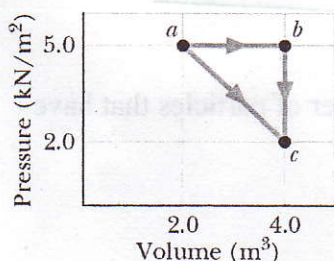
$$W = \int_{V_i}^{V_f} P dV \quad P = \frac{nRT}{V} \rightarrow W = \int_{V_i}^{V_f} \left(\frac{nRT}{V} \right) dV$$

$$W = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln \left(\frac{V_f}{V_i} \right) \rightarrow \text{for one mole } n=1$$

$$Q = W = RT \ln \left(\frac{V_f}{V_i} \right)$$

Problem 8

One mole of an ideal diatomic gas goes from a to c along the diagonal path in the figure below. During the transition, (a) what is the change in internal energy of the gas, and (b) how much energy is added to the gas as heat? (c) How much heat is required if the gas goes from a to c along the indirect path abc?



$$\Delta E_{\text{int}} = n C_v \Delta T \quad \text{diatomic } C_v = \frac{5}{2} R$$

$$\Delta E_{\text{int}} = Q - W$$

$$(a) \Delta E_{\text{int}} = \frac{5}{2} n R \Delta T$$

$$pV = nRT \rightarrow \Delta(pV) = nR \Delta T$$

$$\Delta E_{\text{int}} = \frac{5}{2} \Delta(pV) = \frac{5}{2} (p_c V_c - p_a V_a)$$

$$p_c = 2.0 \times 10^3 \text{ Pa} \quad V_c = 4.0 \text{ m}^3$$

$$p_a = 5.0 \times 10^3 \text{ Pa} \quad V_a = 2.0 \text{ m}^3$$

$$\Delta E_{\text{int}} = -5.0 \times 10^3 \text{ J}$$

$$(b) \Delta E_{\text{int}} = Q - W \rightarrow Q = \Delta E_{\text{int}} + W$$

$$W = \text{area under curve} = 7.0 \times 10^3 \text{ J}$$

$$Q = (-5.0 \times 10^3 \text{ J}) + (7.0 \times 10^3 \text{ J}) = 2.0 \times 10^3 \text{ J}$$

$$(c) (\Delta E_{\text{int}})_{a \rightarrow b \rightarrow c} = (\Delta E_{\text{int}})_{a \rightarrow c} = -5.0 \times 10^3 \text{ J}$$

$$W = \text{area under curve} = 10 \times 10^3 \text{ J}$$

$$Q = \Delta E_{\text{int}} + W \rightarrow Q = 5.0 \times 10^3 \text{ J}$$

Problem 9

Suppose 12.0 g of oxygen (O_2) is heated at constant atmospheric pressure from $25.0^\circ C$ to $125^\circ C$.

(a) How many moles of oxygen are present? (See Table 19-1 for the molar mass.) (b) How much energy is transferred to the oxygen as heat? (The molecules rotate but do not oscillate.) (c) What fraction of the heat is used to raise the internal energy of the oxygen?

$$M_{\text{sam}} = 12.0 \text{ g}$$

$$M_{O_2} = 32.0 \text{ g/mol}$$

$$n = ?$$

$$(a) \quad n = \frac{M_{\text{sam}}}{M} = \frac{12.0 \text{ g}}{32.0 \text{ g/mol}} = \boxed{0.375 \text{ mol}}$$

$$(b) \quad Q = n C_p \Delta T \quad C_p = C_v + R = \frac{7}{2} R \text{ for diatomic}$$

$$\Delta T = 100.0^\circ C = 100.0 \text{ K}$$

$$Q = \frac{7}{2} n R \Delta T = (\frac{7}{2})(0.375 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(100 \text{ K})$$

$$\boxed{Q = 1.09 \times 10^3 \text{ J}}$$

$$(c) \quad \text{fraction} = \frac{\Delta E_{\text{int}}}{Q} = \frac{n C_v \Delta T}{n C_p \Delta T} = \frac{\frac{5}{2} R}{\frac{7}{2} R} = \boxed{\frac{5}{7}}$$

Problem 10

When 1.0 mol of oxygen (O_2) gas is heated at constant pressure starting at $0^\circ C$, how much energy must be added to the gas as heat to double its volume? (The molecules rotate but do not oscillate.)

$$pV = nRT \rightarrow p = \frac{nRT}{V}$$

$C_v = \frac{5}{2} R + C_p = \frac{7}{2} R$
for a diatomic gas that
doesn't oscillate

$$\text{for constant pressure: } \frac{T_1}{V_1} = \frac{T_2}{V_2}$$

$$\frac{V_2}{V_1} = \frac{T_2}{T_1} \rightarrow \frac{V_2}{V_1} = 2 = \frac{T_2}{T_1} \rightarrow T_2 = 2T_1$$

$2T_1$

$$\text{for constant pressure: } Q = n C_p \Delta T = n (\frac{7}{2} R) (T_2 - T_1)$$

$$Q = \frac{7}{2} n R T_1 = \frac{7}{2} (1 \text{ mol}) (8.31 \text{ J/mol}\cdot\text{K}) (273 \text{ K})$$

$$\boxed{Q = 8 \times 10^3 \text{ J}}$$

Problem 11

The volume of an ideal gas is adiabatically reduced from 200 L to 74.3 L. The initial pressure and temperature are 1.00 atm and 300 K. The final pressure is 4.00 atm. (a) Is the gas monatomic, diatomic, or polyatomic? (b) What is the final temperature? (c) How many moles are in the gas?

for an adiabatic process: $P_i V_i^\gamma = P_f V_f^\gamma \rightarrow \left(\frac{V_i}{V_f}\right)^\gamma = P_f/P_i$

$$\left(\frac{200\text{ L}}{74.3\text{ L}}\right)^\gamma = \frac{4.00\text{ atm}}{1.00\text{ atm}} \rightarrow \left(\frac{200}{74.3}\right)^\gamma = 4.00$$

$$\gamma \log\left(\frac{200}{74.3}\right) = \log 4 \rightarrow \gamma = \frac{\log 4}{\log(200/74.3)} = 1.4 = 7/5$$

so gas is diatomic

(b) $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \rightarrow T_2 = \frac{P_2 V_2}{P_1 V_1} T_1 = \frac{(4.00\text{ atm})(74.3\text{ L})}{(1.00\text{ atm})(200\text{ L})} (300\text{ K})$

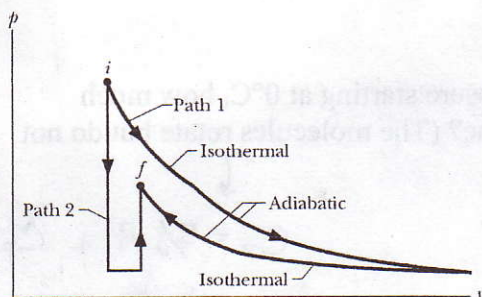
$T_2 = 446\text{ K}$

(c) from $n = \frac{PV}{RT} \rightarrow$

$n = 8.10\text{ moles}$

Problem 12

The figure below shows two paths that may be taken by a gas from an initial point *i* to a final point *f*. Path 1 consists of an isothermal expansion (work is 50 J in magnitude), an adiabatic expansion (work is 40 J in magnitude), an isothermal compression (work is 30 J in magnitude), and then an adiabatic compression (work is 25 J in magnitude). What is the change in the internal energy of the gas if the gas goes from point *i* to point *f* along path 2?



$$(\Delta E_{\text{int}})_{\text{path 2}} = (\Delta E_{\text{int}})_{\text{path 1}}$$

path 1 \rightarrow 2 isotherms + 2 adiabats

↓

$$\Delta T = 0 \text{ so } \Delta E_{\text{int}} = 0$$

for adiabats $\rightarrow Q = 0$ so $\Delta E_{\text{int}} = -W$

adiabatic expansion: $W = 40\text{ J}$ so $\Delta E_{\text{int}} = -40\text{ J}$

adiabatic compression: $W = -25\text{ J}$ so $\Delta E_{\text{int}} = 25\text{ J}$

$$(\Delta E_{\text{int}})_{\text{path 1}} = 0\text{ J} + (-40\text{ J}) + 0\text{ J} + (25\text{ J}) = -15\text{ J}$$

$(\Delta E_{\text{int}})_{\text{path 2}} = -15\text{ J}$