Answer Key

terms of the angular separation of lines of different wavelength ($D = \Delta\theta/\Delta\lambda$) and in terms of the grating properties ($D = m/d\cos\theta$). You should know the meaning of the resolving power and how to calculate it in terms of the difference in wavelength ($R = \lambda/\Delta\lambda$) and in terms of grating properties ($R = Nm$).

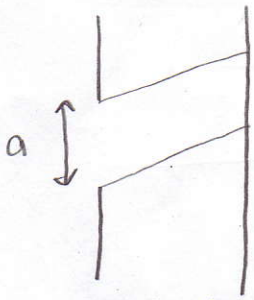
You should know that the diffraction angle for the first minimum of a circular aperture is given by $\theta_R = \sin^{-1}(1.22\lambda/d)$, where d is the diameter of the aperture. You should also know that this expression also gives the Rayleigh criterion for the resolution of two far-away objects. If $d \gg \lambda$ then θ_R is given in radians by $\theta_R = 1.22\lambda/d$.

You should be able to use the Bragg condition for diffraction from a crystal: $2d\sin\theta = m\lambda$. Remember that θ is measured from the reflecting planes, not their normal. Some problems ask you to use the Bragg condition to find the atomic separation in a crystal.

Questions and Example Problems from Chapter 36

Question 1

You are conducting a single-slit diffraction experiment with light of wavelength λ. What appears, on a distant viewing screen, at a point at which the top and bottom rays through the slit have a path length difference equal to (a) 5λ and (b) 4.5λ?



$a\sin\theta = m\lambda \longrightarrow \Delta L = m\lambda \quad m = 1, 2, \ldots$

(a) $\boxed{\text{dark fringe}}$ ($m = 5$ minimum)

(b) $\boxed{\text{bright fringe}}$ (approximately the maximum between $m = 4$ + $m = 5$ minimum)

Question 2

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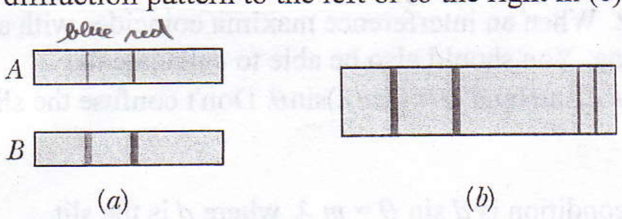
(a) Figure a below shows the lines produced by diffraction gratings A and B using light of the same wavelength; the lines are of the same order and appear at the same angles θ. Which grating has the greater number of rulings? (b) Figure b below shows lines of two orders produced by a single diffraction grating using light of two wavelengths, both in the red region of the spectrum. Which lines, the left pair or right pair, are in the order with greater m? Is the center of the diffraction pattern to the left or to the right in (c) Fig. a and (d) Fig. b?



$\Delta\theta_{hw} = \dfrac{\lambda}{N d\cos\theta}$

bigger N, smaller $\Delta\theta_{hw}$

(a) $\boxed{A}$

(b) $\boxed{\text{left}}$

$D = \dfrac{m}{d\cos\theta} \rightarrow$ bigger m, bigger dispersion

(c) $\boxed{\text{left}}$ (d) $\boxed{\text{right}}$

Problem 1

Monochromatic light of wavelength 441 nm is incident on a narrow slit. On a screen 2.00 m away, the distance between the second diffraction minimum and the central maximum is 1.50 cm. (a) Calculate the angle of diffraction θ of the second minimum. (b) Find the width of the slit.

$\lambda = 441\times10^{-9}\,m$ $\tan\theta = y_m/D \longrightarrow$ for small angles $\theta = y_m/D$

$D = 2.00\,m$

$y_m = 1.50\times10^{-2}\,m$ (a) $\theta = \dfrac{0.0150\,m}{2.00\,m} \longrightarrow \boxed{\theta = 0.0075 \text{ rad or } 0.430°}$

(b) $m = 2$ $a\sin\theta = m\lambda$

$a = ?$ $a = \dfrac{m\lambda}{\sin\theta} = \dfrac{2(441\times10^{-9}\,m)}{\sin 0.430°} \longrightarrow \boxed{a = 1.18\times10^{-4}\,m}$

Problem 2

A slit 1.00 mm wide is illuminated by light of wavelength 589 nm. We see a diffraction pattern on a screen 3.00 m away. What is the distance between the first two diffraction minima on the same side of the central diffraction maximum?

$$\left.\begin{array}{l}\sin\theta = m\lambda/a \\ \tan\theta = y_m/D\end{array}\right\} \quad y_m = m\lambda D/a \text{ for small angles}$$

$$\left.\begin{array}{l}y_2 = 2\lambda D/a \\ y_1 = 1\lambda D/a\end{array}\right\rangle \quad \Delta y = y_2 - y_1 = \lambda D/a$$

$$\Delta y = \frac{(589\times10^{-9}\,m)(3.00\,m)}{(1.00\times10^{-3}\,m)} \longrightarrow \boxed{\begin{array}{l}\Delta y = 1.77\times10^{-3}\,m \\ \quad = 1.77\,mm\end{array}}$$

Problem 3

A single-slit diffraction experiment is set up with light of wavelength 420 nm, incident perpendicularly on a slit of width 5.10 μm. The viewing screen is 3.20 m distant. On the screen, what is the distance between the center of the diffraction pattern and the second diffraction minimum?

$\lambda = 420\times10^{-9}\,m$ $a\sin\theta = m\lambda \longrightarrow \boxed{\sin\theta = m\lambda/a}$

$a = 510\times10^{-6}\,m$ $\tan\theta = y_m/D$

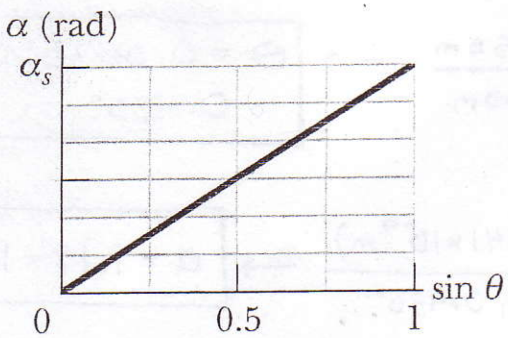
$D = 3.20\,m$ for small angles, $\tan\theta \approx \sin\theta \approx \theta$

$\Delta y = ?$ $m\lambda/a = y_m/D \longrightarrow y_m = \dfrac{m\lambda D}{a}$

$$y_m = \frac{(2)(420\times10^{-9}\,m)(3.20\,m)}{510\times10^{-6}\,m} \longrightarrow \boxed{y_m = 5.27\,mm}$$

Problem 4

The figure below gives α versus the sine of the angle θ in a single-slit diffraction experiment using light of wavelength 610 nm. The vertical axis scale is set by as $\alpha_s = 12$rad. What are (a) the slit width, (b)the total number of diffraction minima in the pattern (count them on both sides of the center of the diffraction pattern), (c) the least angle for a minimum, and (d) the greatest angle for a minimum.



$\lambda = 610\,\text{nm}$

when $\sin\theta = 1 \rightarrow \alpha = 12$ rad.

$$\alpha = \frac{\pi a}{\lambda}\sin\theta \longrightarrow (12\,\text{rad}) = \pi a/\lambda\,(1)$$

$$a = \frac{(12\,\text{rad})\,\lambda}{\pi} = \frac{(12\,\text{rad})(610\,\text{nm})}{\pi} \longrightarrow a = 2330\,\text{nm}$$

$$\boxed{a = 2.33\times10^{-6}\,\text{m}}$$

(b) $a\sin\theta = m\lambda \rightarrow \sin\theta = m\lambda/a \qquad m\lambda/a < 1$

$$m < a/\lambda \longrightarrow m < \frac{2330\,\text{nm}}{610\,\text{nm}} \longrightarrow m < 3.82$$

$\boxed{\text{6 minima total}}$ $\longrightarrow$ $m = 1, 2, + 3$ on either side

(c) least angle $\rightarrow$ solve for θ with $m = 1$

$$a\sin\theta = m\lambda \rightarrow \sin\theta = \frac{m\lambda}{a} = \frac{(1)(610\,\text{nm})}{2330\,\text{nm}} = 0.262$$

$$\boxed{\theta = 15.2^\circ}$$

(d) greatest angle $\rightarrow$ solve for θ with $m = 3$

$$\sin\theta = (3)(610\,\text{nm})/2330\,\text{nm} = 0.785 \longrightarrow \boxed{\theta = 51.8^\circ}$$

Problem 5

A double-slit system with individual slit widths of 0.030 mm and a slit separation of 0.18 mm is illuminated with 500 nm light directed perpendicular to the plane of the slits. What is the total number of bright fringes appearing between the two first order minima of the diffraction pattern? (Do not count the fringes that coincide with the minima of the diffraction pattern.)

$a = 0.030\,mm$

$d = 0.18\,mm$

$m = d/a \rightarrow$ gives order of interference fringe that coincides with the first diffraction minima

$$m = \frac{0.18\,mm}{0.030\,mm} = 6$$

$m = 6$ fringe is removed

$\Rightarrow$ we see the $m = 0$ and $m = 1, 2, 3, 4, \,\&\, 5$ fringes on either side

$\boxed{\text{11 bright fringes}}$

Problem 6

In a double-slit experiment, the slit separation d is 2.00 times the slit width w. How many bright interference fringes are in the central diffraction envelope?

$d = 2.00w = 2.00a$

$\Rightarrow$ location of first minima ($m=1$) in diffraction pattern: $a \sin\theta = \lambda$

$\sin\theta = \lambda/a$

$\Rightarrow$ location of maxima (bright fringes) in double-slit interference: $d \sin\theta = m\lambda$

✓ put into eq. for maxima

$d(\lambda/a) = m\lambda$

$m = d/a = \frac{2.00a}{a} = 2.00 \longrightarrow$ since bright fringes corresponding to $m=2$ are in diffraction minima, these fringes are removed

$\boxed{\#\text{ bright fringes} = 3}$

↓ central bright fringe ($m=0$) + $m=1$ fringe on either side

Problem 7

Light of wavelength 600 nm is incident normally on a diffraction grating. Two adjacent maxima occur at angles given by sin θ = 0.2 and sin θ = 0.3. The fourth-order maxima are missing. (a) What is the separation between adjacent slits? (b) What is the smallest slit width this grating can have? (c) Which orders of intensity maxima are produced by the grating, assuming the values derived in (a) and (b)?

diffraction grating $\longrightarrow$ $d \sin\theta = m\lambda$ adjacent maxima are separated by $\Delta m = 1$

(a) $d(0.2) = m\lambda$ (1)

$d(0.3) = (m+1)\lambda$ (2)

subtract (1) from (2)

$(0.1)\,d = \lambda$

$d = 10\lambda = 6000\text{ nm} = \boxed{6.0\times10^{-6}\text{ m}}$

(b) $m = 4$ maximum missing $\rightarrow$ corresponds to first min. in diffraction pattern

$d/a = 4$

$a = d/4 = \boxed{1.5\times10^{-6}\text{ m}}$

(c) $\sin\theta = m\lambda/d$ $\quad m\lambda/d \leq 1 \rightarrow m \leq d/\lambda \rightarrow m \leq \dfrac{6.00\times10^{-6}\text{ m}}{600\times10^{-9}\text{ m}}$

$m \leq 10$ observable orders $\Rightarrow$ $\boxed{m = 0, 1, 2, 3, 5, 6, 7, 9}$ $\rightarrow$ $m = 8$ is missing because falls on 2^{nd} diffraction min.

Problem 8

A diffraction grating 20.0 mm wide has 6000 rulings. (a) Calculate the distance d between adjacent rulings. (b) At what angles θ will intensity maxima occur on a viewing screen if the radiation incident on the grating has a wavelength of 589 nm?

$w = 20.0\text{mm} = 20.0\times10^{-3}\text{ m}$

$N = 6000$

grating spacing $d = w/N$

(a) $d = w/N = \dfrac{20.0\times10^{-3}\text{ m}}{6000} \longrightarrow \boxed{d = 3.33\times10^{-6}\text{ m}}$

(b) $\sin\theta = m\lambda/d$ $\quad m = 0, 1, 2, \ldots$

since $\sin\theta \leq 1$, we must have $m\lambda/d \leq 1$ or $m \leq d/\lambda$

$m \leq \dfrac{3.33\times10^{-6}\text{ m}}{589\times10^{-9}\text{ m}} \longrightarrow \underline{\underline{m \leq 5.7}}$

$\theta = \sin^{-1}\left[\dfrac{m(589\times10^{-9}\text{m})}{3.33\times10^{-6}\text{m}}\right] \longrightarrow$ $\boxed{\begin{array}{l} m=0\ \theta=0^\circ;\ m=1\ \theta=10.2^\circ; \\ m=2\ \theta=20.7^\circ;\ m=3\ \theta=32.2^\circ; \\ m=4\ \theta=45^\circ;\ m=5\ \theta=62.2^\circ \end{array}}$

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Problem 9
A grating has 600 rulings/mm and is 5.0 mm wide. (a) What is the smallest wavelength interval it can resolve in the third order at $\lambda = 500$ nm? (b) How many higher orders of maxima can be seen?

$$\left.\begin{aligned} N/w &= 600 \text{ rulings/mm} \\ w &= 5.0 \text{ mm} \end{aligned}\right\} N = 3000$$

(a) $\mathcal{R} = \dfrac{\lambda_{ave}}{\Delta\lambda}$ $\quad \mathcal{R} = Nm \longrightarrow \dfrac{\lambda_{ave}}{\Delta\lambda} = Nm \longrightarrow \Delta\lambda = \dfrac{\lambda_{ave}}{Nm}$

$$\Delta\lambda = \frac{(500\text{nm})}{(3000)(3)} = \boxed{5.6\times10^{-11}\text{m} = 56\text{ pm}}$$

(b) $m\lambda/d \le 1 \longrightarrow m \le d/\lambda$ $\quad d = w/N = 1\text{mm}/600 \times \dfrac{1\text{m}}{10^3\text{mm}} = 1.67\times10^{-6}\text{m}$

$$m \le (1.67\times10^{-6}\text{m}) / 500\times10^{-9}\text{m} \rightarrow m \le 3.34$$

$\boxed{\text{m = 3 is highest order, so no higher orders can be seen}}$

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Problem 10
The *D* line in the spectrum of sodium is a doublet with wavelengths 589.0 and 589.6 nm. Calculate the minimum number of lines needed in a grating that will resolve this doublet in the second-order spectrum.

$$\mathcal{R} = \frac{\lambda_{avg}}{\Delta\lambda} = Nm$$

$$\lambda_{avg} = \frac{589.0\text{nm} + 589.6\text{nm}}{2} = \underline{\underline{589.3\text{nm}}}$$

$$\Delta\lambda = 589.6\text{nm} - 589.0\text{nm} = \underline{\underline{0.60\text{nm}}}$$

$m = 2$

$N = ?$ $\qquad N = \dfrac{\lambda_{avg}}{m\Delta\lambda}$

$$N = \frac{(589.3\text{nm})}{2(0.60\text{nm})} \longrightarrow \boxed{N = 491}$$

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Problem 11

An x-ray beam of a certain wavelength is incident on a NaCl crystal, at 30.0° to a certain family of reflecting planes of spacing 39.8 pm. If the reflection from those planes is of the first order, what is the wavelength of the x rays?

$\theta = 30.0°$ $\qquad 2d\sin\theta = m\lambda \quad m = 1, 2, 3, \ldots$

$d = 39.8\text{ pm}$ $\qquad$ first order $\rightarrow$ $m = 1$

$= 39.8\times10^{-12}\text{ m}$

$m = 1$ $\qquad 2d\sin\theta = (1)\lambda$

$\lambda = ?$ $\qquad \lambda = 2d\sin\theta$

$$\lambda = 2(39.8\times10^{-12}\text{ m})\sin 30.0°$$

$$\boxed{\lambda = 3.98\times10^{-11}\text{ m} = 39.8\text{ pm}}$$

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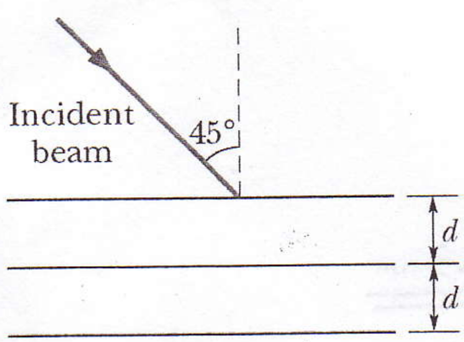
Problem 12

In the figure below, an x-ray beam of wavelengths from 95.0 pm to 140 pm is incident at 45° to a family of reflecting planes with spacing d = 275 pm. At which wavelengths will these planes produce intensity maxima in their reflections?



$\lambda = 95.0\text{ pm} - 140\text{ pm}$ $\qquad \text{pm} = 10^{-12}\text{ m}$

$\theta = 45°$

$d = 275\text{ pm}$

maxima: $2d\sin\theta = m\lambda \quad m = 1, 2, 3, \ldots$

$$\lambda = \frac{2d\sin\theta}{m} = \frac{2(275\text{ pm})\sin 45°}{m}$$

$$\lambda = \frac{389\text{ pm}}{m} \quad m = 1, 2, 3, \ldots$$

$$\lambda = \underset{m=1}{389\text{ pm}},\ \underset{m=2}{194\text{ pm}},\ \underset{m=3}{130\text{ pm}},\ \underset{m=4}{97.2\text{ pm}},\ \ldots$$

$$\boxed{\lambda = 97.2\text{ pm} \ \& \ 130\text{ pm}}$$

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Problem 13

The two headlights of an approaching automobile are 1.4 m apart. At what (a) angular separation and (b) maximum distance will the eye resolve them? Assume that the pupil diameter is 5.0 mm, and use a wavelength of 550 nm for the light. Also assume that diffraction effects alone limit the resolution so that Rayleigh's criterion can be applied.

$d = 5.0 \times 10^{-3}\,m$

$\lambda = 550 \times 10^{-9}\,m$

$x = 1.4\,m$

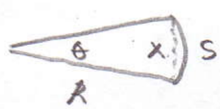
$r = ?$

(a) $\theta_R = 1.22\lambda/d$

$$\theta_R = \frac{1.22(550 \times 10^{-9}\,m)}{5.0 \times 10^{-3}\,m}$$

$$\theta_R = 1.34 \times 10^{-4}\,rad$$

(b)



$s = R\theta \longrightarrow x = R\theta$ for small angles

$$R = x/\theta = \frac{1.4\,m}{1.34 \times 10^{-4}} \longrightarrow \boxed{R = 1.0 \times 10^{4}\,m = 10\,Km}$$

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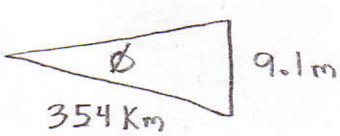
Problem 14

In June 1985, a laser beam was sent out from the Air Force Optical Station on Maui, Hawaii, and reflected back from the shuttle Discovery as it sped by, 354 km overhead. The diameter of the central maximum of the beam at the shuttle position was said to be 9.1 m, and the beam wavelength was 500 nm. What is the effective diameter of the laser aperture at the Maui ground station? (Hint: A laser beam spreads only because of diffraction; assume a circular exit aperture.)



for small angles: $\phi = \dfrac{9.1\,m}{354 \times 10^{3}\,m} = \boxed{2.57 \times 10^{-5}\,rad}$

$\Rightarrow$ for a circular aperture, $\sin\theta = \dfrac{1.22\lambda}{d} \longrightarrow \theta = \dfrac{1.22\lambda}{d}$ for small angles

$\theta = 1.22\lambda/d$ gives angle from the central axis to any point on the circular minimum so in our case, $\underline{\phi = 2\theta}$

$$\phi = 2\theta = 2(1.22\lambda/d) \longrightarrow d = \frac{2(1.22\lambda)}{\phi}$$

$$d = \frac{2(1.22)(500 \times 10^{-9}\,m)}{2.57 \times 10^{-5}\,rad} \longrightarrow \boxed{d = 0.047\,m}$$