For example, the wave function for a free particle traveling in the positive x direction is $\psi(x) = Ae^{ikx}$, where k is related to the momentum p of the particle by $p = hk/2\pi$. Since $e^{i\alpha} = \cos\alpha + i\sin\alpha$, the free-particle wave function can be written

$\psi = Ae^{ikx} = A\cos(kx) + iA\sin(kx)$.

Both the real and imaginary parts are sinusoidal and both have the same wavelength λ, related to k by $k = 2\pi/\lambda$.

The complex conjugate of a complex number has the same real part as the original number but its imaginary part is the negative of the imaginary part of the original number. Thus, the complex conjugate of $\psi = \psi_R + i\psi_I$ is $\psi^* = \psi_R - \psi_I$ and the complex conjugate of e^{ikx} is e^{-ikx}.

The square of the magnitude of a complex number is found by multiplying the number by its complex conjugate: $|\psi|^2 = \psi\psi^*$. Thus, the square of the magnitude of ψ is

$|\psi|^2 = (\psi_R + i\psi_I)(\psi_R - i\psi_I) = \psi_R^2 + \psi_I^2$.

It is the sum of squares of the real and imaginary parts. The square of the magnitude of the free-particle wave function is

$|Ae^{ikx}|^2 = Ae^{ikx}Ae^{-ikx} = A^2e^0 = A^2$,

where we have assumed A is real.

If a particle has energy E, its complete wave function Ψ is the product of a coordinate-dependent function and a time-dependent function. The time-dependent function has the form $e^{i\omega t}$, where ω is the angular frequency. Since $\omega = 2\pi f$ and $f = E/h$, where f is the frequency, the angular frequency is related to the energy by $\omega = 2\pi E/h$. Thus, $\Psi = \psi e^{i\omega t}$. Ψ and ψ lead to the same probability density:

$|\Psi|^2 = |\psi e^{i\omega t}|^2 = \psi\psi^* e^{i\omega t}e^{-i\omega t} = \psi\psi^* = |\psi|^2$.

Questions and Example Problems from Chapter 38

Question 1

Of the following statements about the photoelectric effect, which are true and which are false? (a) The greater the frequency of the incident light is, the greater is the stopping potential. (b) The greater the intensity of the incident light is, the greater is the cutoff frequency. (c) The greater the work function of the target material is, the greater is the stopping potential. (d) The greater the work function of the target material is, the greater is the cutoff frequency. (e) The greater the frequency of the incident light is, the greater is the maximum kinetic energy of the ejected electrons. (f) The greater the energy of the photons is, the smaller is the stopping potential

(a) $\boxed{\text{true}}$ → greater f, greater KE_{max}

(b) $\boxed{\text{false}}$

(c) $\boxed{\text{false}}$

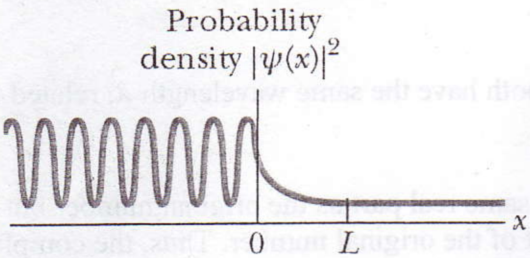
(d) $\boxed{\text{true}}$ → $\Phi = hf_0$

(e) $\boxed{\text{true}}$ → $KE_{max} = hf - \Phi$

(f) $\boxed{\text{false}}$

Question 2

In the figure below, why are the minima in the values of $|\psi|^2$ greater than zero?



$\Rightarrow$ the amplitude of the reflected wave is less than the amplitude of the incident wave so we do not get fully destructive interference

Question 3

The following nonrelativistic particles all have the same kinetic energy. Rank them in order of their de Broglie wavelengths, greatest first: electron, alpha particle, neutron.

$$KE = \tfrac{1}{2}mv^2 = p^2/2m \longrightarrow p = \sqrt{2mKE}$$

since KE is same for each, the bigger m, the great p

de Broglie wavelength: $\lambda = h/p$ (bigger p, smaller λ)

$p_\alpha > p_n > p_e$

$$\boxed{\lambda_e > \lambda_n > \lambda_\alpha}$$

Problem 1

An ultraviolet lamp emits light of wavelength 400 nm, at the rate (power) of 400 W. An infrared lamp emits light of wavelength 700 nm, also at the rate of 400 W. (a) Which lamp emits photons at the greater rate and (b) what is that greater rate?

rate at which photons are emitted = $\dfrac{\text{rate at which energy is emitted}}{\text{energy per photon}}$

$R = P/E \quad E = hf = hc/\lambda$

$R = P\lambda/hc$ (a) greater $\lambda \rightarrow$ greater R

700nm lamp has greater rate

(b) $R = \dfrac{P\lambda}{hc} = \dfrac{(400\,J/s)(700\,nm)}{(1240\,eV\cdot nm)(1.6\times10^{-19}\,J/eV)}$ $\quad hc = 1240\,eV\cdot nm$

$$\boxed{R = 1.41\times10^{21}\ \text{photons/s}}$$

Problem 2

A spectral emission line is electromagnetic radiation that is emitted in a wavelength range narrow enough to be taken as a single wavelength. One such emission line that is important in astronomy has a wavelength of 21 cm. What is the photon energy in the electromagnetic wave at that wavelength?

$$E = hf = \frac{hc}{\lambda} \rightarrow E = \frac{(6.63\times10^{-34}\,J\cdot s)(3.0\times10^{8}\,m/s)}{(0.21\,m)}$$

$$E = 9.47\times10^{-25}\,J\left(\frac{1\,eV}{1.602\times10^{-19}\,J}\right) = \boxed{5.9\times10^{-6}\,eV}$$

or using $hc = 1240\,eV\cdot nm$:

$$E = \frac{hc}{\lambda} = \frac{1240\,eV\cdot nm}{21\times10^{7}\,nm} = \boxed{5.9\times10^{-6}\,eV}$$

Problem 3

The stopping potential for electrons emitted from a surface illuminated by light of wavelength 491 nm is 0.710 V. When the incident wavelength is changed to a new value, the stopping potential is 1.43 V. (a) What is this new wavelength? (b) What is the work function for the surface?

$\lambda_1 = 491\,nm$

$V_{stop_1} = 0.710\,V$

$\lambda_2 = ?$

$V_{stop_2} = 1.43\,V$

$$KE_{max} = eV_{stop} \rightarrow KE_{max_1} = 0.710\,eV$$

$$KE_{max_2} = 1.43\,eV$$

$$KE_{max} = hf - \Phi = hc/\lambda - \Phi$$

$\Phi = hc/\lambda - KE_{max} \rightarrow \Phi$ is a constant for surface

$$hc/\lambda_1 - KE_{max_1} = hc/\lambda_2 - KE_{max_2} \rightarrow hc/\lambda_2 = hc/\lambda_1 - KE_{max_1} + KE_{max_2}$$

$$hc/\lambda_2 = \frac{1240\,eV\,nm}{491\,nm} - 0.710\,eV + 1.43\,eV \rightarrow hc/\lambda_2 = 3.25\,eV$$

$$\lambda_2 = hc/3.25\,eV = \frac{1240\,eV\cdot nm}{3.25\,eV} \longrightarrow \boxed{\lambda_2 = 382\,nm}$$

(b) $$\Phi = hc/\lambda - KE_{max} = \frac{1240\,eV\cdot nm}{491\,nm} - 0.710\,eV \rightarrow \boxed{\Phi = 1.82\,eV}$$

Problem 4
The wavelength associated with the cutoff frequency for silver is 325 nm. Find the maximum kinetic energy of electrons ejected from a silver surface by ultraviolet light of wavelength 254 nm

$\lambda_0 = c/f_0 = 325\,nm \longrightarrow$ cutoff frequency $f_0 = c/\lambda_0$

since this is the minimum frequency that will eject an electron, $\Phi = hf_0 = hc/\lambda_0$

$hf = K_{max} + \Phi$

$hc/\lambda = K_{max} + hc/\lambda_0 \longrightarrow K_{max} = hc/\lambda - hc/\lambda_0$

$$K_{max} = \frac{1240\,eV\cdot nm}{254\,nm} - \frac{1240\,eV\cdot nm}{325\,nm} \longrightarrow \boxed{K_{max} = 1.07\,eV}$$

Problem 5
In a photoelectric experiment using a sodium surface, you find a stopping potential of 1.85 V for a wavelength of 300 nm and a stopping potential of 0.820 V for a wavelength of 400 nm. From these data find (a) a value for the Planck constant, (b) the work function ϕ for sodium, and (c) the cutoff wavelength λ_0 for sodium.

$V_{stop} = 1.85\,V$ for $\lambda = 300\,nm \longrightarrow eV_{stop1} = hc/\lambda_1 - \Phi \quad (1)$

$V_{stop} = 0.820\,V$ for $\lambda = 400\,nm \longrightarrow eV_{stop2} = hc/\lambda_2 - \Phi \quad (2)$

(a) subtract (2) from (1) $\longrightarrow e(V_1 - V_2) = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$

$$h = \frac{e(V_1 - V_2)}{c\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)} \longrightarrow h = \frac{e(1.85V - 0.820V)}{(3.0\times10^{17}\,nm/s)\left(\frac{1}{300nm} - \frac{1}{400nm}\right)} = \boxed{4.12\times10^{-15}\,eV\cdot s}$$

(b) add (1)+(2) $\longrightarrow e(V_1 + V_2) = hc\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right) - 2\Phi$

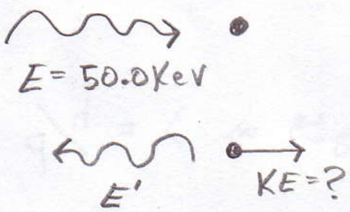
$$\Phi = \frac{hc\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right) - e(V_1 + V_2)}{2} = \frac{(1240\,eV\cdot nm)\left(\frac{1}{300nm} + \frac{1}{400nm}\right) - 2.67\,eV}{2}$$

$\Phi = 2.27\,eV$ (c) $\Phi = hf_0 = hc/\lambda_0 \longrightarrow \lambda_0 = \frac{hc}{\Phi} = \frac{1240\,eV\cdot nm}{2.27\,eV}$

$\boxed{\lambda_0 = 546\,nm}$

Problem 6

Consider a collision between an x-ray photon of initial energy 50.0 keV and an electron at rest, in which the photon is scattered backward and the electron is knocked forward. (a) What is the energy of the back-scattered photon? (b) What is the kinetic energy of the electron?



$$\phi = 180^\circ$$

$$h/m_e c = 2.43\times10^{-12}\ \text{m}$$

$$\Delta\lambda = h/m_e c\,(1-\cos\phi) = (2.43\times10^{-12}\ \text{m})(1-\cos 180^\circ)$$

$$= 4.86\times10^{-12}\ \text{m}$$

(a) $E = hc/\lambda \rightarrow \lambda = hc/E = \dfrac{1240\ \text{eV}\cdot\text{nm}}{50.0\times10^{3}\ \text{eV}} = 0.0248\ \text{nm} = 2.48\times10^{-11}\ \text{m}$

$\lambda' = \lambda + \Delta\lambda = 2.97\times10^{-11}\ \text{m} \rightarrow E' = hc/\lambda' = \dfrac{(1240\ \text{eV}\cdot\text{nm})}{0.0297\ \text{nm}}$

$E' = 4.18\times10^{4}\ \text{eV} = \boxed{41.8\ \text{eV}}$

(b) from conservation of energy $\rightarrow KE = E - E'$

$KE = 50.0\ \text{KeV} - 41.8\ \text{KeV} = \boxed{8.2\ \text{KeV}}$

Problem 7

Gamma rays of photon energy 0.511 MeV are directed onto an aluminum target and are scattered in various directions by loosely bound electrons there. (a) What is the wavelength of the incident gamma rays? (b) What is the wavelength of gamma rays scattered at 90.0° to the incident beam? (c) What is the photon energy of the rays scattered in this direction?

$E = 0.511\ \text{MeV}$

$= 0.511\times10^{6}\ \text{eV}$

(a) $E = hf = hc/\lambda \rightarrow \lambda = hc/E$

$\lambda = \dfrac{1240\ \text{eV}\cdot\text{nm}}{0.511\times10^{6}\ \text{eV}} \rightarrow \lambda = 2.43\times10^{-3}\ \text{nm}$

$= \boxed{2.43\ \text{pm}}$

(b) $\Delta\lambda = \dfrac{h}{m_e c}(1-\cos\phi) = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(9.11\times10^{-31}\ \text{Kg})(3.0\times10^{8}\ \text{m/s})}(1-\cos 90.0^\circ)$

$\Delta\lambda = 2.43\ \text{pm} = \lambda' - \lambda \rightarrow \lambda' = \lambda + \Delta\lambda = \boxed{4.86\ \text{pm}}$

(c) $E = hf' = hc/\lambda'$

$E = \dfrac{1240\ \text{eV}\cdot\text{nm}}{4.86\times10^{3}\ \text{nm}} \longrightarrow \boxed{E = 0.255\ \text{MeV}}$

Problem 8
A bullet of mass 40 g travels at 1000 m/s. Although the bullet is clearly too large to be treated as a matter wave, determine what Eq. 38-13 predicts for its de Broglie wavelength.

$m = 40 g = 40.0 \times 10^{-3}$ Kg

$v = 1000$ m/s

$\lambda = ?$

de Broglie wavelength $\Rightarrow \lambda = h/p$

$$\lambda = h/mv$$

$$\lambda = \frac{(6.63 \times 10^{-34} J \cdot s)}{(4.0 \times 10^{-2} Kg)(1000 m/s)}$$

$$\boxed{\lambda = 1.7 \times 10^{-35} m}$$

Problem 9
A nonrelativistic particle is moving three times as fast as an electron. The ratio of the de Broglie wavelength of the particle to that of the electron is 1.813×10^{-4}. By calculating its mass, identify the particle.

$V_{particle} = 3 V_{electron}$

de Broglie wavelength $\lambda = h/p = h/mv$

$$\frac{\lambda_{particle}}{\lambda_{electron}} = 1.813 \times 10^{-4} \longrightarrow \frac{\frac{h}{m_p V_p}}{\frac{h}{m_e V_e}} = 1.813 \times 10^{-4}$$

$$\left(\frac{m_e}{m_p}\right)\underbrace{\left(\frac{V_e}{V_p}\right)}_{= 1/3} = 1.813 \times 10^{-4} \longrightarrow \left(\frac{m_e}{m_p}\right) = 3(1.813 \times 10^{-4}) = 5.439 \times 10^{-4}$$

$$m_p = \frac{m_e}{5.439 \times 10^{-4}} = \frac{9.11 \times 10^{-31} Kg}{5.439 \times 10^{-4}} \longrightarrow \boxed{m_p = 1.6749 \times 10^{-27} Kg}$$

particle is a neutron

Problem 10
Imagine playing baseball in a universe (not ours!) where the Planck constant is 0.60 J·s. What would be the uncertainty in the position of a 0.50 kg baseball that is moving at 20 m/s along an axis if the uncertainty in the speed is 1.0 m/s?

$h = 0.60\ J\cdot s$

$m = 0.50 Kg$

$\Delta X = ?$

$\Delta V = 1.0 m/s$

$\Delta p = \Delta(mv) = m\Delta V$

$\Delta p = (0.50 Kg)(1.0 m/s) = 0.50 Kg\, m/s$

uncertainty principle: $\Delta X \Delta p_x \geq \hbar$

$\Delta X \Delta p_x \geq h/2\pi$

$$\Delta X \geq \frac{h}{2\pi \Delta p_x}$$

$$\Delta X \geq \frac{(0.60\ J\cdot s)}{2\pi(0.50 Kg m/s)} \longrightarrow \boxed{\Delta X \geq 0.19 m}$$

Problem 11
A 1500 kg car moving at 20 m/s approaches a hill that is 24 m high and 30 m long. Although the car and hill are clearly too large to be treated as matter waves, determine what Eq. 38-21 predicts for the transmission coefficient of the car, as if it could tunnel through the hill as a matter wave. Treat the hill as a potential energy barrier where the potential energy is gravitational.

$m = 1500 Kg$

$v = 20 m/s$

$h = 24 m$

$L = 30 m$

$U = mgh = (1500 Kg)(9.8 m/s^2)(24 m) = 3.53\times10^5 J$

$E = \frac{1}{2}mv^2 = \frac{1}{2}(1500 Kg)(20 m/s)^2 = 3.0\times10^5 J$

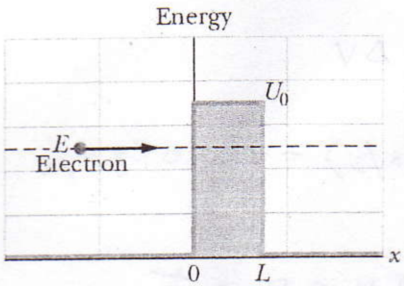
$T = e^{-2KL}$ $\quad K = \sqrt{\frac{8\pi^2 m(U-E)}{h^2}}$

$$K = \sqrt{\frac{8\pi^2(1500 Kg)(3.53\times10^5 J - 3.0\times10^5 J)}{(6.63\times10^{-34} J\cdot s)^2}}$$

$K = 1.28\times10^{38} m^{-1}$

$$\boxed{T = e^{-2KL} \approx 0}$$

Problem 12
Consider a potential energy barrier like that of the figure below but whose height U_0 is 6.0 eV and whose thickness L is 0.70 nm. What is the energy of an incident electron whose transmission coefficient is 0.0010?



$$U_0 = 6.0\,\text{eV}$$
$$L = 0.70\,\text{nm}$$
$$T = 0.0010$$
$$E = ?$$

$$T \approx e^{-2KL} = e^{-2L\left[\sqrt{\frac{8\pi^2 m (U_0 - E)}{h^2}}\right]}$$

$$\ln T = -2L\sqrt{\frac{8\pi^2 m (U_0 - E)}{h^2}}$$

$$\frac{(\ln T)^2}{4L^2} = \frac{8\pi^2 m (U_0 - E)}{h^2}$$

$$(U_0 - E) = \frac{h^2 (\ln T)^2}{(4L^2)(8\pi^2 m)} \longrightarrow E = U_0 - \frac{h^2 (\ln T)^2}{32\pi^2 L^2 m}$$

* useful trick: multiply fraction by c^2/c^2 and use $hc = 1240\,\text{eV}\cdot\text{nm}$ and $m_e c^2 = 0.511\,\text{MeV}$

$$E = U_0 - \frac{(hc)^2 (\ln T)^2}{32\pi^2 L^2 (m_e c^2)}$$

$$E = 6.0\,\text{eV} - \frac{(1240\,\text{eV}\cdot\text{nm})^2 (\ln 0.0010)^2}{32\pi^2 (0.70\,\text{nm})^2 (0.511\times10^6\,\text{eV})}$$

$$\boxed{E = 5.1\,\text{eV}}$$