Physics 4C
Chapter 19: The Kinetic Theory of Gases

"Whether you think you can or think you can't, you're usually right." – Henry Ford

"The only thing in life that is achieved without effort is failure." – Source unknown

"We are what we repeatedly do. Excellence, therefore, is not an act, but a habit." – Aristotle

Reading: *pages 507- 529*

Outline:

⇒ *Avogadro's number*
⇒ *ideal gases*
    ideal gas law
    work done at constant volume, pressure, and temperature
⇒ *rms speed and translational KE*
⇒ *mean free path*
⇒ *distribution of molecular speeds*
    average, rms, and most probable speeds
⇒ *molar specific heats*
    internal energy
    molar specific heat at constant volume and pressure
    degrees of freedom (diatomic and polyatomic gases)
⇒ *adiabatic expansion*
    free expansions
⇒ *review of four special processes*
    isothermal, isobaric, isochoric, adiabatic

Problem Solving Techniques

You should know the relationships between the mass of a molecule, the mass of a mole of molecules, and the total mass of a collection of molecules. If a gas contains N molecules, each of mass m, the total mass is Nm. Since there are N_A molecules in a mole, the gas contains N/N_A moles. The molar mass M is the mass of N_A molecules: $M = N_A m$. If the gas contains n moles, its total mass is nM. Note that this is exactly the same as Nm.

Many problems require you to use the ideal gas law $pV = nRT$ to compute one of the quantities that appear in it. You should also know how to compute changes in one of the quantities for various processes. For a change in volume (and temperature) at constant pressure, $p\Delta V = nR\Delta T$; for a change in pressure (and temperature) at constant volume, $V\Delta p = nR\Delta T$; and for a change in volume (and pressure) at constant temperature, $p\Delta V = - V\Delta p$. In each case, the amount of gas was assumed to remain constant.

You should understand how to compute the work done by a gas during a given process. Use $W = \int p\,dV$. First, you will need to find the functional form of the dependence of the pressure p on the volume V for the process. The ideal gas law is often helpful here. Use $p = nRT/V$. If the temperature is constant, the integral can be evaluated immediately. If the temperature is not constant you need to know how it changes.

You should know how the root-mean-square speed of molecules in a gas depends on the temperature: $v_{rms} = \sqrt{3RT/M}$, where M is the molar mass. You should also know how the average translational kinetic energy depends on the temperature: $K_{ave} = \frac{3}{2}RT/N_A$. The translational kinetic energy per mole is $\frac{3}{2}RT$, the energy of n moles is $\frac{3}{2}nRT$, and the translational kinetic energy for N molecules is $\frac{3}{2}NRT/N_A = \frac{3}{2}NkT$. For monatomic molecules the last two expressions give the internal energy. For diatomic and polyatomic molecules other terms, giving the rotational energy, must be added. Other problems deal with the mean free path.

You should be able to solve problems involving the first law of thermodynamics: $\Delta E_{int} = Q - W$. For some of these you need to know the specific heat or the molar specific heat. Remember that the change in the internal energy is given by $\Delta E_{int} = nC_V\Delta T$ and the energy input as heat is given by $Q = nC\Delta T$, where C is the molar specific heat for the process.

Some problems deal with ideal gases as they undergo adiabatic processes. Remember that the combination pV^γ remains constant for such processes. Here γ is the ratio of the specific heats: $\gamma = C_p/C_V$. The ideal gas law is still valid and for some problems $pV = nRT$ must be solved simultaneously with pV^γ = constant.

Mathematical Skills

This chapter introduces some of the ideas of statistics, the most important of which are the average and root-mean-square of a collection of numbers. To find the average, add the numbers and divide the result by the number of terms in the sum. To find the root-mean-square, add the squares of the numbers, divide the sum by the number of terms, and take the square root of the result. The root-mean-square is the square root of the average of the squares of the numbers.

For practice, suppose the speeds of five molecules are 350 m/s, 225 m/s, 432 m/s, 375 m/s, and 450 m/s. Find their average and root-mean-square speeds. Your answers should be v_{ave} = 366 m/s and v_{rms} = 375 m/s. Notice that the average and root-mean-square values are different. In fact, the root-mean-square speed is greater than the average speed.

Integrals of the Maxwell distribution function.

This chapter contains several integrals that are difficult to evaluate using only a knowledge of introductory calculus. They are associated with the average and mean-square speed for molecules with a Maxwellian speed distribution. The average speed is given by

$$v_{\text{avg}} = 4\pi\left[\frac{M}{2\pi RT}\right]^{3/2} \int_0^\infty v^3 e^{-Mv^2/2RT}\,\mathrm{d}v$$

and the mean-square speed is given by

$$v^2_{\text{avg}} = 4\pi\left[\frac{M}{2\pi RT}\right]^{3/2} \int_0^\infty v^4 e^{-Mv^2/2RT}\,\mathrm{d}v$$

Most standard integral tables list the following integrals:

$$\int_0^\infty x^{2a} e^{-px^2}\,\mathrm{d}x = \frac{(2a-1)!!}{2(2p)^a}\sqrt{\frac{\pi}{p}}$$

and

$$\int_0^\infty x^{2a+1} e^{-px^2}\,\mathrm{d}x = \frac{a!}{2p^{a+1}}$$

Here p is any positive number and a is any integer. $a!$ is the factorial of a; that is, $a! = 1\cdot 2\cdot 3\cdot 4\ldots a$. $(2a-1)!!$ is the product of all odd integers from 1 to $2a-1$; that is, $(2a-1)!! = 1\cdot 3\cdot 5\cdot 7\ldots(2a-1)$. We use the first integral when x to an *even* power multiplies the exponential in the integrand and the second when x to an *odd* power multiplies the exponential.

To evaluate the integral for the average speed of a Maxwellian distribution, substitute $v = x$ and $M/2RT = p$ into the equation for v_{avg}. You should obtain

$$v_{\text{avg}} = 4\pi\left[\frac{M}{2\pi RT}\right]^{3/2} \int_0^\infty x^3 e^{-px^2}\,\mathrm{d}x$$

The integral is the same as the second integral taken from the tables, with $a = 1$. So

$$v_{\text{avg}} = 4\pi\left[\frac{M}{2\pi RT}\right]^{3/2} \frac{1}{2p^2} = 4\pi\left[\frac{M}{2\pi RT}\right]^{3/2} \frac{2R^2T^2}{M^2} = \sqrt{\frac{8RT}{\pi M}}$$

To evaluate the integral for the mean-square speed of a Maxwellian distribution, make the same substitutions to obtain

$$v^2_{\text{avg}} = 4\pi\left[\frac{M}{2\pi RT}\right]^{3/2} \int_0^\infty x^4 e^{-px^2}\,\mathrm{d}x$$

The integral is the same as the first one taken from the tables, with *a* = 2. So

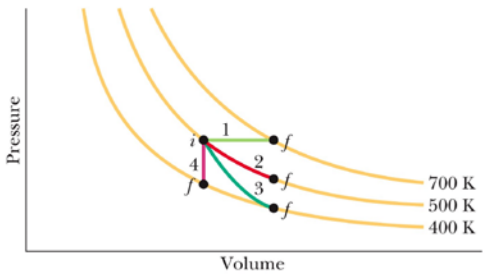
$$v_{avg}^2 = 4\pi\left[\frac{M}{2\pi RT}\right]^{3/2}\frac{3}{2(2p)^2}\sqrt{\frac{\pi}{p}} = 4\pi\left[\frac{M}{2\pi RT}\right]^{3/2}\frac{3R^2T^2}{2M^2}\sqrt{\frac{2\pi RT}{M}} = \frac{3RT}{M}$$

Thus, the root-mean-square speed is $\sqrt{3RT/M}$.

Questions and Example Problems from Chapter 19

Question 1

(a) Rank the four paths in the figure below according to the work done by the gas, greatest first.
(b) Rank paths 1, 2, and 3 according to the change in the internal energy of the gas, most positive first and most negative last.

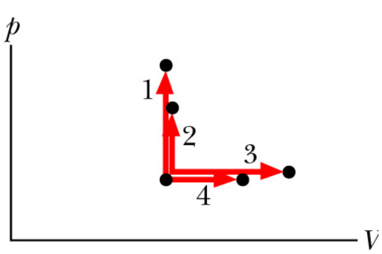


Question 2

An ideal diatomic gas, with molecular rotation but not oscillation, loses energy as heat Q. Is the resulting decrease in the internal energy of the gas greater if the loss occurs in a constant-volume process or in a constant-pressure process?

Question 3

A certain amount of energy is to be transferred as heat to 1 mol of a monatomic gas (a) at constant pressure and (b) at constant volume, and to 1 mol of a diatomic gas (c) at constant pressure and (d) at constant volume. The figure below shows four paths from an initial point to four final points on a p-V diagram. Which path goes with which process? (e) Are the molecules of the diatomic gas rotating?



Problem 1

A quantity of ideal gas at 10.0°C and 100 kPa occupies a volume of 2.50 m^3. (a) How many moles of the gas are present? (b) If the pressure is now raised to 300 kPa and the temperature is raised to 30.0°C, how much volume does the gas occupy? Assume no leaks.

Problem 2

The best laboratory vacuum has a pressure of about 1.00×10^{-18} atm, or 1.01×10^{-13} Pa. How many gas molecules are there per cubic centimeter in such a vacuum at 293 K?

Problem 3
Air that initially occupies 0.140 m^3 at a gauge pressure of 103.0 kPa is expanded isothermally to a pressure of 101.3 kPa and then cooled at constant pressure until it reaches its initial volume. Compute the work done by the air. (Gauge pressure is the difference between the actual pressure and atmospheric pressure.)

Problem 4
At what temperature do atoms of helium gas have the same rms speed as molecules of hydrogen gas at 20.0 ^{o}C? (The molar masses are given in Table 19-1).

Problem 5

At 20°C and 750 torr pressure, the mean free paths for argon gas (Ar) and nitrogen gas (N_2) are $\lambda_{Ar} = 9.9 \times 10^{-6}$ cm and $\lambda_{N2} = 27.5 \times 10^{-6}$ cm. (a) Find the ratio of the effective diameter of argon to that of nitrogen. What is the mean free path of argon at (b) 20°C and 150 torr, and (c) -40°C and 750 torr?

Problem 6

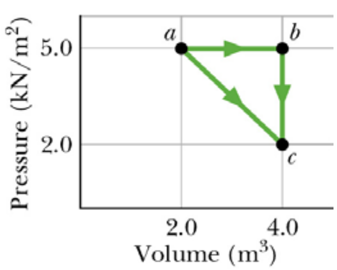
Twenty-two particles have speeds as follows (N_i represents the number of particles that have speed v_i):

N_i	2	4	6	8	2
v_i (cm/s)	1.0	2.0	3.0	4.0	5.0

(a) Compute their average speed v_{avg}. (b) Compute their root-mean-square speed v_{rms}. (c) Of the five speeds shown, which is the most probable speed v_P?

Problem 7
One mole of an ideal gas undergoes an isothermal expansion. Find the energy added to the gas as heat in terms of the initial and final volumes and the temperature. (Hint: Use the first law of thermodynamics.)

Problem 8
One mole of an ideal diatomic gas goes from a to c along the diagonal path in the figure below. During the transition, (a) what is the change in internal energy of the gas, and (b) how much energy is added to the gas as heat? (c) How much heat is required if the gas goes from a to c along the indirect path abc?



Problem 9
Suppose 12.0 g of oxygen (O_2) is heated at constant atmospheric pressure from 25.0°C to 125°C. (a) How many moles of oxygen are present? (See Table 19-1 for the molar mass.) (b) How much energy is transferred to the oxygen as heat? (The molecules rotate but do not oscillate.) (c) What fraction of the heat is used to raise the internal energy of the oxygen?

Problem 10
When 1.0 mol of oxygen (O_2) gas is heated at constant pressure starting at 0°C, how much energy must be added to the gas as heat to double its volume? (The molecules rotate but do not oscillate.)

Problem 11
The volume of an ideal gas is adiabatically reduced from 200 L to 74.3 L. The initial pressure and temperature are 1.00 atm and 300 K. The final pressure is 4.00 atm. (a) Is the gas monatomic, diatomic, or polyatomic? (b) What is the final temperature? (c) How many moles are in the gas?

Problem 12
The figure below shows two paths that may be taken by a gas from an initial point *i* to a final point *f*. Path 1 consists of an isothermal expansion (work is 50 J in magnitude), an adiabatic expansion (work is 40 J in magnitude), an isothermal compression (work is 30 J in magnitude), and then an adiabatic compression (work is 25 J in magnitude). What is the change in the internal energy of the gas if the gas goes from point *i* to point *f* along path 2?