Physics 4C
Chapter 38: Photons and Matter Waves

"Nothing can bring you peace but yourself." – Ralph Waldo Emerson

"To live as fully, as completely as possible, to be happy … is the true aim and end to life."
Llewelyn Powers

"To most folks are about as happy as they make up their minds to be." – Abraham Lincoln

"There can be no happiness if the things we believe in are different from the things we do."
Freya Stark

Reading: *pages 1057 – 1077*

Outline:

⇒ *intro to quantum mechanics*
- *blackbody radiation*

⇒ *photons*
⇒ *photoelectric effect*
- *stopping potential*
- *cutoff frequency and work function*
- *photoelectric equation*

⇒ *momentum of a photon*
- *Compton shift*

⇒ *light as a probability wave (PowerPoint)*
⇒ *matter waves*
⇒ *Schrodinger's equation in one-dimension*
- *free particles*
- *probability density*

⇒ *Heisenberg's Uncertainty Principle*
⇒ *barrier tunneling*

Problem Solving Techniques

You should know the relationship between the energy E of a photon and the frequency f of the wave associated with it: $E = hf$. You should also know the relationship between the magnitude of the photon momentum and the wavelength of the wave: $p = h/\lambda$. In both classical and quantum physics the momentum and energy are related by $E = pc$. This follows immediately from $f\lambda = c$ and the relationships given above.

You should understand that a collection of N monochromatic photons (in a light beam, for example) has a total energy of Nhf. If a uniform beam of monochromatic photons has cross-sectional area A then the rate at which energy is transmitted through the area is $P = Rhf$, where R is the number of photons that pass through per unit time. The intensity is $I = Rhf/A$.

The photoelectric effect is described by $hf = K_{max} + \Phi$, where f is the frequency of the incident radiation, K_{max} is the kinetic energy of the most energetic photoelectron ejected, and Φ is the work function of the target material. You should also know that the value of K_{max} is found by measuring the stopping potential V_0 and that $K_{max} = eV_0$. The work function can be measured experimentally by finding the frequency of the light for which the stopping potential is zero. Then $hf = \Phi$.

Upon scattering from an electron initially at rest, the wavelength associated with a photon changes from λ to $\lambda' = \lambda + \Delta\lambda$, where $\Delta\lambda = (h/mc)(1 - \cos\phi)$. Here m is the mass of an electron, h is the Plank constant, c is the speed of light, and ϕ is the scattering angle (measured from the direction of incidence). The energy of the photon is reduced from $E = hf = hc/\lambda$ to $E' = hf' = hc/\lambda'$. The energy lost by the photon appears as an increase in the kinetic energy of the electron.

You should know the relationship between the energy of a particle and the frequency of its wave ($E = hf$), the relationship between the momentum of a particle and the wavelength of its wave ($p = h/\lambda$), and the relationship between the energy and momentum of a particle ($E = p^2/2m$). The latter expression is valid for a *nonrelativistic* particle of mass m. The analogous expressions for a photon are $E = hf$, $p = h/\lambda$, and $E = pc$. Only the last is different for photons and electrons.

Some problems deal with solutions to Schröodinger's equation and with the nature of the probability density $|\Psi|^2$ when Ψ is complex. Remember that the wave function for a free particle traveling in the positive x direction is $\psi = Ae^{ikx}$, where $k = 2\pi/\lambda = 2\pi p/h$, where p is the momentum of the particle and λ is the wavelength of its wave.

Some problems deal with the uncertainty principle $\Delta x \cdot \Delta p \geq h$. Usually Δx or Δp is given and you are asked to compute the other quantity.

Barrier tunneling problems usually involve a computation of the transmission coefficient $T = e^{-2kL}$, where $k = \sqrt{8\pi^2 m(U_0 - E)/h^2}$. Here L is the width of the barrier (a length), U_0 is the height of the barrier (an energy), and E is the kinetic energy of the particle incident on the barrier. T gives the probability that the particle will tunnel through the barrier. If N particles, all with the same mass and energy, are incident on a barrier, then on average, TN tunnel through. The others are reflected.

Mathematical Skills

Matter wave functions are often complex quantities. This means they can be written as the sum of a real and an imaginary part: $\psi(x) = \psi_R(x) + i\psi_I(x)$, where ψ_R is the real part and ψ_I is the imaginary part. The symbol i stands for $\sqrt{-1}$.

For example, the wave function for a free particle traveling in the positive x direction is $\psi(x) = Ae^{ikx}$, where k is related to the momentum p of the particle by $p = hk/2\pi$. Since $e^{i\alpha} = \cos\alpha + i\sin\alpha$, the free-particle wave function can be written

$\psi = Ae^{ikx} = A\cos(kx) + iA\sin(kx)$.

Both the real and imaginary parts are sinusoidal and both have the same wavelength λ, related to k by $k = 2\pi/\lambda$.

The complex conjugate of a complex number has the same real part as the original number but its imaginary part is the negative of the imaginary part of the original number. Thus, the complex conjugate of $\psi = \psi_R + i\psi_I$ is $\psi^* = \psi_R - \psi_I$ and the complex conjugate of e^{ikx} is e^{-ikx}.

The square of the magnitude of a complex number is found by multiplying the number by its complex conjugate: $|\psi|^2 = \psi\psi^*$. Thus, the square of the magnitude of ψ is

$|\psi|^2 = (\psi_R + i\psi_I)(\psi_R - i\psi_I) = \psi_R^2 + \psi_I^2$.

It is the sum of squares of the real and imaginary parts. The square of the magnitude of the free-particle wave function is

$|Ae^{ikx}|^2 = Ae^{ikx}Ae^{-ikx} = A^2e^0 = A^2$,

where we have assumed A is real.

If a particle has energy E, its complete wave function Ψ is the product of a coordinate-dependent function and a time-dependent function. The time-dependent function has the form $e^{i\omega t}$, where ω is the angular frequency. Since $\omega = 2\pi f$ and $f = E/h$, where f is the frequency, the angular frequency is related to the energy by $\omega = 2\pi E/h$. Thus, $\Psi = \psi\, e^{i\omega t}$. Ψ and ψ lead to the same probability density:

$|\Psi|^2 = |\psi\, e^{i\omega t}|^2 = \psi\, \psi^*\, e^{i\omega t}e^{-i\omega t} = \psi\, \psi^* = |\psi|^2$.

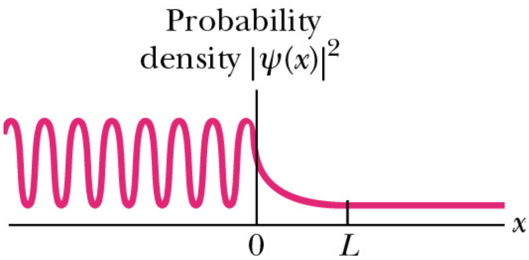
Questions and Example Problems from Chapter 38

Question 1

Of the following statements about the photoelectric effect, which are true and which are false? (a) The greater the frequency of the incident light is, the greater is the stopping potential. (b) The greater the intensity of the incident light is, the greater is the cutoff frequency. (c) The greater the work function of the target material is, the greater is the stopping potential. (d) The greater the work function of the target material is, the greater is the cutoff frequency. (e) The greater the frequency of the incident light is, the greater is the maximum kinetic energy of the ejected electrons. (f) The greater the energy of the photons is, the smaller is the stopping potential

Question 2

In the figure below, why are the minima in the values of $|\psi|^2$ greater than zero?



Question 3

The following nonrelativistic particles all have the same kinetic energy. Rank them in order of their de Broglie wavelengths, greatest first: electron, alpha particle, neutron.

Problem 1

An ultraviolet lamp emits light of wavelength 400 nm, at the rate (power) of 400 W. An infrared lamp emits light of wavelength 700 nm, also at the rate of 400 W. (a) Which lamp emits photons at the greater rate and (b) what is that greater rate?

Problem 2

A spectral emission line is electromagnetic radiation that is emitted in a wavelength range narrow enough to be taken as a single wavelength. One such emission line that is important in astronomy has a wavelength of 21 cm. What is the photon energy in the electromagnetic wave at that wavelength?

Problem 3

The stopping potential for electrons emitted from a surface illuminated by light of wavelength 491 nm is 0.710 V. When the incident wavelength is changed to a new value, the stopping potential is 1.43 V. (a) What is this new wavelength? (b) What is the work function for the surface?

Problem 4

The wavelength associated with the cutoff frequency for silver is 325 nm. Find the maximum kinetic energy of electrons ejected from a silver surface by ultraviolet light of wavelength 254 nm

Problem 5

In a photoelectric experiment using a sodium surface, you find a stopping potential of 1.85 V for a wavelength of 300 nm and a stopping potential of 0.820 V for a wavelength of 400 nm. From these data find (a) a value for the Planck constant, (b) the work function ϕ for sodium, and (c) the cutoff wavelength λ_0 for sodium

Problem 6
Consider a collision between an x-ray photon of initial energy 50.0 keV and an electron at rest, in which the photon is scattered backward and the electron is knocked forward. (a) What is the energy of the back-scattered photon? (b) What is the kinetic energy of the electron?

Problem 7
Gamma rays of photon energy 0.511 MeV are directed onto an aluminum target and are scattered in various directions by loosely bound electrons there. (a) What is the wavelength of the incident gamma rays? (b) What is the wavelength of gamma rays scattered at 90.0° to the incident beam? (c) What is the photon energy of the rays scattered in this direction?

Problem 8
A bullet of mass 40 g travels at 1000 m/s. Although the bullet is clearly too large to be treated as a matter wave, determine what Eq. 38-13 predicts for its de Broglie wavelength.

Problem 9
A nonrelativistic particle is moving three times as fast as an electron. The ratio of the de Broglie wavelength of the particle to that of the electron is 1.813×10^{-4}. By calculating its mass, identify the particle.

Problem 10
Imagine playing baseball in a universe (not ours!) where the Planck constant is 0.60 J·s. What would be the uncertainty in the position of a 0.50 kg baseball that is moving at 20 m/s along an axis if the uncertainty in the speed is 1.0 m/s?

Problem 11
A 1500 kg car moving at 20 m/s approaches a hill that is 24 m high and 30 m long. Although the car and hill are clearly too large to be treated as matter waves, determine what Eq. 38-21 predicts for the transmission coefficient of the car, as if it could tunnel through the hill as a matter wave. Treat the hill as a potential energy barrier where the potential energy is gravitational.

Problem 12

Consider a potential energy barrier like that of the figure below but whose height U_0 is 6.0 eV and whose thickness L is 0.70 nm. What is the energy of an incident electron whose transmission coefficient is 0.0010?