Standard Deviation

Mean (Average)

⇒ Suppose that I gave the same test to two different classes. If I wanted to know how well each class did, what number would I use to compare the two classes?

⇒ I would probably use the average or mean from each class.

⇒ However, there is a major problem with only looking at the average. What is it???

Mean (Average)

⇒ The average (or mean) doesn't give any indication of how spread out the values (test scores) are:

(Class 1)	(Class 2)
79	100
78	99
77	96
75	75
75	68
74	67
74	66
74	63
74	59
70	57

Mean = 75 **Mean = 75**

Deviation from the Mean

⇒ What we would like is some indication of how spread out the test scores are from each class.

⇒ One possibility would be to calculate the average deviation, where the deviation *d* for each value is defined as:

deviation = value - average

⇒ However, there is a major problem with calculating the average deviation. What is it???

Deviation from the Mean

Test Scores	Deviation
79	4
78	3
77	2
75	0
75	0
74	-1
74	-1
74	-1
74	-1
70	-5

Test Scores	Deviation
100	25
99	24
96	21
75	0
68	-7
67	-8
66	-9
63	-12
59	-16
57	-18

⇒ The average deviation is **always** zero!!!

Standard Deviation

⇒ The standard deviation σ is the quantity used to describe how spread out the values are in a given set of data.

⇒ The standard deviation σ is defined as follows:

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(d_i)^2} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \overline{x})^2}$$

Standard Deviation

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(d_i)^2} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \overline{x})^2}$$

1) First, find the mean (average) of all values: $\overline{x}$

2) Then, for each value, find the deviation of that value from the mean and square it:

$$(x_i - \overline{x})^2$$

Standard Deviation

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(d_i)^2} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \overline{x})^2}$$

3) Then, find the average of all of the deviations squared (dividing by N-1 instead of N):

$$\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \overline{x})^2$$

3) Finally, take the square root of the average of the deviations squared:

$$\sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \overline{x})^2}$$

Standard Deviation

Test Scores	Deviation	Deviation^2
79	4	16
78	3	9
77	2	4
75	0	0
75	0	0
74	-1	1
74	-1	1
74	-1	1
74	-1	1
70	-5	25

The sum of all deviations squared is 58

58 / (10-1) = 6.44

$\sqrt{6.4} = 2.5$

⇒ The standard deviation for class 1 is σ = 2.5

Standard Deviation

Test Scores	Deviation	Deviation^2
100	25	625
99	24	576
96	21	441
75	0	0
68	-7	49
67	-8	64
66	-9	81
63	-12	144
59	-16	256
57	-18	324

The sum of all deviations squared is 2560

2560 / (10-1) = 284

$\sqrt{284} = 17$

⇒ The standard deviation for class 2 is σ = 17

Standard Deviation

⇒ The standard deviation σ gives us an idea of how spread out the values in a given data set are.

small σ ⇒ values are close together

large σ ⇒ values are spread out

⇒ To give others an idea of how precise the results of an experiment are, scientific results are usually reported as:

result = average ± σ

Standard Deviation

⇒ The results of the test scores from the two classes would therefore be reported as:

class 1 ⇒ score = 75 ± 2.5

class 2 ⇒ score = 75 ± 17

⇒ Statistically, ~68% of the values in a data set should fall within 1σ of the average. This means that ~68% of the values should fall within the range: average – σ and average + σ.

Standard Deviation

(Class 1)
79
78
77
75
75
74
74
74
74
70

average = 75 σ = 2.5

⇒ ~68% of the value should fall within the range: 75 – 2.5 and 75 + 2.5

⇒ ~68% of the values should fall between 72.5 and 77.5

Standard Deviation

(Class 2)
100
99
96
75
68
67
66
63
59
57

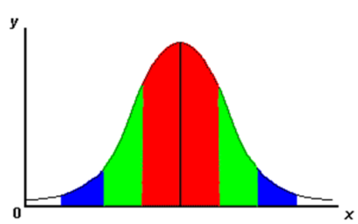
average = 75 σ = 17

⇒ ~68% of the value should fall within the range: 75 – 17 and 75 + 17

⇒ ~68% of the values should fall between 58 and 92

Standard Deviation

⇒ Statistically, ~68% of the values in a data set should fall within 1σ of the average, ~95% of the values in a data set should fall within 2σ of the average, and ~99% of the values in a data set should fall within 3σ of the average.



Is Theory Consistent with Experiment?

⇒ In Physics 11, we will use the following criteria to determine whether an experimental measurement of *x* is consistent with the theoretical prediction x_{thy}:

If $\bar{x} - \sigma \le x_{thy} \le \bar{x} + \sigma$

then the experiment result and the theoretical prediction are consistent.

⇒ That is, if the theoretical result falls within one standard deviation of the mean, the experimental result is consistent with the model.