Physics 11
Chapter 10 HW Solutions

Chapter 10
Conceptual Questions: 11, 13, 17, 21
Problems: 3, 10, 30, 31, 43, 49, 54, 55, 58, 59

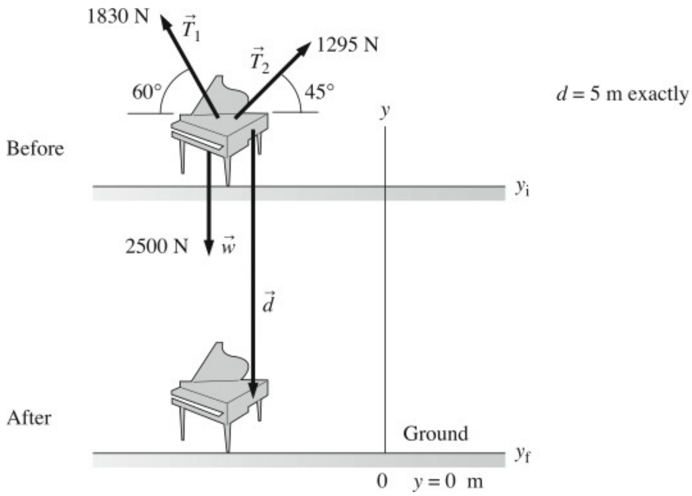
Q10.11. **Reason:** The energies involved here are kinetic energy, gravitational potential energy, elastic potential energy, and thermal energy. For the system to be isolated, we must not have any work being done on the system and no heat being transferred into or out of the system. The ball's kinetic and elastic energy is changing, so we should consider it part of the system. Since its gravitational potential energy is changing, we need to also consider the earth as part of the system. Thermal energy will be generated in the ball and floor when the ball hits the floor, so we must consider both to be part of the system. In as much as the earth itself is not an isolated system (heat can leave the earth) we really should consider the universe the system to consider the system completely isolated.
Assess: In order to have an isolated system no heat can leave or enter the system and all forces must be internal.

Q10.13. **Reason:** **(a)** The work done is $W = Fd$. Both particles experience the same force, so the greater work is done on the particle that undergoes the greater displacement. Particle A, which is less massive than B, will have the greater acceleration and thus travel further during the 1 s interval. Thus more work is done on particle A. **(b)** Impulse is $F\Delta t$. Both particles experience the same force F for the same time interval $\Delta t = 1$ s. Thus the same impulse is delivered to both particles. **(c)** Both particles receive the same impulse, so the change in their momenta is the same, that is, $m_A(v_f)_A = m_B(v_f)_B$. But because $m_A < m_B$, it must be that $(v_f)_A > (v_f)_B$. This result can also be found from kinematics, as in part (a).
Assess: Work is the product of the force and the displacement, while impulse is the product of the force and the time during which the force acts.

Q10.17. **Reason:** **(a)** If the car is to go twice as fast at the bottom, its kinetic energy, proportional to v^2, will be *four times* as great. You thus need to give it four times as much gravitational potential energy at the top. Since gravitational potential energy is linearly proportional to the height h, you'll need to increase the height of the track by a factor of four. **(b)** Using considerations of conservation of energy, as in part (a), we see that the speed of the car at the bottom depends only on the height of the track, not its shape.
Assess: Kinetic energy is proportional to the *square* of the velocity.

Q10.21. **Reason:** As you land, the force of the ground or pad does negative work on your body, transferring out the kinetic energy you have just before impact. This work is $-Fd$, where d is the distance over which your body stops. With the short stopping distance involved upon hitting the ground, the force F will be much greater than it is with the long stopping distance upon hitting the pad.
Assess: For a given amount of work, the force is large when the displacement is small.

P10.3. **Prepare:** Note that not all the forces in this problem are parallel to the displacement. Equation 10.6 gives the work done by a constant force which is not parallel to the displacement: $W = Fd\cos(\theta)$, where W is the work done by the force F at an angle θ to the displacement d. Here the displacement is exactly downwards in the same direction as $\vec{w}$. We will take all forces as having four significant figures (as implied by $T_2 = 1295$ N).



Solve: Refer to the diagram. The angle between the force $\vec{w}$ and the displacement is $0°$, so

$$W_{\vec{w}} = wd\cos\theta = (2500 \text{ N})(5 \text{ m})\cos(0°) = 12.5 \text{ kJ}$$

The angle between the force $\vec{T}_1$ and the displacement is $90° + 60° = 150°$.

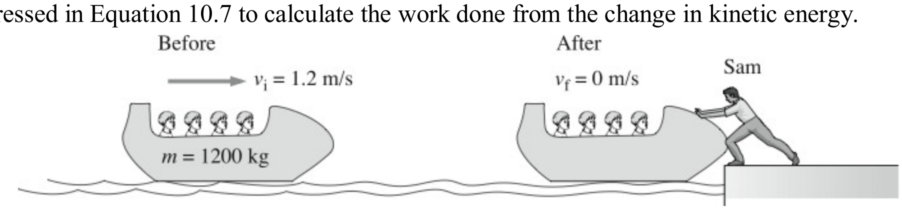
$$W_{\vec{T}_1} = T_1 d\cos\theta = (1830 \text{ N})(5 \text{ m})\cos(150°) = -7.92 \text{ kJ}$$

The angle between the tension $\vec{T}_2$ and the displacement is $90° + 45° = 135°$.

$$W_{\vec{T}_2} = T_2 d\cos\theta = (1295 \text{ N})(5 \text{ m})\cos(135°) = -4.58 \text{ kJ}$$

Assess: Note that the displacement d in all these cases is directed downwards and that it is always the angle between the force and displacement used in the work equation. For example, the angle between $\vec{T}_1$ and $\vec{d}$ is $150°$, not $60°$.

P10.10. Prepare: We will assume that all the work Sam does goes into stopping the boat. We can use conservation of energy as expressed in Equation 10.7 to calculate the work done from the change in kinetic energy.



Solve: Refer to the before and after representation of Sam stopping a boat. Equation 10.7 becomes

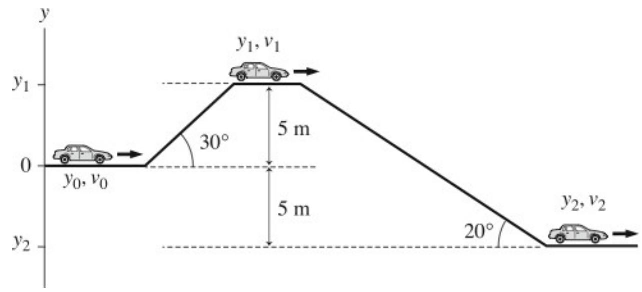
$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W$$

Since the boat is at rest at the end of the process, $v_f = 0$ m/s. Therefore, the final kinetic energy is zero. The work done on the boat is then

$$W = -\frac{1}{2}mv_i^2 = -\frac{1}{2}(1200 \text{ kg})(1.2 \text{ m/s})^2 = -0.86 \text{ kJ}$$

Assess: Note that the work done by Sam on the boat is negative. This is because the force Sam exerts on the boat must be opposite to the direction of motion of the boat to slow it down.

P10.30. Prepare: Assume there is zero rolling friction since friction is not mentioned in the problem. The sum of the kinetic and gravitational potential energy, therefore, does not change during the car's motion. Consider the other side of the hill to be the zero for gravitational potential energy.



Solve: **(a)** The initial energy of the car is

$$K_0 + U_{g0} = \frac{1}{2}mv_0^2 + mgy_0 = \frac{1}{2}(1500 \text{ kg})(10 \text{ m/s})^2 = 7.5\times10^4 \text{ J}$$

The energy of the car at the top of the hill is

$$K_1 + U_{g1} = K_1 + mgy_1 = K_1 + (1500 \text{ kg})(9.80 \text{ m/s}^2)(5.0 \text{ m}) = K_1 + 7.4\times10^4 \text{ J}$$

If the car just wants to make it to the top, then $K_1 = 0$. That is, the car has no velocity at the top of the hill. In other words, a minimum energy of 7.4×10^4 J is needed to get to the top. Since this energy is less than the available energy of 7.5×10^4 J, the car can make it to the top.

(b) The conservation of energy equation $K_0 + U_{g0} = K_2 + U_{g2}$ is

$$7.5\times10^4 \text{ J} = \frac{1}{2}mv_2^2 + mgy_2 \Rightarrow 7.5\times10^4 \text{ J} = \frac{1}{2}(1500 \text{ kg})v_2^2 + (1500 \text{ kg})(9.80 \text{ m/s}^2)(-5.0 \text{ m})$$

$$\Rightarrow v_2 = 14 \text{ m/s}$$

Assess: A higher speed on the other side of the hill is reasonable because the car has increased its kinetic energy and lowered its potential energy compared to its starting values. Note that the shape of the hill is irrelevant because gravitational potential energy depends only on height.

P10.43. **Prepare:** The work done on the car while it is accelerating from rest to the final speed is the change in kinetic energy. Knowing the work done and the time to do this work we can determine the power associated with this work.

Solve: The change in kinetic energy of the car is

$$W = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 = \frac{1}{2}(1000 \text{ kg})(30 \text{ m/s})^2 = 4.5\times10^5 \text{ J}$$

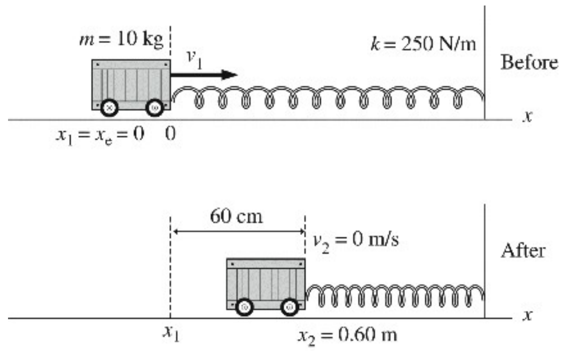
since the initial kinetic energy is zero.

The power associated with this work is

$$P = \frac{W}{\Delta t} = \frac{4.5\times10^5 \text{ J}}{10 \text{ s}} = 45 \text{ kW}$$

Assess: This is reasonable. In most cars only a small fraction of the work done by the engine goes into propelling the car.

P10.31. **Prepare:** Consider the spring as an ideal spring that obeys Hooke's law. We will also assume zero rolling friction during the compression of the spring, so that mechanical energy is conserved. At the maximum compression of the spring, 60 cm, the velocity of the cart will be zero.



The figure shows a before-and-after pictorial representation. The "before" situation is when the cart hits the spring in its equilibrium position. We put the origin of our coordinate system at this equilibrium position of the free end of the spring. This give $x_1 = x_e = 0$ and $x_2 = 60$ cm.

Solve: The conservation of energy equation $K_2 + U_{s2} = K_1 + U_{s1}$ is

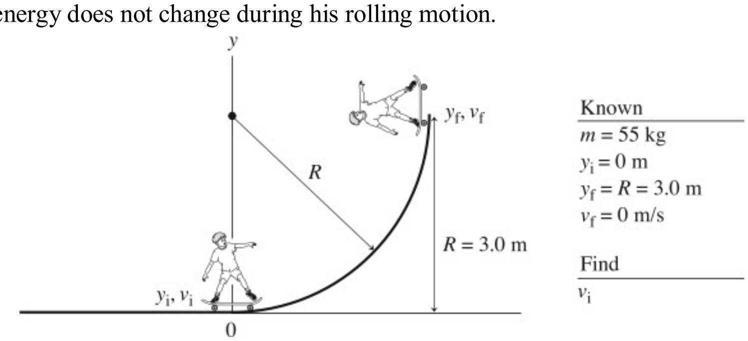
$$\frac{1}{2}mv_2^2 + \frac{1}{2}kx_2{}^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}kx_1{}^2$$

Using $v_2 = 0$ m/s, $x_2 = 0.60$ m, and $x_1 = 0$ m gives:

$$\frac{1}{2}kx_2{}^2 = \frac{1}{2}mv_1^2 \Rightarrow v_1 = \left(\sqrt{\frac{k}{m}}\right)x_2 = \left(\sqrt{\frac{250\text{ N/m}}{10\text{ kg}}}\right)(0.60\text{ m}) = 3.0\text{ m/s}$$

Assess: Elastic potential energy is always measured from the unstretched or uncompressed length of the spring.

P10.49. Prepare: Assuming that the track offers no rolling friction, the sum of the skateboarder's kinetic and gravitational potential energy does not change during his rolling motion.



The vertical displacement of the skateboarder is equal to the radius of the track.

Solve: **(a)** The quantity $K + U_g$ is the same at the upper edge of the quarter-pipe track as it was at the bottom. The energy conservation equation $K_f + U_{gf} = K_i + U_{gi}$ is

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i \Rightarrow v_i^2 = v_f^2 + 2g(y_f - y_i)$$

$$v_i^2 = (0\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(3.0\text{ m} - 0\text{ m}) = 58.8\text{ m/s} \Rightarrow v_i = 7.7\text{ m/s}$$

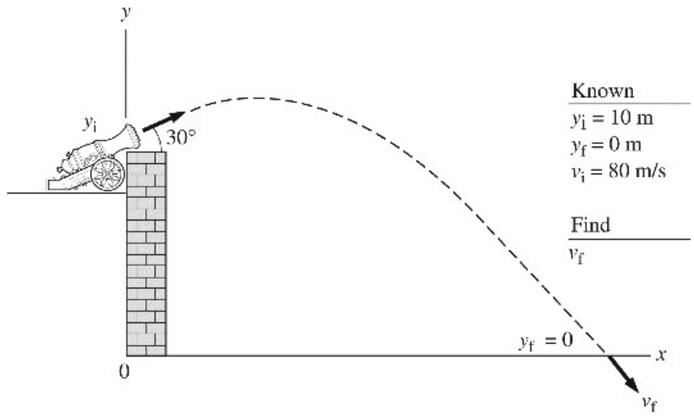
(b) If the skateboarder is in a low crouch, his height above ground at the beginning of the trip changes to 0.75 m. His height above ground at the top of the pipe remains the same since he is horizontal at that point. Following the same procedure as for part (a),

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i \Rightarrow v_i^2 = v_f^2 + 2g(y_f - y_i)$$

$$v_i^2 = (0\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(3.0\text{ m} - 0.75\text{ m}) = 44.1\text{ m/s} \Rightarrow v_i = 6.6\text{ m/s}$$

Assess: Note that we did not need to know the skateboarder's mass, as is the case with free-fall motion. Note that the shape of the track is irrelevant.

P10.54. Prepare: This is case of free-fall, so the sum of the kinetic and gravitational potential energy does not change as the cannon ball falls.



The figure shows a before-and-after pictorial representation. To express the gravitational potential energy, we put the origin of our coordinate system on the ground below the fortress.

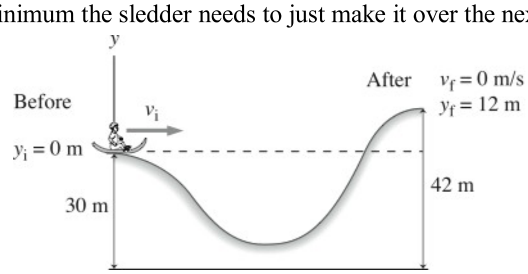
Solve: Using $y_f = 0$ and the equation $K_i + U_{gi} = K_f + U_{gf}$ we get

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f \Rightarrow v_i^2 + 2gy_i = v_f^2$$

$$v_f = \sqrt{v_i^2 + 2gy_i} = \sqrt{(80\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(10\text{ m})} = 81\text{ m/s}$$

Assess: Note that we did not need to use the tilt angle of the cannon, because kinetic energy is a scalar. Also note that using the energy conservation equation, we can find only the magnitude of the final velocity, not the direction of the velocity vector. Note that this method is much easier than using kinematics to calculate the final velocity of the ball.

P10.55. Prepare: Since the hill is frictionless, mechanical energy will be conserved during the sledder's trip. To make it over the next hill, the sledder's velocity must be greater than or equal to zero at the top of the hill. The minimum velocity the sledder can have at the top of the second hill is 0 m/s to just make it over. The corresponding velocity at the top of the initial hill will be the minimum the sledder needs to just make it over the next hill.



Solve: Consider the before and after pictorial representation. We will use the sledder's initial height as the reference for gravitational potential energy. Since there is no friction, the conservation of energy equation, Equation 10.4 reads

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f \Rightarrow v_i^2 = 2gy_f$$

$$v_i = \sqrt{2gy_f} = \sqrt{2(9.80\text{ m/s}^2)(12\text{ m})} = 15\text{ m/s}$$

Where we have used $y_i = 0$ m, and $v_f = 0$ m/s for the sledder to just make it over the second hill. Note that since we are using the top of the first hill as the reference of gravitational potential energy, we must use the height of the top of the second hill above the first for y_f, $y_f = 42\text{ m} - 30\text{ m} = 12\text{ m}$.

Assess: Note the shape of the hill doesn't matter, only the difference in height between the first and second hill is needed, as expected for gravitational potential energy. Since the second hill is higher than the first, we expect that the sledder needs the additional kinetic energy at the initial hill to make up for the additional potential energy needed at the top of the second hill.

P10.58. Prepare: We will take the system to be the person plus the earth. When a person drops from a certain height, the initial potential energy is transformed to kinetic energy. When the person hits the ground, if they land rigidly upright, we assume that all of this energy is transformed into elastic potential energy of the compressed leg bones. The maximum energy that can be absorbed by the leg bones is 200 J; this limits the maximum height.

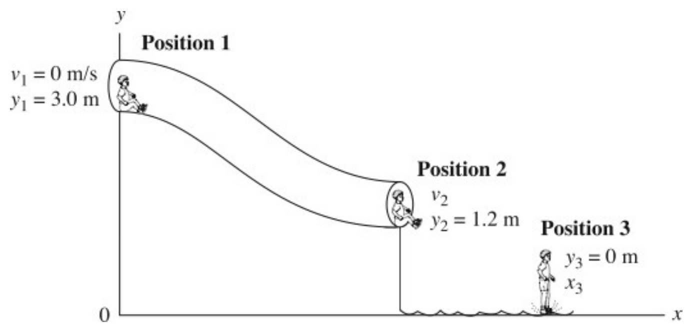
Solve: **(a)** The initial potential energy can be at most 200 J, so the height h of the jump is limited by $mgh = 200\text{ J}$ For $m = 60\text{ kg}$, this limits the height to

$$h = 200\text{ J/mg} = 200\text{ J}/(60\text{ kg})(9.8\text{ m/s}^2) = 0.34\text{ m}$$

(b) If some of the energy is transformed to other forms than elastic energy in the bones, the initial height can be greater. If a person flexes her legs on landing, some energy is transformed to thermal energy. This allows for a greater initial height.

Assess: There are other tissues in the body with elastic properties that will absorb energy as well, so this limit is quite conservative.

P10.59. Prepare: This is a two-part problem. The slide is frictionless, so mechanical energy is conserved. We will calculate the final velocity of the people as they exit the slide and then use that result to calculate how far they travel from the exit before they hit the water.



Solve: Refer to the diagram. Setting the reference for gravitational potential energy to be zero at the bottom of the slide, the energy conservation equation becomes

$$mgy_1 = \frac{1}{2}mv_1^2 \Rightarrow v_1 = \sqrt{2gy_1} = \sqrt{2(9.80\text{ m/s}^2)(3.0\text{ m})} = 7.67\text{ m/s}$$

Note that this result does not depend on the person's mass. We keep an additional significant figure here for the second part of the calculation.

After they leave the slide, they are falling under the influence of gravity. Their initial velocity in the y direction is zero. The time it takes for them to fall to the water can be calculated with ordinary kinematics.

$$\Delta y = v_{0y}\Delta t - \frac{1}{2}a\Delta t^2,\text{ with } v_{0y} = 0\text{ m/s gives}$$

$$\Delta y = -1.2\text{ m} = -\frac{1}{2}(9.80\text{ m/s}^2)\Delta t^2\text{ or}$$

$$\Delta t = \sqrt{\frac{(2)(1.2\text{ m})}{9.80\text{ m/s}^2}} = 0.50\text{ s}$$

Using $v_1 = 7.67\text{ s}$ from the first part of the problem, we find

$$\Delta x = v_1\Delta t = (7.67\text{ m/s})(0.50\text{ s}) = 3.8\text{ m}$$

The mass of the person was not necessary for this part of the calculation either.

Assess: Though this is a two-part problem mechanical energy is conserved throughout the whole process. However we could not use conservation of energy to solve the problem since we are not given the final velocity of the person before they hit the water, which is necessary for the conservation of energy equation. Note that it does not matter what the mass of the person is, they will always travel 3.8 m from the exit of the tube before hitting the water.