Physics 2A
Chapter 14 HW Solutions

Chapter 14
Conceptual Questions: 3, 8, 10, 20
Problems: 4, 7, 13, 19, 21, 29, 45, 50, 59

Q14.3. Reason: We are given the graph of x versus t. However, we want to think about the slope of this graph to answer velocity questions.
(a) When the x versus t graph is increasing, the particle is moving to the right. It has maximum speed when the positive slope of the x versus t graph is greatest. This occurs at 0 s and 4 s.
(b) When the x versus t graph is decreasing, the particle is moving to the left. It has maximum speed when the negative slope of the x versus t graph is greatest. This occurs at 2 s and 6 s.
(c) The particle is instantaneously at rest when the slope of the x versus t graph is zero. This occurs at 1 s, 3 s, 5 s, and 7 s.
Assess: This is reminiscent of material studied in Chapter 2; what is new is that the motion is oscillatory and the graph periodic.

Q14.8. Reason: From the graph the strategy is to determine the period, then use $f = 1/T$. As is done in Figure 14.4, one can measure the period between two crests; in this case it appears to be 2 s.

$$f = \frac{1}{T} = \frac{1}{2\ \text{s}} = 0.5\ \text{Hz}$$

The amplitude is the maximum distance from the equilibrium position. On this graph it appears that $A = 10$ cm.
Assess: The amplitude is *not* the distance from the maximum to the minimum—that would be $2A$. See Figure 14.6.

Q14.10. Reason: The period of a block oscillating on a spring is given in Equation 14.26, $T = 2\pi\sqrt{m/k}$. We are told that $T_1 = 2.0$ s.
(a) In this case the mass is doubled, $m_2 = 2m_1$.

$$\frac{T_2}{T_1} = \frac{2\pi\sqrt{m_2/k}}{2\pi\sqrt{m_1/k}} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{2m_1}{m_1}} = \sqrt{2}$$

So $T_2 = \sqrt{2}T_1 = \sqrt{2}(2.0\ \text{s}) = 2.8$ s.
(b) In this case the spring constant is doubled, $k_2 = 2k_1$.

$$\frac{T_2}{T_1} = \frac{2\pi\sqrt{m/k_2}}{2\pi\sqrt{m/k_1}} = \sqrt{\frac{k_1}{k_2}} = \sqrt{\frac{k_1}{2k_1}} = \frac{1}{\sqrt{2}}$$

So $T_2 = T_1/\sqrt{2} = (2.0\ \text{s})/\sqrt{2} = 1.4$ s.
(c) The formula for the period does not contain the amplitude; that is, the period is independent of the amplitude. Changing (in particular, doubling) the amplitude does not affect the period, so the new period is still 2.0 s.
Assess: It is equally important to understand what *doesn't* appear in a formula. It is quite startling, really, the first time you realize it, that the amplitude doesn't affect the period. But this is crucial to the idea of simple harmonic motion. Of course, if the spring is stretched too far, out of its linear region, then the amplitude would matter.

Q14.20. Reason: The truck is a "driven oscillator." At the intermediate speed where the vertical motion is large and unpleasant, the driving frequency due to hitting the bumps of the washboard is close to the natural (resonance) frequency

(which is determined by the suspension system and the mass of the truck); the resulting large-amplitude motion is resonance.
At speeds either significantly above or below that intermediate speed, the driving frequency of the bumps in the road is either smaller or greater than the resonance frequency and the response of the driven oscillator is small. See Figure 14.24.
Assess: It should be noted that a driven oscillator does oscillate at the driving frequency, whether that happens to be close to the resonance frequency or not. So the frequency of the up-and-down motion of the truck is the frequency with which it hits the regular bumps in the washboard road.
You could try an experiment by varying the natural frequency of your truck by loading it more or less and see if the speed that produces resonance on the same washboard road changes similarly.

P14.4. Prepare: $180° = \pi$ radian. Small angle approximation holds for angles less than 10° or 0.17 rad.
Solve: **(a)** and **(b)**

θ (°)	θ (rad)	$\sin\theta$
0	0	0
2	0.0349	0.0349
4	0.0698	0.0698
6	0.1047	0.1045
8	0.1396	0.1392
10	0.1745	0.1736
12	0.2094	0.2079

(c) 12°
(d) The small-angle approximation is good to three significant figures for θ up to 10°.

P14.7. Model: The air-track glider attached to a spring is in simple harmonic motion. The glider is pulled to the right and released from rest at $t = 0$ s. It then oscillates with a period $T = 2.0$ s and a maximum speed $4v_{max} = 0$ cm/s $= 0.40$ m/s. While the amplitude of the oscillation can be obtained from Equation 14.13, the position of the glider can be obtained from Equation 14.10, $x(t) = A\cos\left(\frac{2\pi t}{T}\right)$.
Solve: **(a)**

$$v_{max} = (2\pi A/T) \Rightarrow A = \frac{v_{max}T}{2\pi} = \frac{(0.40\text{ m/s})(2.0\text{ s})}{2\pi} = 0.127\text{ m} = 0.13\text{ m}$$

(b) The glider's position at $t = 0.25$ s is

$$x_{0.25\text{ s}} = (0.127\text{ m})\cos\left[\frac{2\pi(0.25\text{ s})}{2.0\text{ s}}\right] = 0.090\text{ m} = 9.0\text{ cm}$$

Assess: At t = 0.25 s, which is less than one quarter of the time period, the object has not reached the equilibrium position and is still moving toward the left.

P14.13. Prepare: Solve $a_{max} = (2\pi f)^2 A$ for A.
Solve:
(a)

$$A = \frac{a_{max}}{(2\pi f)^2} = \frac{0.20\text{ m/s}^2}{(2\pi(1.3\text{ Hz}))^2} = 3.0\text{ mm}$$

(b)

$$v_{max} = 2\pi fA = 2\pi(1.3\text{ Hz})(0.0030\text{ m}) = 0.024\text{ m/s}$$

Assess: These both seem like small but possible answers.

P14.19. Prepare: The oscillating mass is in simple harmonic motion. The position of the oscillating mass is given by $x(t) = (2.0\text{ cm})\cos(10t)$, where t is in seconds. We will compare this with Equation 14.10.
Solve: **(a)** The amplitude $A = 2.0$ cm.

(b) The period is calculated as follows:

$$\frac{2\pi}{T} = 10 \text{ rad/s} \Rightarrow T = \frac{2\pi}{10 \text{ rad/s}} = 0.63 \text{ s}$$

(c) The spring constant is calculated from Equation 14.27 as follows:

$$\frac{2\pi}{T} = \sqrt{\frac{k}{m}} \Rightarrow k = m\left(\frac{2\pi}{T}\right)^2 = (0.050 \text{ kg})(10 \text{ rad/s})^2 = 5.0 \text{ N/m}$$

(d) The maximum speed from Equation 14.26 is

$$v_{\text{max}} = 2\pi fA = \left(\frac{2\pi}{T}\right)A = (10 \text{ rad/s})(2.0 \text{ cm}) = 20 \text{ cm/s}$$

(e) The total energy from Equation 14.22 is

$$E = \frac{1}{2}kA^2 = \frac{1}{2}(5.0 \text{ N/m})(0.02 \text{ m})^2 = 1.0 \times 10^{-3} \text{ J}$$

(f) At $t = 0.40$ s, the velocity from Equation 14.12 is

$$v_x = -(20.0 \text{ cm/s})\sin[(10 \text{ rad/s})(0.40 \text{ s})] = 15 \text{ cm/s}$$

Assess: Velocity at $t = 0.40$ s is less than the maximum velocity, as would be expected.

P14.21. Prepare: The mass attached to the spring is in simple harmonic motion.
Solve: (a) The period using Equation 14.27 is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{(0.507 \text{ kg})}{(20 \text{ N/m})}} = 1.00 \text{ s}$$

(b) Using Equation 14.26, the maximum speed $v_{\text{max}} = 2\pi fA = (2\pi/T)A = (2\pi/1.00 \text{ s})(0.10 \text{ m}) = 0.628 \text{ m/s}$.

(c) The total energy from Equation 14.23 is $E = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}(0.507 \text{ kg})(0.628 \text{ m/s})^2 = 0.100 \text{ J}$.

P14.29. Prepare: To complete a whole period, the wrecking ball will have to swing down, up to the other side, back down, and up again to the original position. So the time it takes to swing from maximum height down to lowest height once is one-quarter of a period. We will assume that the wrecking ball is a simple small-angle pendulum and so it's period is given by $T = 2\pi\sqrt{L/g}$.

Solve:

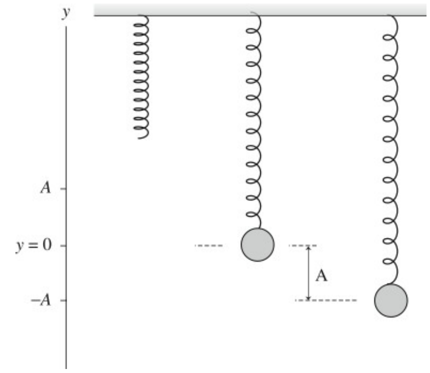
$$\frac{1}{4}T = \frac{1}{4}2\pi\sqrt{\frac{L}{g}} = \frac{\pi}{2}\sqrt{\frac{10 \text{ m}}{9.8 \text{ m/s}^2}} = 1.6 \text{ s}$$

Assess: This *is* enough time to dive out of the way, but it is still wiser to not stand in the way of wrecking balls.

P14.45. Prepare: The vertical oscillations constitute simple harmonic motion. A pictorial representation of the spring and the ball is shown in the following figure. The period and frequency of oscillations are

$$T = \frac{20 \text{ s}}{30 \text{ oscillations}} = 0.6667 \text{ s} \quad \text{and} \quad f = \frac{1}{T} = \frac{1}{0.6667 \text{ s}} = 1.50 \text{ Hz}$$

Since k is known, we can obtain the mass m using Equation 14.27.



Solve: **(a)** The mass can be found as follows:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow m = \frac{k}{(2\pi f)^2} = \frac{15.0 \text{ N/m}}{[2\pi(1.50 \text{ Hz})]^2} = 0.169 \text{ kg}$$

(b) The maximum speed is given by Equation 14.26, $v_{max} = 2\pi fA = 2\pi(1.50 \text{ Hz})(0.0600 \text{ m}) = 0.565 \text{ m/s}$.
Assess: Both the mass of the ball and its maximum speed are reasonable.

P14.50. Prepare: First we figure the mass of the chair alone, then the mass of the chair plus astronaut, then subtract.
Solve:

$$m_{ch} = \frac{T^2}{4\pi^2}k = \frac{(0.901 \text{ s})^2}{4\pi^2}(606 \text{ N / m}) = 12.46 \text{ kg}$$

$$m_{tot} = \frac{T^2}{4\pi^2}k = \frac{(2.09 \text{ s})^2}{4\pi^2}(606 \text{ N / m}) = 67.05 \text{ kg}$$

$m_* = m_{tot} - m_{ch} = 67.05 \text{ kg} - 12.46 \text{ kg} = 54.59 \text{ kg}$ which we report as 54.6 kg.
Assess: This is a reasonable weight for a light astronaut.

P14.59. Prepare: The strategy will be to find how long the block on the left will take to fall to the table in free fall and use that as one-half of a period for the oscillator on the right. The spring is 30 cm long, so before release the spring is neither stretched nor compressed and the block is at its maximum height (not in equilibrium because the force of gravity is still acting on it). To go from maximum height to minimum height takes one-half of a period.
Once we know the period and the mass of the block on the right we can solve for k.
Solve:

$$\Delta y = \frac{1}{2}a_y(\Delta t)^2$$

$$\Delta t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(-0.30 \text{ m})}{-9.8 \text{ m/s}^2}} = 0.247 \text{ s} = \frac{1}{2}T$$

$T = 2(0.247 \text{ s}) = 0.495 \text{ s}$.
Solve Equation 14.27 for k.

$$k = \frac{4\pi^2}{T^2}m = \frac{4\pi^2}{(0.495 \text{ s})^2}(0.050 \text{ kg}) = 8.1 \text{ N/m}$$

Assess: This is a reasonable spring, neither extremely stiff nor extremely loose.
The mass of the block on the left is irrelevant since all objects have the same acceleration in free fall.
The block on the right will pass through its equilibrium position when it falls far enough that $k\Delta y = mg$; that position is halfway down its total descent.