Physics 2A
Chapter 14: Oscillations

"The tragedy of life doesn't lie in not reaching your goal. The tragedy lies in having no goals to reach." **–** Benjamin Mays

"*If you have a goal in life that takes a lot of energy, that requires a lot of work, that incurs a great deal of interest and that is a challenge to you, you will always look forward to waking up to see what the new day brings."* **–** Susan Polis Schultz

"Do it now. You become successful the moment you start moving toward a worthwhile goal."
Unknown

Reading: pages 444 – 469; skip section 14.6

Outline:

⇒ equilibrium and oscillation
    frequency and period
⇒ simple harmonic motion
    Hooke's Law
    mass on a spring
    the pendulum
    angular frequency
    displacement. velocity, and acceleration
⇒ energy in simple harmonic motion
    conservation of mechanical energy
    frequency and period in SHM
⇒ pendulum motion
⇒ resonance

Problem Solving

Many of the problems in this chapter deal with Hooke's law, which states that the restoring force of an ideal spring is given by $F = -kx$. In this equation, F is the force exerted by the spring, k is the spring constant, and x is the displacement of the spring from its unstrained length. The minus sign indicates that the force is always in the opposite direction of the displacement.

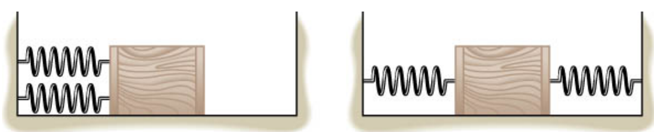
You should be familiar with the definition of the radian, which is measure of angular displacement. The relationship between radians and degrees is given by $1\ rad = \frac{360^\circ}{2\pi} = 57.3^\circ$.
You should also be familiar with the definition of angular velocity ω. It is the angular equivalent of velocity.

Some problems make use of the relationships among angular frequency, frequency, and period for simple harmonic motion: $\omega = 2\pi f$, and $f = 1/T$. Occasionally the period is given indirectly by describing a time interval. You must then know, for example, that the time the oscillator takes to go from maximum displacement in one direction to maximum displacement in the other direction is $T/2$ or the time it takes to go from maximum displacement to zero displacement is $T/4$. If these time intervals or others are given, you should be able to calculate the period, frequency, and angular frequency. You should also know how to find the maximum speed and maximum acceleration in terms of the angular frequency and amplitude: $v_{max} = A\omega$ and $a_{max} = A\omega^2$. Some problems require you to know the relationship between the angular frequency and the appropriate physical properties of the oscillating system: $\omega = \sqrt{k/m}$ for an undamped spring-object system.

Some problems can be solved using the principle of mechanical energy conservation. For a spring-object system, the mechanical energy E is given by: $E = ½\, mv^2 + ½\, kx^2 + mgh$.

Questions and Example Problems from Chapter 14

Question 1
The drawing shows identical springs that are attached to a box in two different ways. Initially, the springs are unstrained. The box is then pulled to the right and released. In each case, the initial displacement of the box is the same. At the moment of release, which box, of either, experiences the greater net force due to the spring? Provide a reason for your answer.



Question 2
Suppose that a grandfather clock (a simple pendulum) is running slowly. That is, the time it takes to complete each cycle is longer than it should be. Should one shorten or lengthen the pendulum to make the clock keep the correct time? Why?

Problem 1
The equilibrium length of a certain spring with a force constant of k = 250 N/m is 0.20 m. (a) What force is required to stretch this spring to twice its equilibrium length? (b) Is the force required to compress the spring to half its length the same as in part (a)? Explain.

SUMMARY

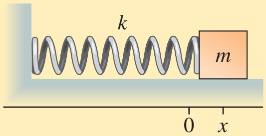
The goal of Chapter 14 has been to understand systems that oscillate with simple harmonic motion.

GENERAL PRINCIPLES

Restoring Forces

SHM occurs when a **linear restoring force** acts to return a system to an equilibrium position.

Mass on spring



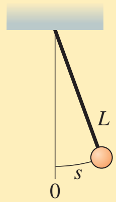
$$(F_{\text{net}})_x = -kx$$

The frequency of a mass on a spring depends on the mass and the spring constant:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Pendulum

$$(F_{\text{net}})_t = -\left(\frac{mg}{L}\right)s$$



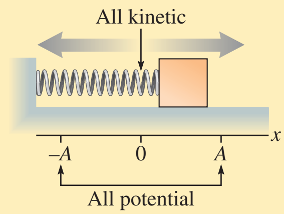
The frequency of a pendulum depends on the length and the free-fall acceleration:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

Energy

If there is no friction or dissipation, kinetic and potential energies are alternately transformed into each other in SHM, with the sum of the two conserved.

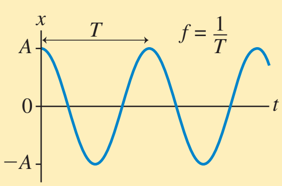
$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}m{v_{\max}}^2 = \frac{1}{2}kA^2$$



IMPORTANT CONCEPTS

Oscillation

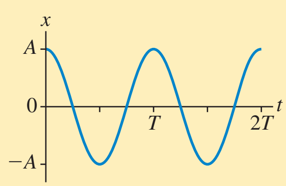
An **oscillation** is a repetitive motion about an equilibrium position. The **amplitude** A is the maximum displacement from equilibrium. The period T is the time for one cycle. We may also characterize an oscillation by its frequency f.



Simple Harmonic Motion (SHM)

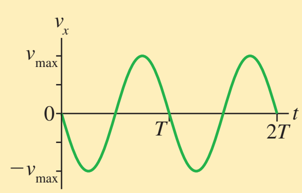
SHM is an oscillation that is described by a sinusoidal function. All systems that undergo SHM can be described by the same functional forms.

Position-versus-time is a cosine function.



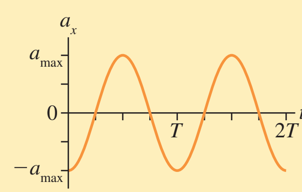
$$x(t) = A\cos(2\pi f t)$$
$$x_{\max} = A$$

Velocity-versus-time is an inverted sine function.



$$v_x(t) = -v_{\max}\sin(2\pi f t)$$
$$v_{\max} = 2\pi f A$$

Acceleration-versus-time is an inverted cosine function.

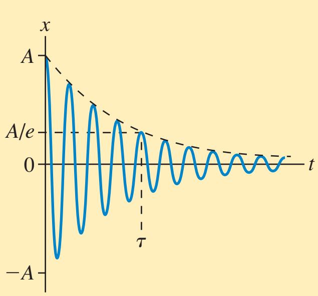


$$a_x(t) = -a_{\max}\cos(2\pi f t)$$
$$a_{\max} = (2\pi f)^2 A$$

APPLICATIONS

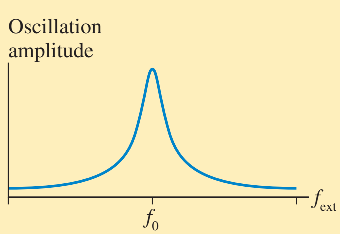
Damping

Simple harmonic motion with damping (due to drag) decreases in amplitude over time. The **time constant** τ determines how quickly the amplitude decays.



Resonance

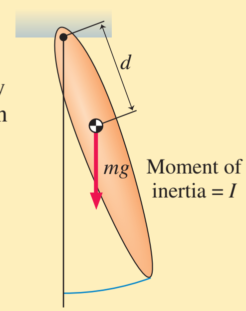
A system that oscillates has a **natural frequency** of oscillation f_0. **Resonance** occurs if the system is driven with a frequency f_{ext} that matches this natural frequency. This may produce a large amplitude of oscillation.



Physical pendulum

A **physical pendulum** is a pendulum with mass distributed along its length. The frequency depends on the position of the center of gravity and the moment of inertia.

The motion of legs during walking can be described using a physical pendulum model.



$$f = \frac{1}{2\pi}\sqrt{\frac{mgd}{I}}$$

Problem 2

An air-track glider attached to a spring oscillates between the 10 cm mark and the 60 cm mark on the track. The glider completes 10 oscillations in 33 s. What are the (a) period, (b) frequency, (c) amplitude, and (d) maximum speed of the glider?

Problem 3

A computer to be used in a satellite must be able to withstand accelerations of up to 25 times the acceleration due to gravity, In a test to see whether it meets this specification, the computer is bolted to a frame that is vibrated back and forth in simple harmonic motion at a frequency of 9.5 Hz. What is the minimum amplitude of vibration that must be used in this test?

Problem 4

A block of mass $m = 0.750$ kg is fastened to an unstrained horizontal spring whose spring constant is $k = 82.0$ N/m. The block is given a displacement of +0.120 m, where the + sign indicates that the displacement is along the +x axis, and then released from rest. (a) What is the force (magnitude and direction) that the spring exerts on the block just before the block is released? (b) Find the frequency of the resulting oscillatory motion. (c) What is the maximum speed of the block? (d) Determine the magnitude of the maximum acceleration of the block?

Problem 5

The shock absorbers in the suspension system of a car are in such bad shape that they have no effect on the behavior of the springs attached to the axles. Each of the identical springs attached to the front axle supports 320 kg. A person pushes down on the middle of the front end of the car and notices that it vibrates through 5 cycles in 3.0 s. Find the spring constant of either spring.

Problem 6

A 200 g mass attached to a horizontal spring oscillates at a frequency of 2.0 Hz. At one instant, the mass is at x = 5.0 cm and has speed v = -30 cm/s. Determine: (a) The period. (b) The amplitude. (c) The maximum speed. (d) The total energy.

Problem 7

An archer pulls the bowstring back for a distance of 0.470 m before releasing the arrow. The bow and string act like a spring whose spring constant is 425 N/m. (a) What is the elastic potential energy of the drawn bow? (b) The arrow has a mass of 0.0300 kg. How fast is it traveling when it leaves the bow?

Problem 8

A 1.00×10^{-2} kg block is resting on a horizontal frictionless surface and is attached to a horizontal spring whose spring constant is 124 N/m. The block is shoved parallel to the spring axis and is given an initial speed of 8.00 m/s, while the spring is initially unstrained. What is the amplitude of the resulting simple harmonic motion?

Problem 9

A 0.40 kg mass is attached to a spring with a force constant of 26 N/m and released from rest a distance of 3.2 cm from the equilibrium position of the spring. What is the speed of the mass when it is halfway to the equilibrium position?

Problem 10

Astronauts on a distant planet set up a simple pendulum of length 1.2 m. The pendulum executes simple harmonic motion and makes 100 complete vibrations in 280 s. What is the acceleration due to gravity?

Problem 11

If the period of a simple pendulum is to be 2.0 s, what should be its length?