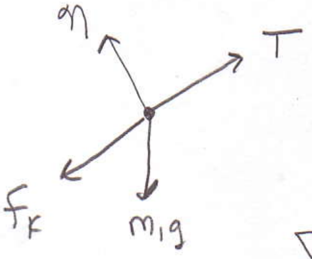
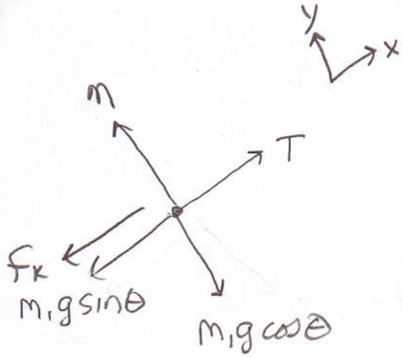
Celebration #1 Review Solutions

Problem 1

block 1:





$$\sum F_y = m a_y = 0$$

$$n - m_1 g \cos\theta = 0$$

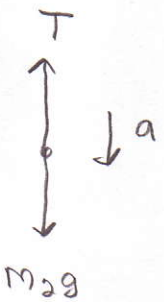
$$n = m_1 g \cos\theta$$

$$\sum F_x = m a_x$$

$$T - f_K - m_1 g \sin\theta = m_1 a \qquad f_K = \mu_K n = \mu_K (m_1 g \cos\theta)$$

$$T - \mu_K m_1 g \cos\theta - m_1 g \sin\theta = m_1 a \quad (1)$$

block 2:



$$\sum F_y = m a_y$$

$$m_2 g - T = m_2 a$$

$$T = m_2 g - m_2 a \quad (2)$$

put (2) → (1) $\quad (m_2 g - m_2 a) - \mu_K m_1 g \cos\theta - m_1 g \sin\theta = m_1 a$

$$m_2 g - \mu_K m_1 g \cos\theta - m_1 g \sin\theta = (m_1 + m_2) a$$

$$m_2 g - m_1 g \sin\theta - (m_1 + m_2) a = \mu_K m_1 g \cos\theta$$

$$\mu_K = \frac{m_2 g - m_1 g \sin\theta - (m_1 + m_2) a}{m_1 g \cos\theta}$$

$$\mu_K = \frac{(20.0\,kg)(9.80\,m/s^2) - (15.0\,kg)(9.80\,m/s^2)\sin 37.0^\circ - (35.0\,kg)(2.00\,m/s^2)}{(15.0\,kg)(9.80\,m/s^2)\cos 37.0^\circ}$$

$$\mu_K = 0.320$$

Problem 2

A 2 kg ball is launched at 40 m/s at an angle of 35.0° above the horizontal.

$V_{ox} = 40\,m/s \cos 35° = \underline{32.8\,m/s}$

$V_{oy} = 40\,m/s \sin 35° = \underline{22.9\,m/s}$

(a) How long does the ball stay in the air? (3 points)

at top of path $V_y = 0\,m/s$

$V_{oy} = 22.9\,m/s$

$a_y = -9.8\,m/s^2$

$t = ?$

$V_y = V_{oy} + at$

$t = \frac{V_y - V_{oy}}{a_y} = \frac{-22.9\,m/s}{-9.8\,m/s^2} = \underline{2.3\,s}$

total time in air = 4.6 s

(b) How far from its launch point does the ball land? (3 points)

$t = 4.6\,s$

$X_o = 0\,m$

$X = ?$

$V_{ox} = 32.8\,m/s$

$X = X_o + V_{ox} t$

$= (32.8\,m/s)(4.6\,s)$

$X = 151\,m$

(c) When is the ball's speed equal to 35 m/s? (3 points)

$V = \sqrt{V_x^2 + V_y^2}$

$V_y = \pm\sqrt{V^2 - V_x^2}$

$= \pm\sqrt{(35\,m/s)^2 - (32.8\,m/s)^2}$

$= \pm 12.2\,m/s$ → this is V_y when $V = 35\,m/s$

$V_y = V_{oy} + at \rightarrow t = \frac{V_y - V_{oy}}{a}$

for $V_y = 12.2\,m/s$ **$t = 1.1\,s$**

for $V_y = -12.2\,m/s$ **$t = 3.6\,s$**

(d) What is the magnitude and direction of the ball's velocity at t = 3.0 s? (3 points)

$V_x = V_{ox} = 32.8\,m/s$

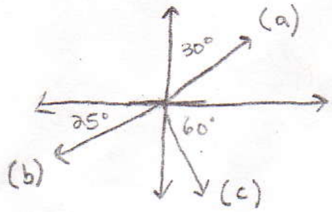
$V_y = V_{oy} + at = 22.9\,m/s + (-9.8\,m/s^2)(3.0\,s)$

$= -6.5\,m/s$

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{(32.8\,m/s)^2 + (-6.5\,m/s)^2}$ **$V = 33.4\,m/s$**

$\theta = \tan^{-1}\left(\frac{-6.5\,m/s}{32.8\,m/s}\right) \Rightarrow$ **$\theta = -11°$ or $349°$**

Problem 3



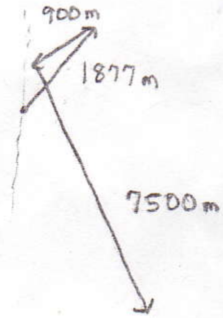
A car drives at a constant speed of 35.0 **mi/hr** at 30° east of north for 2 minutes, then at a constant speed of 20 m/s at 25° south of west for 45 s, and finally at a constant speed of 25.0 m/s at 60° south of east for 5 minutes.

(a) What is the magnitude and direction of the average velocity of the car during this trip? (8 points)

(a) $35\,mi/hr \times \frac{1609\,m}{1\,mi} \times \frac{1\,hr}{3600\,s} = 15.6\,m/s \times 120\,s = 1877\,m$ at 60°

(b) $20\,m/s \times 45\,s = 900\,m$ at 205°

(c) $25.0\,m/s \times 300\,s = 7500\,m$ at 300°



(a) $\vec{r} = (1877\,m \cos 60°)\hat{i} + (1877\,m \sin 60°)\hat{j}$
$= 939\,m\,\hat{i} + 1626\,m\,\hat{j}$

(b) $\vec{r} = (900\,m \cos 205)\hat{i} + (900\,m \sin 205)\hat{j}$
$= -816\,m\,\hat{i} - 380\,m\,\hat{j}$

(c) $\vec{r} = (7500\,m \cos 300)\hat{i} + (7500\,m \sin 300)\hat{j} = 3750\,m\,\hat{i} - 6495\,m\,\hat{j}$

$\Delta\vec{r} = (939\,m - 816\,m + 3750\,m)\hat{i} + (1626\,m - 380\,m - 6495\,m)\hat{j}$

$\Delta\vec{r} = 3873\,m\,\hat{i} - 5249\,m\,\hat{j}$

$\vec{V}_{ave} = \Delta\vec{r}/\Delta t \quad \Delta t = 465\,s$

$\vec{V}_{ave} = 8.33\,m/s\,\hat{i} - 11.3\,m/s\,\hat{j}$

$V = \sqrt{(8.33\,m/s)^2 + (-11.3\,m/s)^2}$
$= 14.0\,m/s$

$\theta = \tan^{-1}\left(\frac{-11.3}{8.33}\right) = -53.6°$

$V = 14.0\,m/s$ at 306°

(b) What is the average speed of the car during this trip? (4 points)

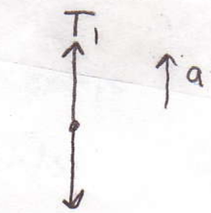
$\text{average speed} = \frac{\text{total distance}}{\text{time}}$

total distance $= 900\,m + 1877\,m + 7500\,m$
$= 10{,}277\,m$

$\text{average speed} = \frac{10{,}277\,m}{465\,s} =$ **22.1 m/s**

Problem 4

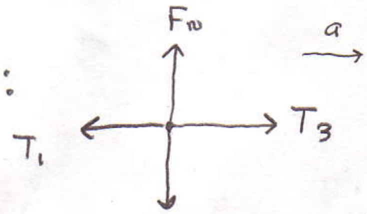
m_1:



$$\sum F_y = ma_y$$

$$T_1 - m_1 g = m_1 a \longrightarrow T_1 = m_1 a + m_1 g \quad (1)$$

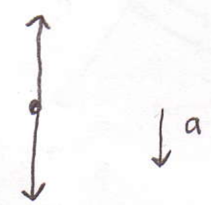
m_2:



$$\sum F_x = ma_x$$

$$T_3 - T_1 = m_2 a \quad (3)$$

m_3:



$$\sum F_y = ma_y$$

$$m_3 g - T_3 = m_3 a \longrightarrow T_3 = m_3 g - m_3 a \quad (2)$$

put (1) + (2) → (3)

$$(m_3 g - m_3 a) - (m_1 a + m_1 g) = m_2 a$$

$$m_3 g - m_1 g = m_1 a + m_2 a + m_3 a$$

$$a = \frac{(m_3 - m_1) g}{(m_1 + m_2 + m_3)} \longrightarrow a = \frac{(2.5\,Kg - 1.5\,Kg)\, 9.8\, m/s^2}{(1.5\,Kg + 3.5\,Kg + 2.5\,Kg)}$$

$$\boxed{a = 1.31\, m/s^2}$$

from (1) $T_1 = m_1(g + a) = (1.5\,Kg)(9.8\,m/s^2 + 1.31\,m/s^2) = \boxed{16.7\,N}$

from (2) $T_3 = m_3(g - a) = (2.5\,Kg)(9.8\,m/s^2 - 1.31\,m/s^2) = \boxed{21.2\,N}$

Problem 5

$v_0 = 8.00 m/s$

$v = 0 m/s$

$x_0 = 0 m$

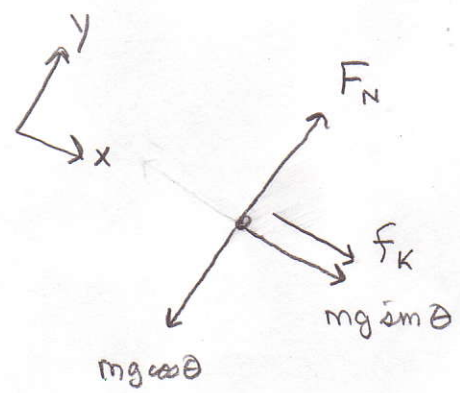
$x = 3.00 m$

$a = ?$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$-v_0^2 = 2ax \longrightarrow a = -v_0^2/2x = \frac{(8.00 m/s)^2}{2(3.00 m)}$$

$$a = -10.7 m/s^2 = 10.7 m/s^2 \text{ down the incline}$$



$$\sum F_x = ma_x$$

$$f_K + mg \sin\theta = ma_x \qquad f_K = \mu_K F_N = \mu_K mg \cos\theta$$

$$\mu_K mg \cos\theta + mg \sin\theta = ma_x$$

$$\mu_K = \frac{a_x - g \sin\theta}{g \cos\theta} \longrightarrow \mu_K = \frac{10.7 m/s^2 - (9.8 m/s^2) \sin 30°}{(9.8 m/s^2) \cos 30°}$$

$$\boxed{\mu_K = 0.68}$$

Problem 6

$y_0 = 0\,m$

$y = -2.2\,m$

$V_{oy} = ?$

$a_y = -9.8\,m/s^2$

$t = 0.30\,s$

$$y = y_0 + V_{oy} t + \tfrac{1}{2} a_y t^2 \quad (y_0 = 0)$$

$$y = V_{oy} t + \tfrac{1}{2} a_y t^2$$

$$V_{oy} = \frac{y - \tfrac{1}{2} a_y t^2}{t}$$

$$V_{oy} = \frac{-2.2\,m - \tfrac{1}{2}(-9.8\,m/s^2)(0.30\,s)^2}{(0.30\,s)}$$

$V_{oy} = -5.86\,m/s$ → ~~speed~~ velocity at top of window

$V < 0$ since object moving downward

↓

How far does an object dropped from rest have to fall so that $V_y = -5.86\,m/s$?

$V_{oy} = 0\,m/s$

$V_y = -5.86\,m/s$

$y_0 = 0\,m$

$y = ?$

$a_y = -9.8\,m/s^2$

$$V_y^2 = V_{oy}^2 + 2a_y(y - y_0) \quad (V_{oy} = 0,\ y_0 = 0)$$

$$V_y^2 = 2a_y y$$

$$y = V_y^2 / 2a_y = \frac{(-5.86\,m/s)^2}{2(-9.8\,m/s^2)}$$

$y = -1.75\,m$

↳ **rock fell 1.75 m**