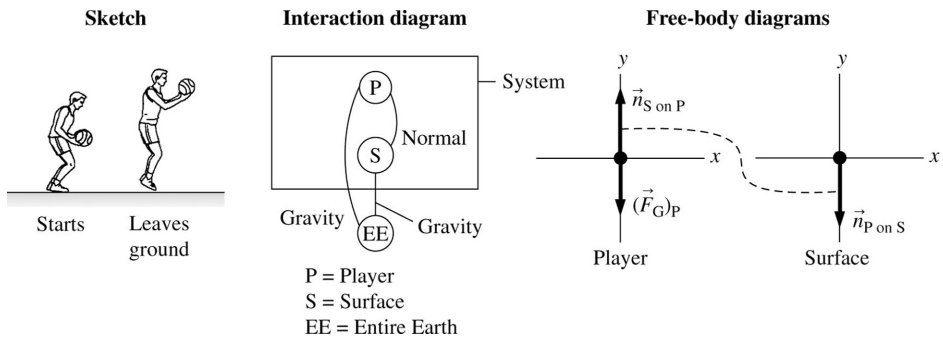
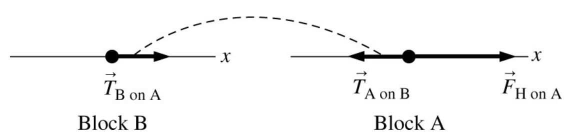
Physics 4A
Chapter 7 HW Solutions

Chapter 7
Conceptual Questions: 4, 13, 15
Exercises & Problems: 8, 24, 25, 31, 32, 34, 38, 39, 40, 42, 48

CQ 7.4. The player pushes down on the floor, which pushes back up on him. The player accelerates upward because the force of his push is greater than the force of gravity.



CQ 7.13.

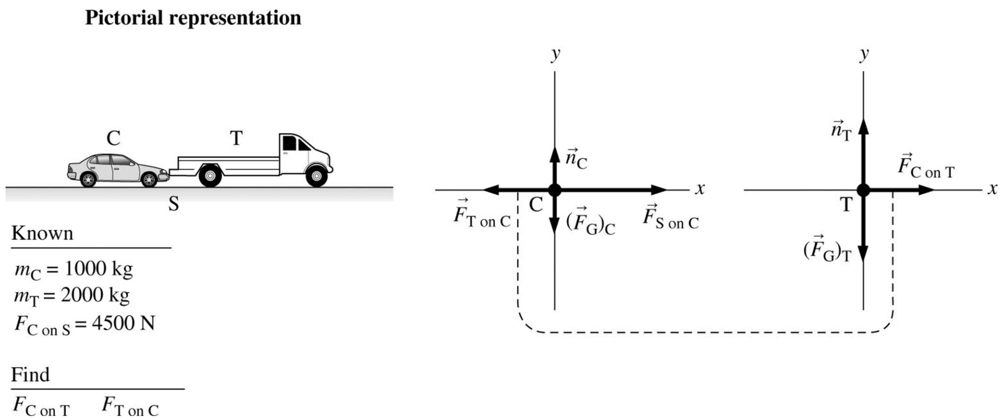


The figure shows the horizontal forces on blocks B and A using the massless-string approximation in the absence of friction. The hand must accelerate both blocks A and B, so more force is required to accelerate the greater mass. Thus the force of the string on B is smaller than the force of the hand on A.

CQ 7.15. Block A's acceleration is greater in case b. In case a, the hanging 10 N must accelerate both the mass of A and its own mass, leading to a smaller acceleration than case b, where the entire 10 N force accelerates the mass of block A.

Case a	Case b
$10\text{ N} = (M_A + M_{10\text{ N}})a$ $a = \dfrac{10\text{ N}}{(M_A + M_{10\text{ N}})}$	$10\text{ N} = M_A a$ $a = \dfrac{10\text{ N}}{M_A}$

7.8. Model: Model the car and the truck as particles denoted by the symbols C and T, respectively. Denote the surface of the ground by the symbol S.
Visualize:



Solve: **(a)** The x-component of Newton's second law for the car gives

$$\Sigma(F_{\text{on C}})_x = F_{\text{S on C}} - F_{\text{T on C}} = m_C a_C$$

The x-component of Newton's second law for the truck gives

$$\Sigma(F_{\text{on T}})_x = F_{\text{C on T}} = m_T a_T$$

Using $a_C = a_T \equiv a$ and $F_{\text{T on C}} = F_{\text{C on T}}$, we get

$$(F_{\text{C on S}} - F_{\text{C on T}})\left(\frac{1}{m_C}\right) = a \text{ and } (F_{\text{C on T}})\left(\frac{1}{m_T}\right) = a$$

Combining these two equations,

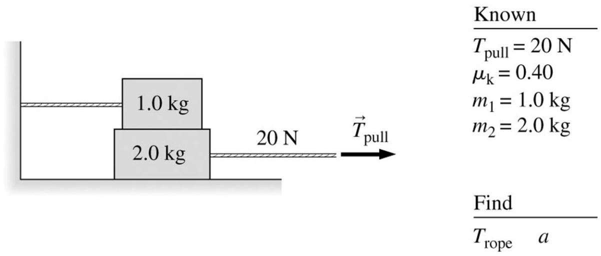
$$(F_{\text{C on S}} - F_{\text{C on T}})\left(\frac{1}{m_C}\right) = (F_{\text{C on T}})\left(\frac{1}{m_T}\right) \Rightarrow F_{\text{C on T}}\left(\frac{1}{m_C} + \frac{1}{m_T}\right) = (F_{\text{C on S}})\left(\frac{1}{m_C}\right)$$

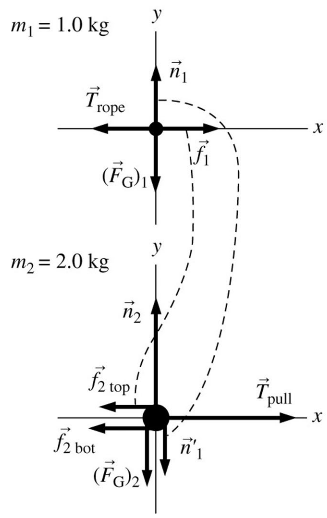
$$F_{\text{C on T}} = (F_{\text{C on S}})\left(\frac{m_T}{m_C + m_T}\right) = (4500\text{ N})\left(\frac{2000\text{ kg}}{1000\text{ kg} + 2000\text{ kg}}\right) = 3000\text{ N}$$

(b) Due to Newton's third law, $F_{\text{T on C}} = 3000$ N.

7.24. Model: The two blocks (1 and 2) form the system of interest and will be treated as particles. The ropes are assumed to be massless, and the model of kinetic friction will be used.
Visualize:

Pictorial representation





Solve: (a) The separate free-body diagrams for the two blocks show that there are two action/reaction pairs. Notice how block 1 both pushes down on block 2 (force $\vec{n}_1'$) and exerts a retarding friction force $\vec{f}_{2\ top}$ on the top surface of block 2. Block 1 is in static equilibrium ($a_1 = 0\ \text{m/s}^2$) but block 2 is accelerating to the right. Newton's second law for block 1 is

$$(F_{\text{net on 1}})_x = f_1 - T_{\text{rope}} = 0\ \text{N} \quad \Rightarrow \quad T_{\text{rope}} = f_1$$
$$(F_{\text{net on 1}})_y = n_1 - m_1 g = 0\ \text{N} \quad \Rightarrow \quad n_1 = m_1 g$$

Although block 1 is stationary, there is a *kinetic* force of friction because there is motion between blocks 1 and 2. The friction model means $f_1 = \mu_k n_1 = \mu_k m_1 g$. Substitute this result into the x-equation to get the tension in the rope:

$$T_{\text{rope}} = f_1 = \mu_k m_1 g = (0.40)(1.0\ \text{kg})(9.8\ \text{m/s}^2) = 3.9\ \text{N}$$

(b) Newton's second law for block 2 is

$$a_x \equiv a = \frac{(F_{\text{net on 2}})_x}{m_2} = \frac{T_{\text{pull}} - f_{2\text{ top}} - f_{2\text{ bot}}}{m_2}$$

$$a_y = 0 \text{ m/s}^2 = \frac{(F_{\text{net on 2}})_y}{m_2} = \frac{n_2 - n_1' - m_2 g}{m_2}$$

Forces $\vec{n}_1$ and $\vec{n}_1'$ are an action/reaction pair, so $n_1' = n_1 = m_1 g$. Substituting into the y-equation gives $n_2 = (m_1 + m_2)g$. This is not surprising because the combined weight of both objects presses down on the surface. The kinetic friction on the bottom surface of block 2 is then

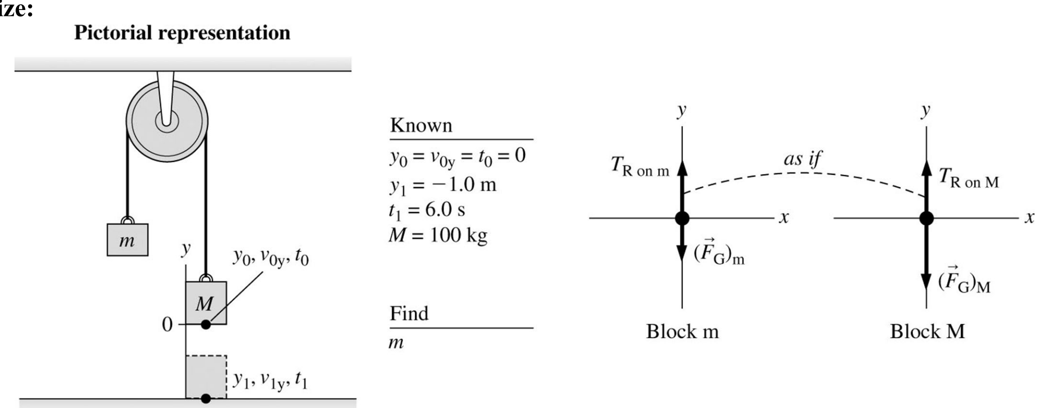
$$f_{2\text{ bot}} = \mu_k n_2 = \mu_k (m_1 + m_2) g$$

The forces $\vec{f}_1$ and $\vec{f}_{2\text{ top}}$ are an action/reaction pair, so $f_{2\text{ bot}} = f_1 = \mu_k m_1 g$. Inserting these friction results into the x-equation gives

$$a = \frac{(F_{\text{net on 2}})_x}{m_2} = \frac{T_{\text{pull}} - \mu_k m_1 g - \mu_k (m_1 + m_2) g}{m_2}$$

$$= \frac{20 \text{ N} - (0.40)(1.0 \text{ kg})(9.8 \text{ m/s}^2) - (0.40)(1.0 \text{ kg} + 2.0 \text{ kg})(9.8 \text{ m/s}^2)}{2.0 \text{ kg}} = 2.2 \text{ m/s}^2$$

7.25. Model: The masses m and M are to be treated in the particle model. We will also assume a massless rope and frictionless pulley, and use the constant-acceleration kinematic equations for m and M.
Visualize:



Solve: Using $y_1 = y_0 + v_{0y}(t_1 - t_0) + \frac{1}{2}a_M(t_1 - t_0)^2$,

$$(-1.0 \text{ m}) = 0 \text{ m} + 0 \text{ m} + \tfrac{1}{2}a_M(6.0 \text{ s} - 0 \text{ s})^2 \quad \Rightarrow \quad a_M = -0.0556 \text{ m/s}^2$$

Newton's second law for m and M gives

$$\Sigma(F_{\text{on m}})_y = T_{\text{R on m}} - (F_G)_m = m a_m \qquad \Sigma(F_{\text{on M}})_y = T_{\text{R on M}} - (F_G)_M = M a_M$$

The acceleration constraint is $a_m = -a_M$. Also, the tensions are an pseudo-action/reaction pair, so $T_{\text{R on m}} = T_{\text{R on M}}$. With these, the second-law equations become

$$T_{\text{R on M}} - Mg = Ma_M$$

$$T_{\text{R on M}} - mg = -ma_M$$

Subtracting the second from the first gives

$$-Mg + mg = Ma_M + ma_M$$

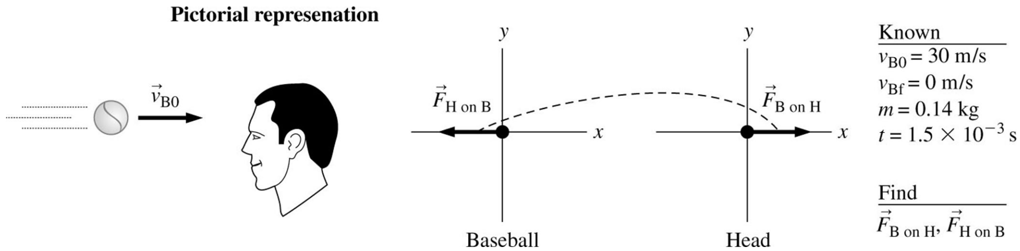
$$m = M\left[\frac{g + a_M}{g - a_M}\right]$$

$$= (100 \text{ kg})\left[\frac{9.8 \text{ m/s}^2 - 0.556 \text{ m/s}^2}{9.8 \text{ m/s}^2 + 0.556 \text{ m/s}^2}\right] = 99 \text{ kg}$$

Assess: Note that $a_m = -a_M = 0.0556\ \text{m/s}^2$. For such a small acceleration, the 1% mass difference seems reasonable.

7.31. Model: We shall only consider horizontal forces. The head and the baseball are the two objects in our system and are treated as particles. We will also use the constant-acceleration kinematic equations.

Visualize:



Solve: **(a)** The ball experiences an average acceleration of

$$a_B = \frac{v_{B1} - v_{B0}}{t} = \frac{-30\ \text{m/s}}{1.5 \times 10^{-3}\ \text{s}} = -20{,}000\ \text{m/s}^2$$

Insert this into Newton's second law to find the force on the baseball:

$$F_{\text{H on B}} = m_B a_B = (0.14\ \text{kg})\left|-20{,}000\ \text{m/s}^2\right| = 2800\ \text{N}$$

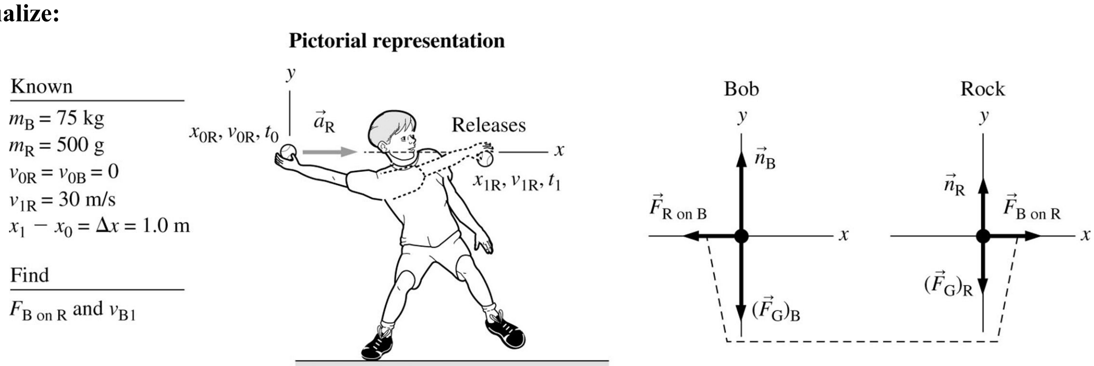
(b) By Newton's third law, the magnitude of the force exerted by the ball on the head is the same as that exerted by the head on the ball. Thus, $F_{\text{B on H}} = 2800$ N.

(c) Because 2800 N < 6000 N, the ball will not fracture your forehead, but will fracture your cheekbone because 2800 N > 1300 N.

Assess: A 90 mph fastball travels at $(90\ \text{mph})(1609.3\ \text{m/mile})(1\ \text{h}/3600\ \text{s}) = 40\ \text{m/s}$, so it will not fracture your forehead, but it will fracture your cheekbone. This explains why baseball helmets protect the cheekbone.

7.32. Model: The rock (R) and Bob (b) are the two objects in our system, and will be treated as particles. We will also use the constant-acceleration kinematic equations.

Visualize:



Solve: **(a)** Bob exerts a forward force $\vec{F}_{\text{B on R}}$ on the rock to accelerate it forward. The rock's acceleration is calculated as follows:

$$v_{1R}^2 = v_{0R}^2 + 2a_{0R}\Delta x \quad \Rightarrow \quad a_R = \frac{v_{1R}^2}{2\Delta x} = \frac{(30\ \text{m/s})^2}{2(1.0\ \text{m})} = 450\ \text{m/s}^2$$

The force is calculated from Newton's second law:

$$F_{\text{B on R}} = m_R a_R = (0.500\ \text{kg})(450\ \text{m/s}^2) = 225\ \text{N}$$

Bob exerts a force of 2.3×10^2 N on the rock.

(b) Because Bob pushes on the rock, the rock pushes back on Bob with a force $\vec{F}_{\text{R on B}}$. Forces $\vec{F}_{\text{R on B}}$ and $\vec{F}_{\text{B on R}}$ are an action/reaction pair, so $F_{\text{R on B}} = F_{\text{B on R}} = 225$ N. The force causes Bob to accelerate backward with an acceleration of

$$a_{\text{B}} = \frac{(F_{\text{net on B}})_x}{m_{\text{B}}} = -\frac{F_{\text{R on B}}}{m_{\text{B}}} = -\frac{225 \text{ N}}{75 \text{ kg}} = -3.0 \text{ m/s}^2$$

This is a rather large acceleration, but it lasts only until Bob releases the rock. We can determine the time interval by returning to the kinematics of the rock:

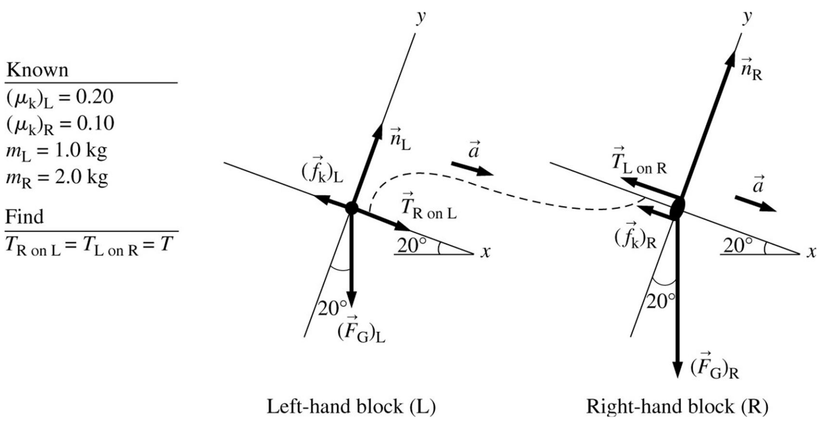
$$v_{1\text{R}} = v_{0\text{R}} + a_{\text{R}}\Delta t = a_{\text{R}}\Delta t \quad \Rightarrow \quad \Delta t = \frac{v_{1\text{R}}}{a_{\text{R}}} = 0.0667 \text{ s}$$

At the end of this interval, Bob's velocity is

$$v_{1\text{B}} = v_{0\text{B}} + a_{\text{B}}\Delta t = a_{\text{B}}\Delta t = -0.20 \text{ m/s}$$

Thus his recoil speed is 0.20 m/s.

7.34. Model: The two blocks form a system of interacting objects. We shall treat them as particles.
Visualize: Please refer to Figure P7.34.



Solve: It is possible that the left-hand block (block L) is accelerating down the slope faster than the right-hand block (block R), causing the string to be slack (zero tension). If that were the case, we would get a zero or negative answer for the tension in the string. Newton's first law applied in the y-direction on block L yields

$$(\Sigma F_{\text{L}})_y = 0 = n_{\text{L}} - (F_{\text{G}})_{\text{L}} \cos(20°) \quad \Rightarrow \quad n_{\text{L}} = m_{\text{L}} g \cos(20°)$$

Therefore

$$(f_{\text{k}})_{\text{L}} = (\mu_{\text{k}})_{\text{L}} m_{\text{L}} g \cos(20°) = (0.20)(1.0 \text{ kg})(9.80 \text{ m/s}^2)\cos(20°) = 1.84 \text{ N}$$

A similar analysis of the forces in the y-direction on block R gives $(f_{\text{k}})_{\text{R}} = 1.84$ N as well. Using Newton's second law in the x-direction for block L gives

$$(\Sigma F_{\text{L}})_x = m_{\text{L}} a = T_{\text{R on L}} - (f_{\text{k}})_{\text{L}} + (F_{\text{G}})_{\text{L}} \sin(20°) \quad \Rightarrow \quad m_{\text{L}} a = T_{\text{R on L}} - 1.84 \text{ N} + m_{\text{L}} g \sin(20°)$$

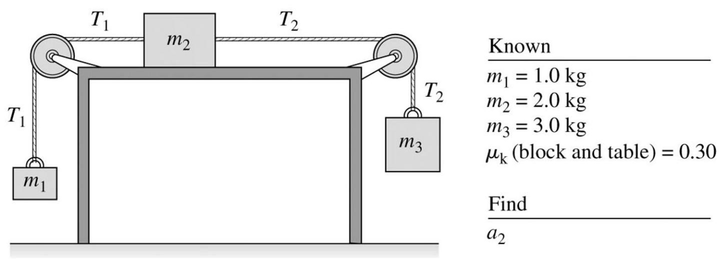
For block R,

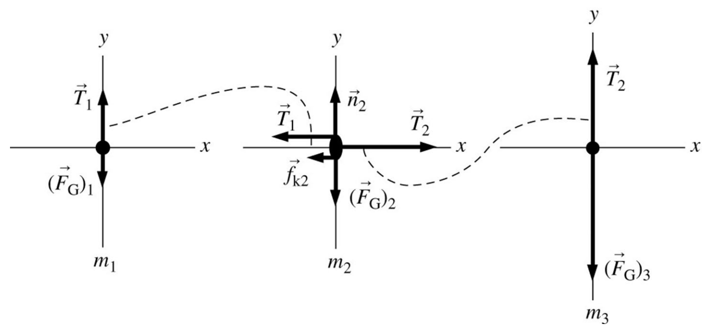
$$(\Sigma F_{\text{R}})_x = m_{\text{R}} a = (F_{\text{G}})_{\text{R}} \sin(20°) - 1.84 \text{ N} - T_{\text{L on R}} \quad \Rightarrow \quad m_{\text{R}} a = m_{\text{R}} g \sin(20°) - 1.84 \text{ N} - T_{\text{L on R}}$$

Solving these two equations in the two unknowns a and $T_{\text{L on R}} = T_{\text{R on L}} \equiv T$, we obtain $a = 2.12 \text{ m/s}^2$ and $T = 0.61$ N.
Assess: The tension in the string is positive, and is about 1/3 of the kinetic friction force on each of the blocks, which is reasonable.

7.38. Model: Assume the particle model for $m_1, m_2,$ and $m_3,$ and the model of kinetic friction. Assume the ropes to be massless, and the pulleys to be frictionless and massless.
Visualize:

Pictorial representation





Solve: Newton's second law for m_1 gives $T_1 - (F_G)_1 = m_1 a_1$. Newton's second law for m_2 gives

$$\Sigma(F_{\text{on } m_2})_y = n_2 - (F_G)_2 = 0 \text{ N} \quad \Rightarrow \quad n_2 = m_2 g$$

$$\Sigma(F_{\text{on } m_2})_x = T_2 - f_{k2} - T = m_2 a_2 \quad \Rightarrow \quad T_2 - \mu_k n_2 - T_1 = m_2 a_2$$

Newton's second law for m_3 gives $T_2 - (F_G)_3 = m_3 a_3$. Since $m_1, m_2,$ and m_3 move together, $a_1 = a_2 = -a_3 \equiv a$. The equations for the three masses thus become

$$T_1 - (F_G)_1 = m_1 a \quad T_2 - \mu_k n_2 - T_1 = m_2 a \quad T_2 - (F_G)_3 = -m_3 a$$

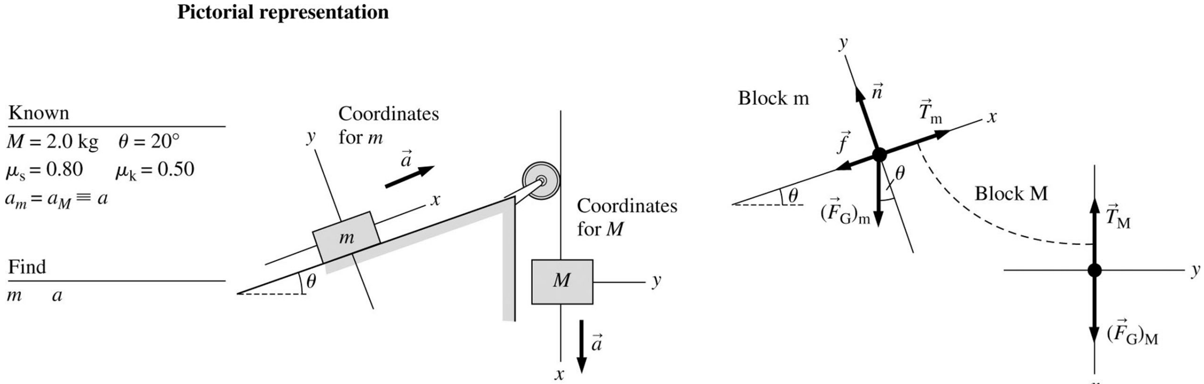
Subtracting the third equation from the sum of the first two equations yields:

$$-(F_G)_1 - \mu_k n_2 + (F_G)_3 = -m_1 g - \mu_k m_2 g + m_3 g = (m_1 + m_2 + m_3)a$$

$$a = \frac{-m_1 g - \mu_k m_2 g + m_3 g}{(m_1 + m_2 + m_3)} = \frac{-1.0 \text{ kg} - (0.30)(2.0 \text{ kg}) + 3.0 \text{ kg}}{1.0 \text{ kg} + 2.0 \text{ kg} + 3.0 \text{ kg}}(9.8 \text{ m/s}^2) = 2.3 \text{ m/s}^2$$

7.39. Model: Assume the particle model for the two blocks, and the model of kinetic and static friction.
Visualize:

Pictorial representation



Solve: (a) If the mass m is too small, the hanging 2.0 kg mass will pull it up the slope. We want to find the smallest mass that will stick as a result of friction. The smallest mass will be the one for which the force of static friction is at its maximum possible value: $f_s = (f_s)_{\max} = \mu_s n$. As long as the mass m is stuck, both blocks are at rest with $\vec{F}_{\text{net}} = 0$ N. In this situation, Newton's second law for the hanging mass M gives

$$(F_{\text{net}})_x = -T_M + Mg = 0 \text{ N} \quad \Rightarrow \quad T_M = Mg = (2.0 \text{ kg})(9.8 \text{ m/s}^2) = 19.6 \text{ N}$$

For the smaller mass *m*,

$$(F_{net})_x = T_m - f_s - mg\sin\theta = 0 \text{ N} \quad (F_{net})_y = n - mg\cos\theta \quad \Rightarrow \quad n = mg\cos\theta$$

For a massless string and frictionless pulley, forces $\vec{T}_m$ and $\vec{T}_M$ act as if they are an action/reaction pair. Thus $T_m = T_M$. Mass m is a minimum when $f_s = (f_s)_{max} = \mu_s n = \mu_s mg\cos\theta.$ Substituting these expressions into the x-equation for m gives

$$T_M - \mu_s mg\cos\theta - mg\sin\theta = 0 \text{ N}$$

$$m = \frac{T_M}{(\mu_s\cos\theta + \sin\theta)g} = \frac{19.6 \text{ N}}{[(0.80)\cos(20°) + \sin(20°)](9.8 \text{ m/s}^2)} = 1.83 \text{ kg}$$

or 1.8 kg to two significant figures.

(b) Because $\mu_k < \mu_S$ the 1.8 kg block will begin to slide up the ramp and the 2.0 kg mass will begin to fall if the block is nudged ever so slightly. In this case, the net force and the acceleration are *not* zero. Notice how, in the pictorial representation, we chose different coordinate systems for the two masses. The magnitudes of the accelerations are the same because the blocks are tied together. Thus, the acceleration constraint is $a_m = a_M \equiv a,$ where a will have a positive value. Newton's second law for block M gives

$$(F_{net})_x = -T + Mg = Ma_M = Ma$$

For block m we have

$$(F_{net})_x = T - f_k - mg\sin\theta = T - \mu_k mg\cos\theta - mg\sin\theta = ma_m = ma$$

In writing these equations, we used Newton's third law to obtain $T_m = T_M = T.$ Also, notice that the x-equation and the friction model for block m don't change, except for μ_s becoming μ_k, so we already know the expression for f_k from part (a). Notice that the tension in the string is *not* the gravitational force *Mg*. We have two equations with the two unknowns *T* and *a*:

$$Mg - T = Ma \quad T - (\mu_k\cos\theta + \sin\theta)mg = ma$$

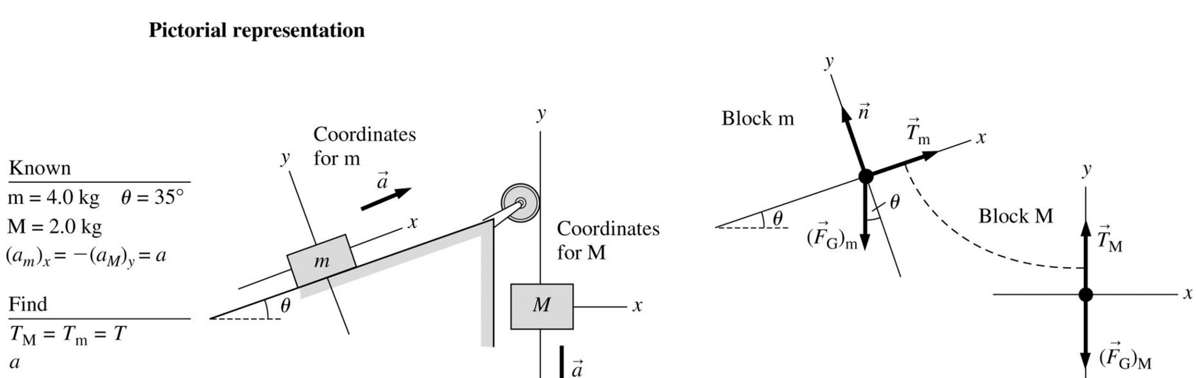
Adding the two equations to eliminate *T* gives

$$Mg - (\mu_k\cos\theta + \sin\theta)mg = Ma + ma$$

$$a = g\frac{M - (\mu_k\cos\theta + \sin\theta)m}{M + m}$$

$$= (9.8 \text{ m/s}^2)\frac{2.0 \text{ kg} - [(0.50)\cos(20°) + \sin(20°)](1.83 \text{ kg})}{2.0 \text{ kg} + 1.83 \text{ kg}} 1.3 \text{ m/s}^2$$

7.40. Model: Assume the particle model for the two blocks and use the friction model.
Visualize:



Solve: (a) The slope is frictionless, so the blocks stay in place *only* if held. Once m is released, the blocks will move one way or the other. As long as m is held, the blocks are in static equilibrium with $\vec{F}_{net} = 0$ N. In this case, Newton's second law for the hanging block M is

$$(F_{\text{net on M}})_y = T_M - Mg = 0 \text{ N} \quad \Rightarrow \quad T_M = Mg = 19.6 \text{ N}$$

Because the string is massless and the pulley is frictionless, $T_M = T_m = T = 20$ N (to two significant figures).

(b) The free-body diagram shows box m after it is released. Whether it moves up or down the slope depends on whether the acceleration a is positive or negative. The acceleration constraint is $(a_m)_x - (a_M)_y \equiv a$ Newton's second law for each system gives

$$(F_{\text{net on m}})_x = T - mg\sin\theta = m(a_m)_x = ma \quad (F_{\text{net on M}})_y = T - Mg = M(a_M)_y = -Ma$$

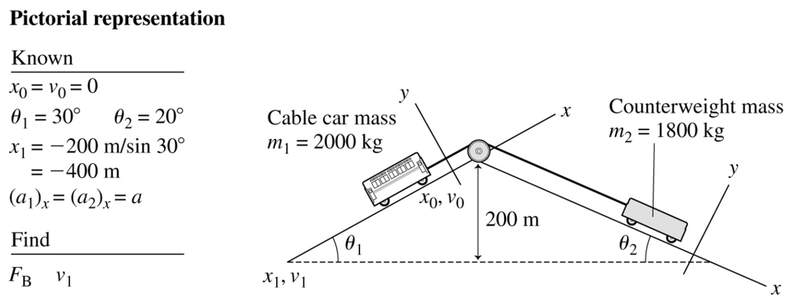
We have two equations in two unknowns. Subtract the second from the first to eliminate T:

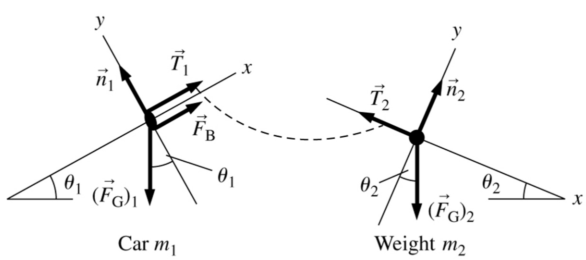
$$-mg\sin\theta + Mg = (m+M)a \quad \Rightarrow \quad a = \frac{M - m\sin\theta}{M+m}g = \frac{2.0\text{ kg} - (4.0\text{ kg})\sin(35°)}{2.0\text{ kg} + 4.0\text{ kg}} = -0.48\text{ m/s}^2$$

Since $a < 0$ m/s^2, the box accelerates *down* the slope.

(c) It is now straightforward to compute $T = Mg - Ma = 21$ N. Notice how the tension is *larger* than when the blocks were motionless.

7.42. Model: Use the particle model for the cable car and the counterweight. Assume a massless cable.
Visualize:





Solve: (a) Notice the separate coordinate systems for the cable car (object 1) and the counterweight (object 2). Forces $\vec{T}_1$ and $\vec{T}_2$ act as if they are an action/reaction pair. The braking force $\vec{F}_B$ works with the cable tension $\vec{T}_1$ to allow the cable car to descend at a constant speed. Constant speed means dynamic equilibrium, so $\vec{F}_{net} = 0$ N for both systems. Newton's second law applied to the cable car gives

$$(F_{\text{net on 1}})_x = T_1 + F_B - m_1 g\sin\theta_1 = 0\text{ N} \quad (F_{\text{net on 1}})_y = n_1 - m_1 g\cos\theta_1 = 0\text{ N}$$

Newton's second law applied to the counterweight gives

$$(F_{\text{net on 2}})_x = m_2 g\sin\theta_2 - T_2 = 0\text{ N} \quad (F_{\text{net on 2}})_y = n_2 - m_2 g\cos\theta_2 = 0\text{ N}$$

From the x-equation for the counterweight, $T_2 = m_2 g\sin\theta_2$. Because we can neglect the pulley's friction and the cable is assumed to be massless, $T_1 = T_2$. Thus the x-equation for the cable car then becomes

$$F_B = m_1 g\sin\theta_1 - T_1 = m_1 g\sin\theta_1 - m_2 g\sin\theta_2 = 3770\text{ N} = 3.8\text{ kN}$$

(b) If the brakes fail, then $F_B = 0$ N. The car will accelerate down the hill on one side while the counterweight accelerates up the hill on the other side. Both will have *negative* accelerations because of the direction of the acceleration vectors. The constraint is $a_{1x} = a_{2x} = a$, where a will have a negative value. Using $T_1 = T_2 = T$, the two x-equations are

$$(F_{\text{net on 1}})_x = T - m_1 g\sin\theta_1 = m_1 a_{1x} = m_1 a \quad (F_{\text{net on 2}})_x = m_2 g\sin\theta_2 - T = m_2 a_{2x} = m_2 a$$

Note that the y-equations aren't needed in this problem. Add the two equations to eliminate T:

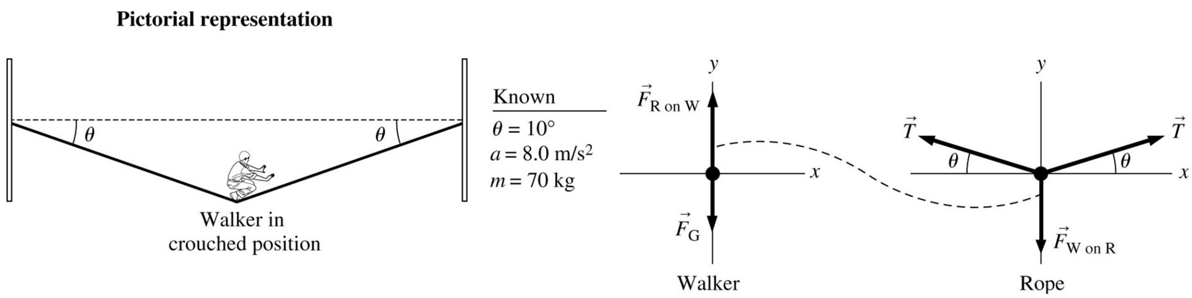
$$-m_1 g\sin\theta_1 + m_2 g\sin\theta \;= (m_1+m_2)a \quad\Rightarrow\quad a = -\frac{m_1\sin\theta_1 - m_2\sin\theta_2}{m_1+m_2}g = -0.991\text{ m/s}^2$$

Now we have a problem in kinematics. The speed at the bottom is calculated as follows:

$$v_1^2 = v_0^2 + 2a(x_1 - x_0) = 2ax_1 \quad\Rightarrow\quad v_1 = \sqrt{2ax_1} = \sqrt{2(-0.991\text{ m/s}^2)(-400\text{ m})} = 28\text{ m/s}$$

Assess: A speed of approximately 60 mph as the cable car travels a distance of 2000 m along a frictionless slope of 30° is reasonable.

7.48. Model: Use the particle model for the tightrope walker and the rope. The rope is assumed to be massless, so the tension in the rope is uniform.
Visualize:



Solve: Newton's second law applied to the tightrope walker gives

$$F_{\text{R on W}} - F_G = ma \quad\Rightarrow\quad F_{\text{R on W}} = m(a+g) = (70\text{ kg})(8.0\text{ m/s}^2 + 9.8\text{ m/s}^2) = 1.25\times10^3\text{ N}$$

Newton's second law applied to the rope gives

$$\Sigma(F_{\text{on R}})_y = T\sin\theta + T\sin\theta - F_{\text{W on R}} = 0\text{ N} \quad\Rightarrow\quad T = \frac{F_{\text{W on R}}}{2\sin(10°)} = \frac{F_{\text{R on W}}}{2\sin(10°)} = \frac{1.25\times10^3\text{ N}}{2\sin(10°)} = 3.6\times10^3\text{ N}$$

We used $F_{\text{W on R}} = F_{\text{R on W}}$ because they are an action/reaction pair.