Physics 4A
Chapter 8 HW Solutions

Chapter 8
Conceptual Questions: 2, 3, 9
Exercises & Problems: 1, 12, 18, 25, 30, 35, 46, 57

CQ 8.2. The free-body diagram **(a)** is correct. The forces acting on the car at the bottom of the hill are the downward gravitational force and an upward normal force. The car can be considered to be in circular motion about a point above the bottom center of the valley, which requires a net force toward the center of the circle. In this case, the circle center is above the car, so the normal force is greater than the gravitational force.

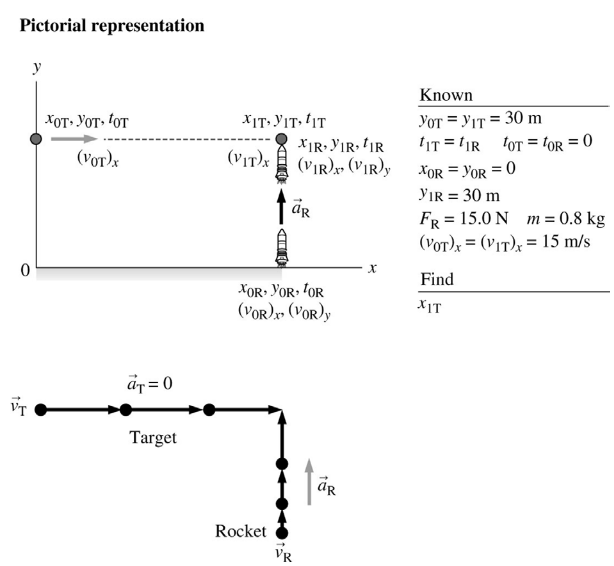
CQ 8.3. $T_c > T_a = T_d > T_b$. Use $T = \frac{mv^2}{r}$. For **(a)**, $T_a = \frac{mv^2}{r}$. For **(b)**, $T_b = \frac{mv^2}{2r} = \frac{1}{2}T_a$. For **(c)**, $T_c = \frac{(2m)v^2}{r} = 2T_a$.

For **(d)**, $T_d = \frac{(2m)v^2}{2r} = T_a$.

CQ 8.9. When the gravitational force on the ball is greater than the required centripetal force $\frac{mv^2}{r}$, the ball is no longer in circular motion. As the figure shows, at the top of the circle the net force on the ball is $(F_r)_{net} = F_G + T = \frac{mv^2}{r}$. When the string goes slack, $T = 0$, leaving $F_G = \frac{mv^2}{r}$. If the velocity is not high enough to make this equality true, the equation above becomes an inequality, $F_G > \frac{mv^2}{r}$, and the ball begins to fall downward since the net force downward is greater than the centripetal force required for circular motion.

8.1. Model: The model rocket and the target will be treated as particles. The kinematics equations in two dimensions apply.

Visualize:



Solve: For the rocket, Newton's second law along the y-direction is

$$(F_{\text{net}})_y = F_R - mg = ma_R$$

$$\Rightarrow a_R = \frac{1}{m}(F_R - mg) = \frac{1}{0.8\ \text{kg}}[15\ \text{N} - (0.8\ \text{kg})(9.8\ \text{m/s}^2)] = 8.95\ \text{m/s}^2$$

Using the kinematic equation $y_{1R} = y_{0R} + (v_{0R})_y(t_{1R} - t_{0R}) + \frac{1}{2}a_R(t_{1R} - t_{0R})^2$,

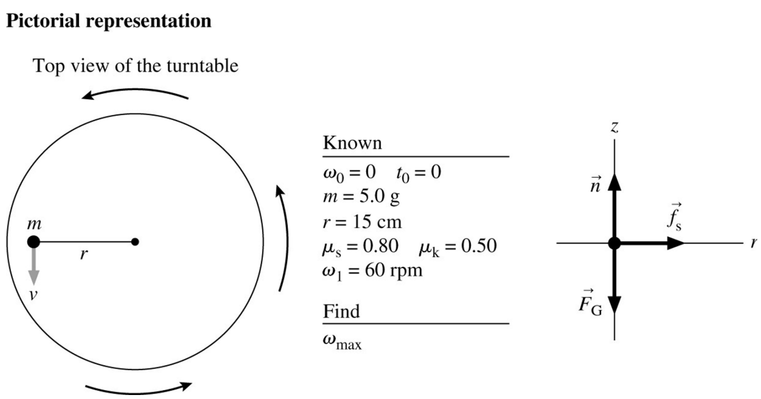
$$30\ \text{m} = 0\ \text{m} + 0\ \text{m} + \tfrac{1}{2}(8.95\ \text{m/s}^2)(t_{1R} - 0\ \text{s})^2 \Rightarrow t_{1R} = 2.589\ \text{s}$$

For the target (noting $t_{1T} = t_{1R}$),

$$x_{1T} = x_{0T} + (v_{0T})_x(t_{1T} - t_{0T}) + \tfrac{1}{2}a_T(t_{1T} - t_{0T})^2 = 0\ \text{m} + (15\ \text{m/s})(2.589\ \text{s} - 0\ \text{s}) + 0\ \text{m} = 39\ \text{m}$$

You should launch when the target is 39 m away.
Assess: The rocket is to be fired when the target is at x_{0T}. For a net acceleration of approximately $9\ \text{m/s}^2$ in the vertical direction and a time of 2.6 s to cover a vertical distance of 30 m, a horizontal distance of 39 m is reasonable.

8.12. Model: Use the particle model and static friction model for the coin, which is undergoing circular motion.
Visualize:



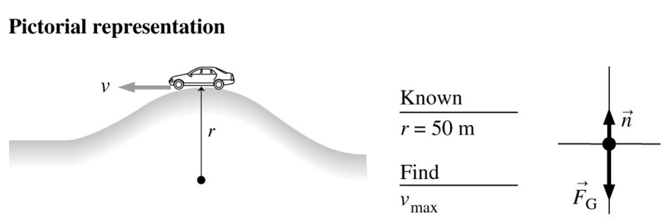
Solve: The force of static friction is $f_s = \mu_s n = \mu_s mg$. This force is equivalent to the maximum centripetal force that can be applied without sliding. That is,

$$\mu_s mg = m\frac{v_t^2}{r} = m(r\omega_{\text{max}}^2) \Rightarrow \omega_{\text{max}} = \sqrt{\frac{\mu_s g}{r}} = \sqrt{\frac{(0.80)(9.8\ \text{m/s}^2)}{0.15\ \text{m}}} = 7.23\ \text{rad/s}$$

$$= 7.23\frac{\text{rad}}{\text{s}} \times \frac{1\ \text{rev}}{2\pi\ \text{rad}} \times \frac{60\ \text{s}}{1\ \text{min}} = 69\ \text{rpm}$$

So, the coin will stay still on the turntable.
Assess: A rotational speed of approximately 1 rev per second for the coin to stay stationary seems reasonable.

8.18. Model: Use the particle model for the car which is undergoing circular motion.
Visualize:



Solve: The car is in circular motion with the center of the circle below the car. Newton's second law at the top of the hill is

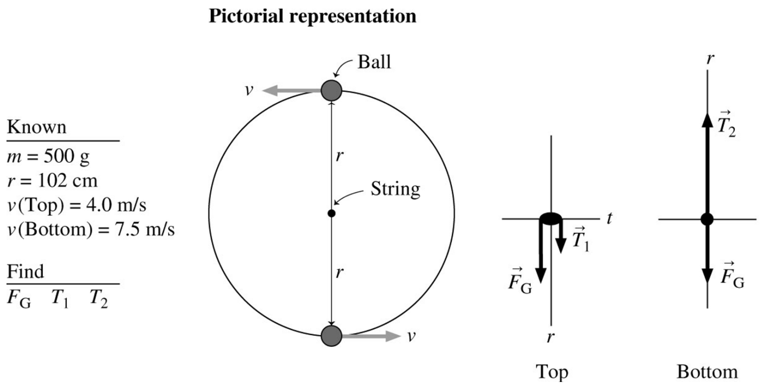
$$\sum F_r = (F_G)_r - n_r = mg - n = ma_r = \frac{mv^2}{r} \Rightarrow v^2 = r\left(g - \frac{n}{m}\right)$$

Maximum speed is reached when $n = 0$ and the car is beginning to lose contact with the road.

$$v_{max} = \sqrt{rg} = \sqrt{(50 \text{ m})(9.8 \text{ m/s}^2)} = 22 \text{ m/s}$$

Assess: A speed of 22 m/s is equivalent to 49 mph, which seems like a reasonable value.

8.25. Model: Model the ball as a particle that is moving in a vertical circle.
Visualize:



Solve: **(a)** The ball's gravitational force $F_G = mg = (0.500 \text{ kg})(9.8 \text{ m/s}^2) = 4.9 \text{ N}$.
(b) Newton's second law at the top is

$$\sum F_r = T_1 + F_G = ma_r = m\frac{v^2}{r}$$

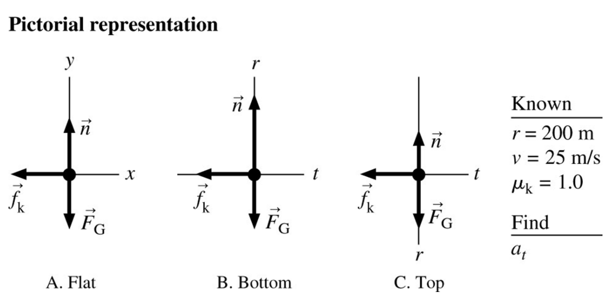
$$\Rightarrow T_1 = m\left(\frac{v^2}{r} - g\right) = (0.500 \text{ kg})\left[\frac{(4.0 \text{ m/s})^2}{1.02 \text{ m}} - 9.8 \text{ m/s}^2\right] = 2.9 \text{ N}$$

(c) Newton's second law at the bottom is

$$\sum F_r = T_2 - F_G = \frac{mv^2}{r}$$

$$\Rightarrow T_2 = m\left(g + \frac{v^2}{r}\right) = (0.500 \text{ kg})\left[9.8 \text{ m/s}^2 + \frac{(7.5 \text{ m/s})^2}{1.02 \text{ m}}\right] = 32 \text{ N}$$

8.30. Model: Use the particle model for the car and the model of kinetic friction.
Visualize:



Solve: We will apply Newton's second law to all three cars.
Car A:

$$\sum F_x = n_x + (f_k)_x + (F_G)_x = 0 \text{ N} - f_k + 0 \text{ N} = ma_x$$

$$\sum F_y = n_y + (f_k)_y + (F_G)_y = n + 0 \text{ N} - mg = 0 \text{ N}$$

The y-component equation means $n = mg$. Since $f_k = \mu_k n$, we have $f_k = \mu_k mg$. From the x-component equation,

$$a_x = \frac{-f_k}{m} = \frac{-\mu_k mg}{m} = -\mu_k g = -9.8 \text{ m/s}^2$$

Car B: Car B is in circular motion with the center of the circle above the car.

$$\sum F_r = n_r + (f_k)_r + (F_G)_r = n + 0 \text{ N} - mg = ma_r = \frac{mv^2}{r}$$

$$\sum F_t = n_t + (f_k)_t + (F_G)_t = 0 \text{ N} - f_k + 0 \text{ N} = +ma_t$$

From the r-equation

$$n = mg + \frac{mv^2}{r} \Rightarrow f_k = \mu_k n = \mu_k m\left(g + \frac{v^2}{r}\right)$$

Substituting back into the t-equation,

$$a_t = -\frac{f_k}{m} = -\frac{\mu_k m}{m}\left(g + \frac{v^2}{r}\right) = -\mu_k\left(9.8 \text{ m/s}^2 + \frac{(25 \text{ m/s})^2}{200 \text{ m}}\right) = -12.9 \text{ m/s}^2$$

Car C: Car C is in circular motion with the center of the circle below the car.

$$\sum F_r = n_r + (f_k)_r + (F_G)_r = -n + 0 \text{ N} + mg = ma_r = \frac{mv^2}{r}$$

$$\sum F_t = n_t + (f_k)_t + (F_G)_t = 0 \text{ N} - f_k + 0 \text{ N} = ma_t$$

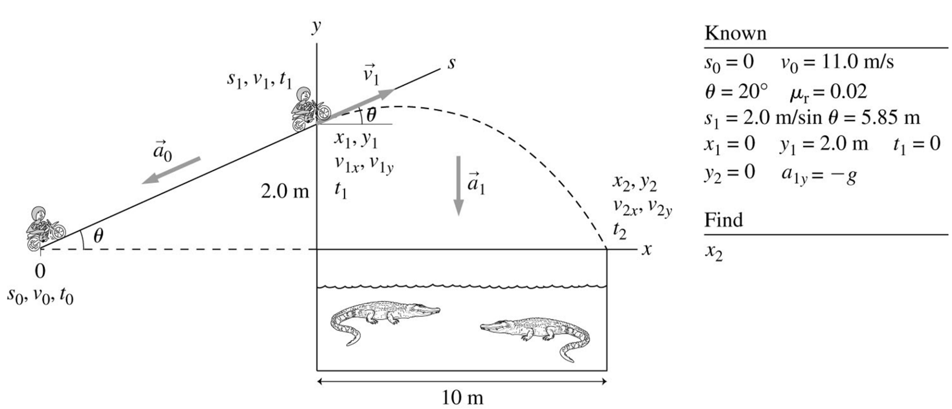
From the r-equation $n = m(g - v^2/r)$. Substituting this into the t-equation yields

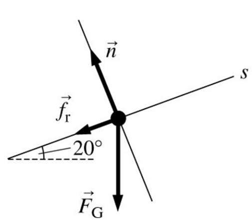
$$a_t = \frac{-f_k}{m} = \frac{-\mu_k n}{m} = -\mu_k (g - v^2/r) = -6.7 \text{ m/s}^2$$

8.35. Model: Treat the motorcycle and rider as a particle.

Visualize: This is a two-part problem. Use an s-axis parallel to the slope for the first part, regular xy-coordinates for the second. The motorcycle's final velocity at the top of the ramp is its initial velocity as it becomes airborne.

Pictorial representation





Solve: The motorcycle's acceleration on the ramp is given by Newton's second law:

$$(F_{\text{net}})_s = -f_{\text{r}} - mg\sin 20° = -\mu_{\text{r}} n - mg\sin 20° = -\mu_{\text{r}} mg\cos 20° - mg\sin 20° = ma_0$$

$$a_0 = -g(\mu_{\text{r}}\cos 20° + \sin 20°) = -(9.8 \text{ m/s}^2)((0.02)\cos 20° + \sin 20°) = -3.536 \text{ m/s}^2$$

The length of the ramp is $s_1 = (2.0 \text{ m})/\sin 20° = 5.85 \text{ m}$. We can use kinematics to find its speed at the top of the ramp:

$$v_1^2 = v_0^2 + 2a_0(s_1 - s_0) = v_0^2 + 2a_0 s_1$$

$$\Rightarrow v_1 = \sqrt{(11.0 \text{ m/s})^2 + 2(-3.536 \text{ m/s}^2)(5.85 \text{ m})} = 8.92 \text{ m/s}$$

This is the motorcycle's initial speed into the air, with velocity components $v_{1x} = v_1\cos 20° = 8.38 \text{ m/s}$ and $v_{1y} = v_1\sin 20° = 3.05 \text{ m/s}$. We can use the y-equation of projectile motion to find the time in the air:

$$y_2 = 0 \text{ m} = y_1 + v_{1y}t_2 + \tfrac{1}{2}a_{1y}t_2^2 = 2.0 \text{ m} + (3.05 \text{ m/s})t_2 - (4.90 \text{ m/s}^2)t_2^2$$

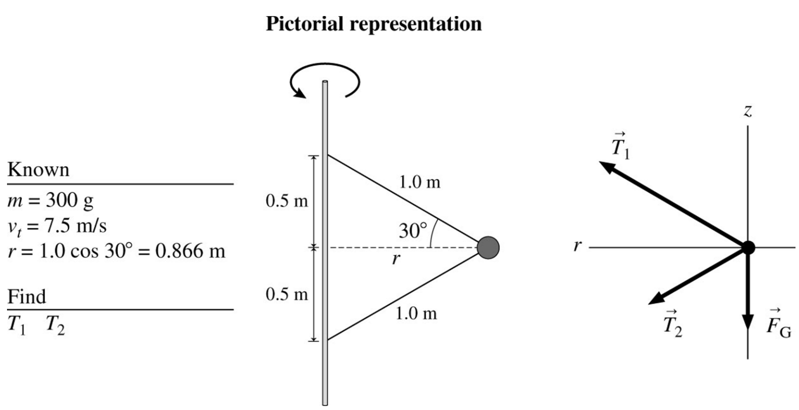
This quadratic equation has roots $t_2 = -0.399 \text{ s}$ (unphysical) and $\mathbf{t_2 = 1.021 \text{ s}}$. The x-equation of motion is thus

$$x_2 = x_1 + v_{1x}t_2 = 0 \text{ m} + (8.38 \text{ m/s})t_2 = 8.6 \text{ m}$$

$8.56 \text{ m} < 10.0 \text{ m}$, so it looks like crocodile food.

8.46. Model: Use the particle model for a sphere revolving in a horizontal circle.

Visualize:



Solve: Newton's second law in the r- and z-directions is

$$\Sigma(F)_r = T_1\cos 30° + T_2\cos 30° = \frac{mv_t^2}{r} \qquad \Sigma(F)_z = T_1\sin 30° - T_2\sin 30° - F_{\text{G}} = 0 \text{ N}$$

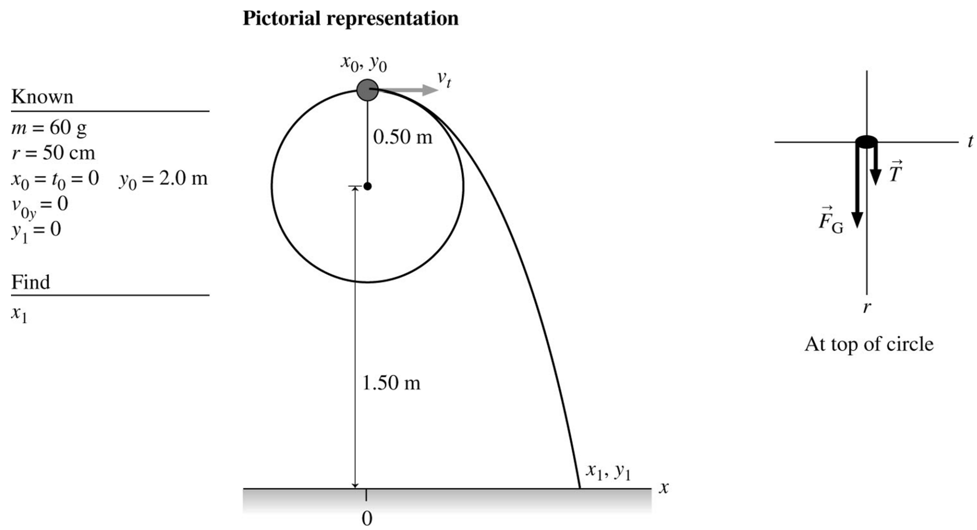
Using $r = (1.0 \text{ m})\cos 30° = 0.886 \text{ m}$, these equations become

$$T_1 + T_2 = \frac{mv_t^2}{r\cos 30°} = \frac{(0.300 \text{ kg})(7.5 \text{ m/s})^2}{(0.866 \text{ m})(0.866)} = 22.5 \text{ N}$$

$$T_1 - T_2 = \frac{mg}{\sin 30°} = \frac{(0.300 \text{ kg})(9.8 \text{ m/s}^2)}{(0.5)} = 5.88 \text{ N}$$

Solving for T_1 and T_2 yields $T_1 = 14.2 \text{ N} \approx 14 \text{ N}$ and $T_2 = 8.3 \text{ N}$.

8.57. Model: Use the particle model for a ball in motion in a vertical circle and then as a projectile.
Visualize:



Solve: For the circular motion, Newton's second law along the r-direction is

$$\sum F_r = T + F_G = \frac{mv_t^2}{r}$$

Since the string goes slack as the particle makes it over the top, $T = 0$ N. That is,

$$F_G = mg = \frac{mv_t^2}{r} \Rightarrow v_t = \sqrt{gr} = \sqrt{(9.8 \text{ m/s}^2)(0.5 \text{ m})} = 2.21 \text{ m/s}$$

The ball begins projectile motion as the string is released. The time it takes for the ball to hit the floor can be found as follows:

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2 \Rightarrow 0 \text{ m} = 2.0 \text{ m} + 0 \text{ m} + \tfrac{1}{2}(-9.8 \text{ m/s}^2)(t_1 - 0 \text{ s})^2 \Rightarrow t_1 = 0.639 \text{ s}$$

The place where the ball hits the ground is

$$x_1 = x_0 + v_{0x}(t_1 - t_0) = 0 \text{ m} + (+2.21 \text{ m/s})(0.639 \text{ s} - 0 \text{ s}) = +1.41 \text{ m}$$

The ball hits the ground 1.4 m to the right of the point beneath the center of the circle.