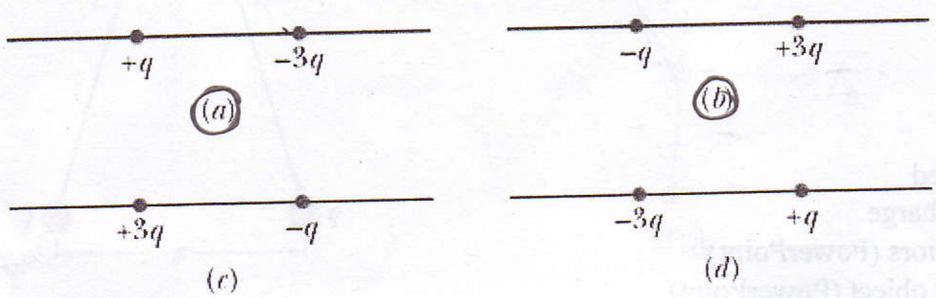
Answer Key

Other problems deal with conducting spheres and you will need to apply the shell theorems. In addition, you should know that when two identical conducting spheres touch each other, the total charge is shared equally between them.

Finally, you should know the relationship between charge and current: current is the charge that moves into or out of a region per unit time.

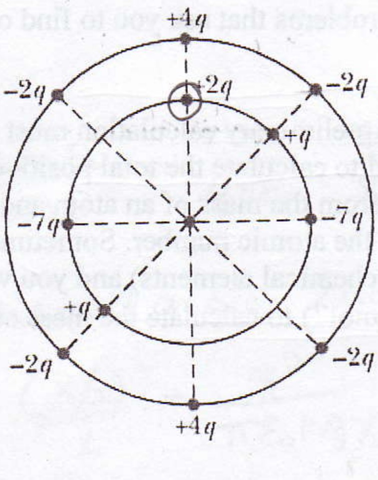
Questions and Example Problems from Chapter 21

Question 1
The figure below shows four situations in which charged particles are fixed in place on an axis. In which situation(s) is there a point to the left of the particles where an electron will be in equilibrium?



situations a and b

Question 2
In the figure to the right, a central particle of charge -q is surrounded by two circular rings of charged particles, of radii r and R, with R > r. What are the magnitude and direction of the net electrostatic force on the central particle due to the other particles?



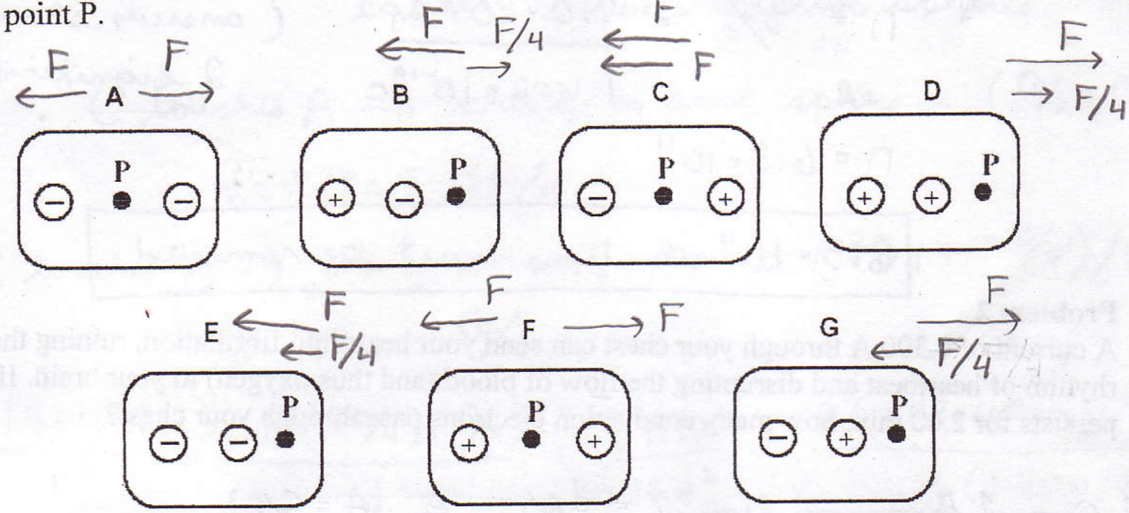
$$F = \frac{k|q_1||q_2|}{r^2}$$

$$F = \frac{2kq^2}{r^2} \text{ upward}$$

Question 3

Given below are seven arrangements of two electric charges. In each figure, a point labeled P is also identified. All of the charges are the same size, 20 C, but they can be either positive or negative. The charges and point P all lie on a straight line. The distances between adjacent items, either between two charges or between a charge and point P, are all 5 cm. There are no other charges in this region.

For this problem, we are going to place a +5 C charge at point P. Rank these arrangements from greatest to least on the basis of the strength of the electric force on the +5 C charge when it is placed at point P. That is, put first the arrangement that will exert the strongest force on the +5 C charge at point P, and put last the arrangement that will exert the weakest force on the +5 C charge when it is placed at point P.



Strongest 1 $\boxed{C}$ 2 $\boxed{DE}$ 3 $\boxed{BG}$ 4 $\boxed{AF}$ 5 ______ 6 ______ 7 ______ Weakest

Or, all of these arrangements exert the same strength force on the +5 C charge. ________

Or, all of these arrangements will exert zero force on the +5 C charge. ________

Please carefully explain your reasoning.

the force from the two electric charges is either F or F/4 where $F = \dfrac{K(20C)(5C)}{(0.050\,m)^2}$

A: 0

B: 3F/4

C: 2F

D: 5F/4

E: 5F/4

F: 0

G: 3F/4

Problem 1
How many electrons would have to be removed from a coin to leave it with a charge of $+1.0 \times 10^{-7}$ C?

$q = 1.0 \times 10^{-7} C$

$q = ne \quad n = \pm 1, \pm 2, \pm 3, \ldots \qquad e = 1.602 \times 10^{-19} C$

$n = q/e = \frac{1.0 \times 10^{-7} C}{1.602 \times 10^{-19} C}$ (answer should have 2 significant figures)

$n = 6.3 \times 10^{11}$

$\boxed{6.3 \times 10^{11} \text{ electrons must be removed}}$

Problem 2
A current of 0.300 A through your chest can send your heart into fibrillation, ruining the normal rhythm of heartbeat and disrupting the flow of blood (and thus oxygen) to your brain. If that current persists for 2.00 min, how many conduction electrons pass through your chest?

$1 C = 1 A \cdot s$ (from $i = dq/dt$ or $A = C/s$)

$dq = i\,dt \rightarrow q = it$

$q = (0.300 A)(120 s) = 36.0 C$

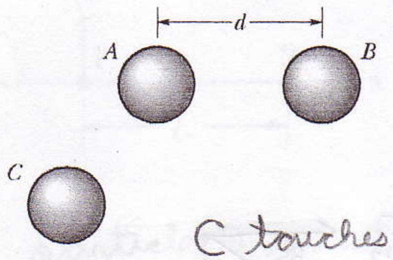
$q = ne \rightarrow n = q/e = \frac{36.0 C}{1.602 \times 10^{-19} C} = 2.25 \times 10^{20}$

$\boxed{36.0 C \rightarrow \text{charge on } 2.25 \times 10^{20} \text{ electrons}}$

($1.0 C \rightarrow$ charge on 6.24×10^{18} electrons)

Problem 3 → will help with PLC #1, question 8

The initial charges on the three identical metal spheres in the figure below are the following: sphere A, Q; sphere B, -Q/4; and sphere C, Q/2, where $Q = 2.00 \times 10^{-14}$ C. Spheres A and B are fixed in place, with a center-to-center separation of d = 1.20 m, which is much larger than the spheres. Sphere C is touched first to sphere A and then to sphere B and then it is removed. What then is the magnitude of the electrostatic force between spheres A and B?



* when two identical conducting spheres are placed in contact, the charge will divide equally between the two spheres

C touches A → charge on each sphere is $(Q/2 + Q)/2$

$$q_C = q_A = 3Q/4$$

then C touches B → charge on each sphere is $(3Q/4 + -Q/4)/2$

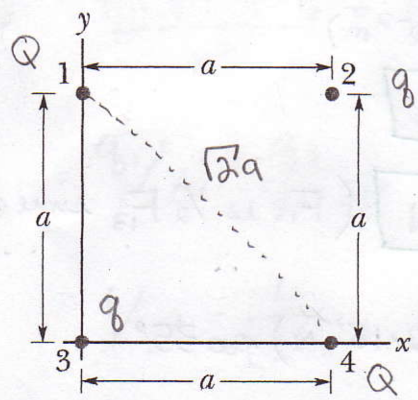
$$q_C = q_B = Q/4$$

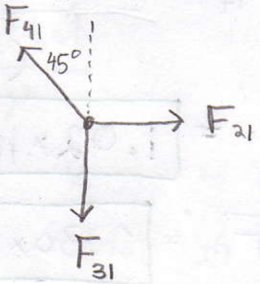
$$F_{AB} = \frac{K|q_A||q_B|}{r^2} = \frac{K(3Q/4)(Q/4)}{d^2} = \frac{3KQ^2}{16d^2} = \frac{3(8.99\times10^9\,\mathrm{Nm^2/C^2})(2.00\times10^{-14}\,\mathrm{C})^2}{16(1.20\,\mathrm{m})^2}$$

$$= \boxed{4.68\times10^{-19}\,\mathrm{N}}$$

Problem 4

In the figure below, four particles form a square. The charges are $q_1 = q_4 = Q$ and $q_2 = q_3 = q$. What is Q/q if the net electrostatic force on particles 1 and 4 is zero?





* q + Q must have opposite signs

$$F = \frac{K|q_1||q_2|}{r^2}$$

$$F_{21} = \frac{K|q||Q|}{a^2} \quad F_{31} = \frac{K|q||Q|}{a^2} \quad F_{41} = \frac{KQ^2}{(\sqrt{2}a)^2}$$

$$\Sigma F_x = F_{21} - F_{41}\sin 45° = \frac{K|q||Q|}{a^2} - \left(\frac{KQ^2}{2a^2}\right)\underbrace{\left(\frac{\sqrt{2}}{2}\right)}_{=\sin 45} = 0$$

$$q - \frac{Q}{2}\left(\frac{\sqrt{2}}{2}\right) = 0 \longrightarrow q = \frac{\sqrt{2}Q}{4} \longrightarrow Q/q = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

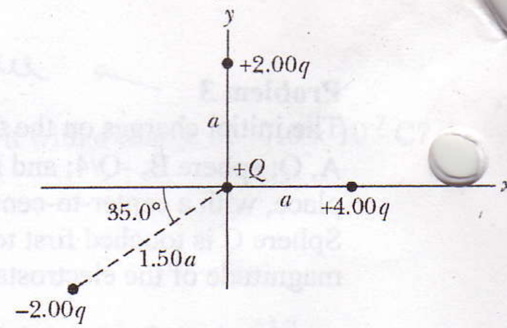
$$\boxed{Q/q = -2\sqrt{2} = -2.83}$$

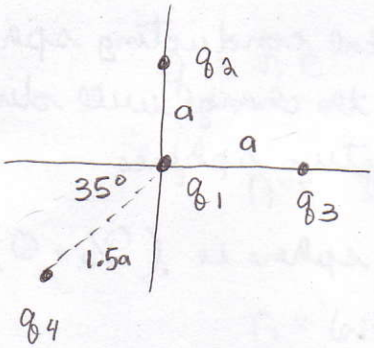
$$\Sigma F_y = F_{41}\cos 45° - F_{31} = \frac{KQ^2}{2a^2}\underbrace{\left(\frac{\sqrt{2}}{2}\right)}_{=\cos 45} - \frac{K|q||Q|}{a^2} = 0$$ → gives same answer as above

$$\boxed{Q/q = -2.83}$$

Problem 5

In the figure to the right, if $Q = +3.20 \times 10^{-19}$ C, $q = 1.60 \times 10^{-19}$ C, and $a = 2.00$ cm, what is the electrostatic force on the particle at the origin due to the other charged particles?





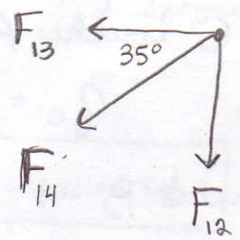
$q_1 = 3.20\times10^{-19}$ C

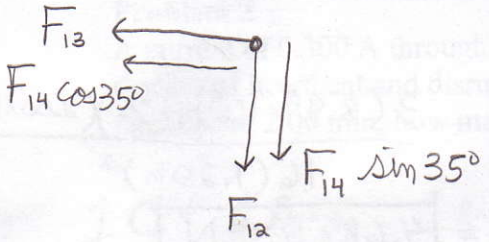
$q_2 = 3.20\times10^{-19}$ C

$q_3 = 6.40\times10^{-19}$ C

$q_4 = -3.20\times10^{-19}$ C

$a = 2.0\times10^{-2}$ m





$$F_{13} = \frac{(8.99\times10^9\ \mathrm{Nm^2/C^2})(3.20\times10^{-19}\mathrm{C})(6.40\times10^{-19}\mathrm{C})}{(2.0\times10^{-2}\mathrm{m})^2} = \boxed{4.60\times10^{-24}\ \mathrm{N}}$$

$$F_{14} = \frac{(8.99\times10^9\ \mathrm{Nm^2/C^2})(3.20\times10^{-19}\mathrm{C})(3.20\times10^{-19}\mathrm{C})}{(3.0\times10^{-2}\mathrm{m})^2} = \boxed{1.02\times10^{-24}\ \mathrm{N}}$$

$F_{12} = \boxed{2.30\times10^{-24}\ \mathrm{N}}$ (F_{12} is ½ F_{13} since $q_2 = q_3/2$)

$$\sum \vec{F}_{1x} = -F_{13} - F_{14}\cos 35° = -4.60\times10^{-24}\mathrm{N} - (1.02\times10^{-24}\mathrm{N})\cos 35°$$
$$= -5.44\times10^{-24}\ \mathrm{N}$$

$$\sum \vec{F}_{1y} = -F_{12} - F_{14}\sin 35° = -2.30\times10^{-24}\mathrm{N} - (1.02\times10^{-24}\mathrm{N})\sin 35°$$
$$= -2.89\times10^{-24}\ \mathrm{N}$$

$$\vec{F}_1 = (-5.44\times10^{-24}\mathrm{N})\hat{i} - (2.89\times10^{-24}\mathrm{N})\hat{j}$$

$$F = \sqrt{(-5.44\times10^{-24}\mathrm{N})^2 + (-2.89\times10^{-24}\mathrm{N})^2} = \boxed{6.16\times10^{-24}\ \mathrm{N}}$$

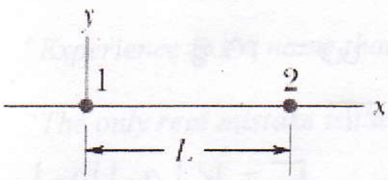
$$\theta = \tan^{-1}\left(\frac{-2.89\times10^{-24}\mathrm{N}}{-5.44\times10^{-24}\mathrm{N}}\right) = 28.0° \rightarrow$$ wrong quadrant so add 180° $\boxed{\theta = 208°}$

$q_1 = +q$ $L = 9.00\,cm$

$q_2 = +4.00\,q$

Problem 6

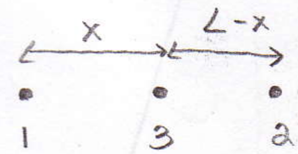
In the figure below, particle 1 of charge +q and particle 2 of charge +4.00 q are held at separation L = 9.00 cm on an x axis. If particle 3 of charge q_3 is to be located such that the three particles remain in place when released, what must be (a) the ratio q_3/q and the (b) x and (c) y coordinates of particle 3?



note: in order for all 3 particles to remain in place, we must have $\sum \vec{F} = 0$ for each particle

$\Rightarrow$ particle 3 must lie between the two particles or else the forces from particles 1 and 2 would lie in the same direction and then $\sum \vec{F}_3 \neq 0$ and particle 3 would move

$\Rightarrow$ let q_3 be a distance x from q_1:



$\sum \vec{F}_3 = \vec{F}_{13} + \vec{F}_{23} = 0 \rightarrow$ forces must have same magnitude but opposite directions

$$\frac{K|q_1||q_3|}{x^2} = \frac{K|q_2||q_3|}{(L-x)^2}$$

$$q_1/x^2 = \frac{q_2}{(L-x)^2} \longrightarrow \frac{q_1}{x^2} = \frac{4q_1}{(L-x)^2} \quad (\text{since } q_2 = 4q_1)$$

$$1/x^2 = \frac{4}{(L-x)^2} \longrightarrow \frac{1}{x} = \frac{2}{L-x} \longrightarrow L-x = 2x$$

$$x = L/3 = 3.00\,cm$$

(b) $\boxed{x = 3.00\,cm}$ (c) $\boxed{y = 0}$

$\Rightarrow$ to get q_3/q look at the net force on q_1 with $x = L/3$ and set $\sum \vec{F}_1 = 0$

$$\sum \vec{F}_1 = \frac{K|q_1||q_3|}{x^2} - \frac{K|q_1||q_2|}{L^2} = 0 \nearrow \frac{q\,q_3}{x^2} = \frac{q(4q)}{L^2}$$

$$q_3/q = \frac{4x^2}{L^2} = \frac{4(L/3)^2}{L^2} = 4/9 \longrightarrow \boxed{q_3/q = -4/9}$$

* q_3/q must be negative because q_3 must have opposite sign as

Problem 7

In the figure below, two tiny conducting balls of identical mass m and identical charge q hang from nonconducting threads of length L. Assume that θ is so small that tan θ can be replaced by its approximate equal, sin θ. (a) Show that

$$x = \left(\frac{q^2 L}{2\pi\varepsilon_0 mg}\right)^{1/3}$$

gives the equilibrium separation x of the balls.

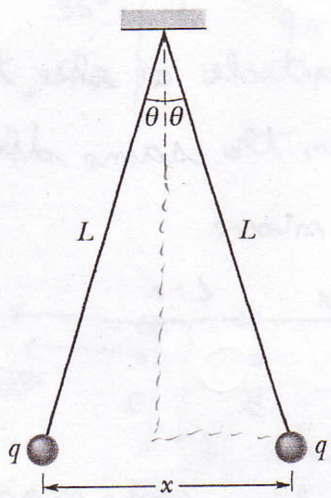
three forces act on each ball

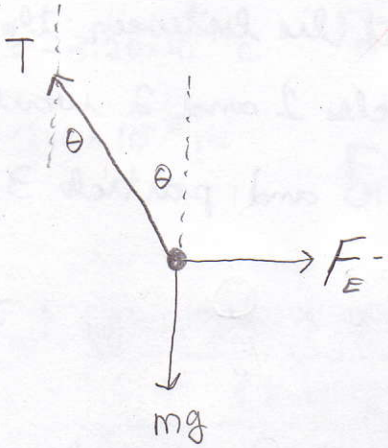
gravity $w = mg$

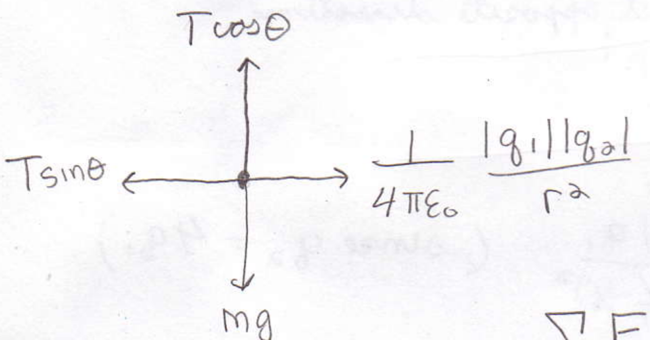
tension T

electric force $F_E = \frac{K|q_1||q_2|}{r^2}$

$F_E = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}$







$\sum F_y = 0 \longrightarrow T\cos\theta - mg = 0$

$T = mg/\cos\theta$

$\sum F_x = 0 \longrightarrow T\sin\theta = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}$ $\quad |q_1| = q,\ |q_2| = q \quad r = x/2$

$\left(\frac{mg}{\cos\theta}\right)\sin\theta = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(x/2)^2}$

$\frac{\sin\theta}{\cos\theta} = \frac{1}{4\pi\varepsilon_0}\frac{4q^2}{x^2}\frac{1}{mg} \longrightarrow \tan\theta = \frac{q^2}{\pi\varepsilon_0 mg x^2}$

for small angles $\tan\theta \approx \sin\theta \rightarrow$ from SOHCAHTOA $\sin\theta = \frac{(x/2)}{L}$

$\frac{(x/2)}{L} = \frac{q^2}{\pi\varepsilon_0 mg x^2} \longrightarrow x^3 = \frac{q^2 L}{2\pi\varepsilon_0 mg}$

$$\boxed{x = \left(\frac{qL^2}{2\pi\varepsilon_0 mg}\right)^{1/3}}$$