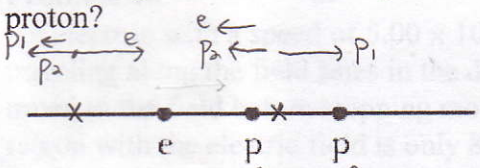
You should know how to calculate the electric field of a continuous distribution of charge. Use the linear or area charge density to write an expression for the charge in an infinitesimal region, then carry out an integration over the entire charge distribution. Don't forget that the field is a vector and you must evaluate an integral for each component or, in special cases, use symmetry to show a field component vanishes.

Some problems deal with the trajectories of charges in electric fields. If the field is uniform, the acceleration of the charge is constant and you may use the kinematic equations of constant motion. The problems are quite similar to projectile motion problems; however, the acceleration is now due to the force of electric field rather than to gravity.

You should know how to calculate the torque of a uniform electric field on an electric dipole and the potential energy of a dipole in an electric field. In some cases, you are asked for the work that is done by the field on a dipole when the dipole turns. This is the negative of the change in the potential energy.

Questions and Example Problems from Chapter 22

Question 1
The figure below shows two protons and an electron that are evenly spaced on an axis. Where on the axis (other than at an infinite distance) is there a point at which their net electric field is zero: to the left of the particles, to their right, between the two protons, or between the electron and the nearer proton?

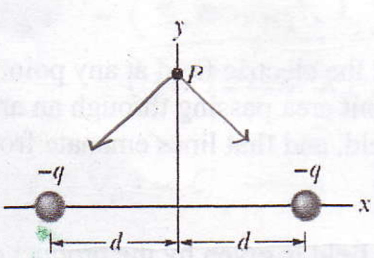


2 point:

1) to the left of the particle

2) between the protons

Question 2
In the figure below, two particles of charge -q are arranged symmetrically about the y axis; each produces an electric field at point P on that axis. (a) Are the magnitudes of the fields at P equal? (b) Is each electric field directed toward or away from the charge producing it? (c) Is the magnitude of the net electric field at P equal to the sum of the magnitudes E of the two field vectors (is it equal to 2E)? (d) Do the x components of those two field vectors add or cancel? (e) Do their y components add or cancel? (f) Is the direction of the net field at P that of the canceling components or the adding components? (g) What is the direction of the net field?



(a) yes

(b) towards

(c) no

(d) cancel

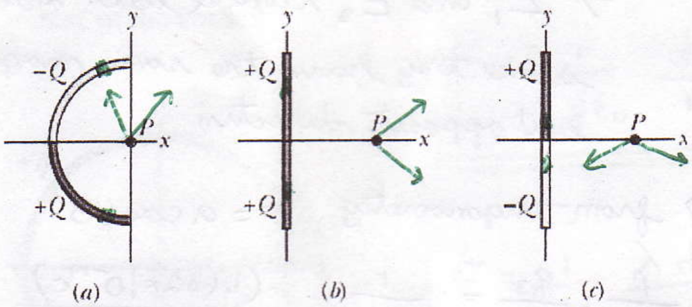
(e) add

(f) adding components

(g) $-\hat{j}$ direction

Question 3

The figure below shows three nonconducting rods, one circular and two straight. Each has a uniform charge of magnitude Q along its top half and another along its bottom half. For each rod, what is the direction of the net electric field at point P?



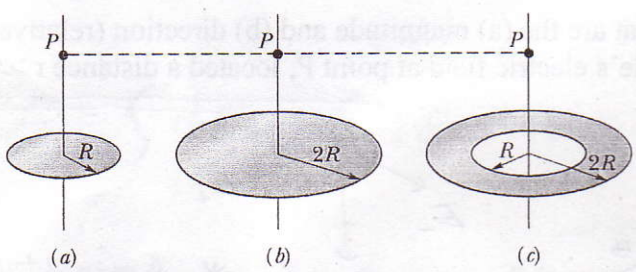
(a) $\boxed{+\hat{j} \text{ direction}}$

(b) $\boxed{+\hat{i} \text{ direction}}$

(c) $\boxed{-\hat{j} \text{ direction}}$

Question 4

The figure below shows two disks and a flat ring, each with the same uniform charge Q. Rank the objects according to the magnitude of the electric field they create at points P, greatest first.



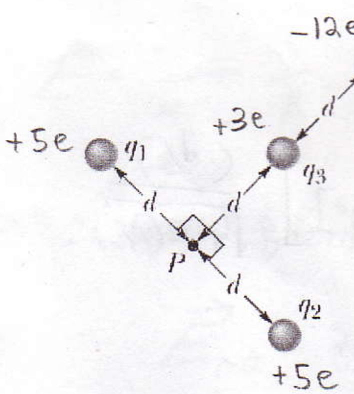
for a ring of charge:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q z}{(z^2+R^2)^{3/2}}$$

$\boxed{a, b, c}$

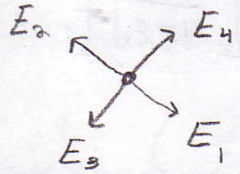
Problem 1

In the figure below, the four particles are fixed in place and have charges $q_1 = q_2 = +5e$, $q_3 = +3e$, and $q_4 = -12e$. Distance $d = 5.0\ \mu m$. What is the magnitude of the net electric field at point P due to the particles?



⇒ for a point charge: $E = \frac{1}{4\pi\varepsilon_0} \frac{|q|}{r^2}$

$\vec{E}$ points away from a + charge & towards a − charge



$\vec{E}$ from q_1 and q_2 cancel each other out

Y X

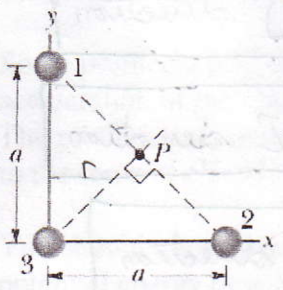
$$\Sigma \vec{E} = \vec{E}_4 - \vec{E}_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{|-12e|}{(2d)^2} - \frac{|+3e|}{d^2}\right)$$

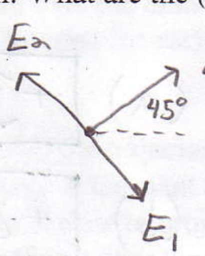
$$= \frac{1}{4\pi\varepsilon_0}\left(3e/d^2 - 3e/d^2\right) = 0$$

$$\boxed{\vec{E}_{net} = 0}$$

Problem 2

In the figure below, the three particles are fixed in place and have charges $q_1 = q_2 = +e$ and $q_3 = +2e$. Distance $a = 6.00\ \mu m$. What are the (a) magnitude and (b) direction of the net electric field at point P due to the particles?





$\Rightarrow \vec{E}_1$ and $\vec{E}_2$ cancel each other out since they have the same magnitude but opposite direction

$\Rightarrow$ from trigonometry, $r = a\cos 45°$

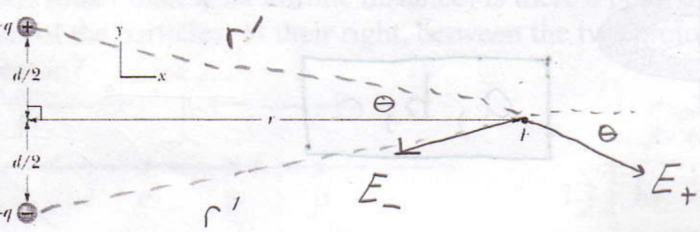
$$\sum \vec{E} = \vec{E}_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{2(1.602\times10^{-19}\,C)}{(6.00\times10^{-6}\,m\cdot\cos45°)^2}$$

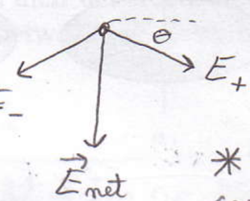
$$\boxed{E = 1.60\times10^{2}\ N/C}$$

(b) from diagram, we can see that $\vec{E}$ is at $\boxed{\theta = 45°}$

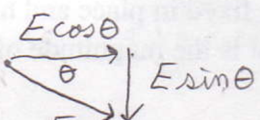
Problem 3

The figure below shows an electric dipole. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the dipole's electric field at point P, located a distance $r \gg d$?





∗ horizontal components cancel, vertical components add



$$r' = \sqrt{r^2 + (d/2)^2}$$

$$\sin\theta = \frac{(d/2)}{\sqrt{r^2+(d/2)^2}} \qquad E_{net} = 2E\sin\theta$$

$$= 2\left[\frac{1}{4\pi\varepsilon_0}\frac{q}{[r^2+(d/2)^2]}\right]\frac{(d/2)}{\sqrt{r^2+(d/2)^2}}$$

$$E_{net} = \frac{1}{4\pi\varepsilon_0}\frac{qd}{[r^2+(d/2)^2]^{3/2}}$$

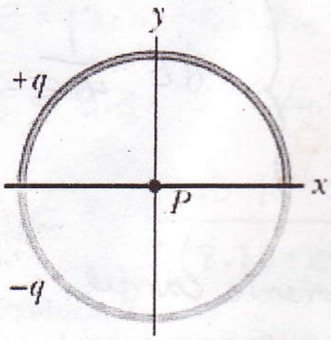
for $r \gg d$ $\quad [r^2+(d/2)^2]^{3/2} \approx [r^2]^{3/2} = r^3$

(a) $\boxed{|\vec{E}_{net}| = \frac{1}{4\pi\varepsilon_0}\frac{qd}{r^3}}$

(b) $\boxed{-y \text{ direction}}$

$$\boxed{\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{qd}{r^3}\,\hat{j}}$$

Problem 4
In the figure below, two curved plastic rods, one of charge +q and the other of charge -q, form a circle of radius R = 8.50 cm in an xy plane. The x axis passes through both of the connecting points, and the charge is distributed uniformly on both rods. If q = 15.0 pC, what are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the electric field produced at P, the center of the circle?

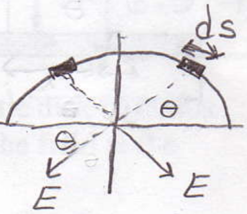


$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} \qquad \lambda = q/L = q/s$$

$$q = \lambda s \rightarrow dq = \lambda\, ds$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}$$

first look at upper + rod:



* x components cancel, y component add

$$E = \int dE_y = \int dE \sin\theta = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda \sin\theta\, ds}{r^2}$$ 2 variables: θ + s

⇒ for θ in radians, $s = R\theta \rightarrow ds = R\,d\theta$

$$E = \frac{1}{4\pi\varepsilon_0}\int_0^{\pi} \frac{\lambda \sin\theta\, R\, d\theta}{R^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{R}\int_0^{\pi} \sin\theta\, d\theta = (\quad)\left[-\cos\theta\right]_0^{\pi}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{R} - \left[-1-(1)\right] = \frac{1}{4\pi\varepsilon_0}\left(\frac{2\lambda}{R}\right) \text{ in the } -y \text{ direction } (-\hat{j})$$

* $\vec{E}$ field from curved rod of charge $-q$ is same as upper rod

$$\vec{E}_{net} = 2\left[\frac{1}{4\pi\varepsilon_0}\left(\frac{2\lambda}{R}\right)\right](-\hat{j}) = \frac{1}{4\pi\varepsilon_0}\frac{4\lambda}{R}(-\hat{j})$$

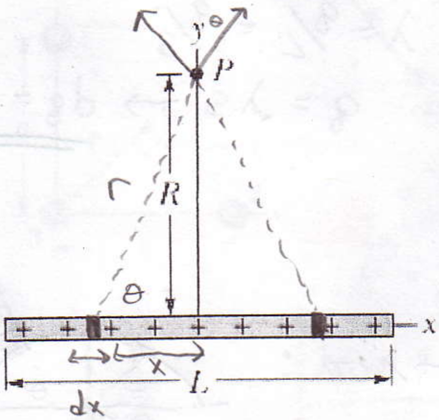
$R = 0.0850\text{m}$

$$\lambda = q/L = \frac{q}{\pi R} = \frac{15.0\times10^{-12}\text{C}}{\pi(0.0850\text{m})} = 5.62\times10^{-11}\text{ C/m}$$

$$\vec{E}_{net} = \frac{(8.99\times10^{9}\text{Nm}^2/\text{C}^2)\,4\,(5.62\times10^{-11}\text{C/m})}{0.0850\text{m}}(-\hat{j}) = \boxed{(23.8\text{ N/C})(-\hat{j})}$$

Problem 5

In the figure below, positive charge q = 7.81 pC is spread uniformly along a thin nonconducting rod of length L = 14.5 cm. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the electric field at point P, at distance R = 6.00 cm from the rod along its perpendicular bisector?



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

$$\lambda = q/L \rightarrow q = \lambda L$$

$$dq = \lambda\, dx$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{r^2}$$

* horizontal components cancel, vertical components add

$$E = \int dE_y = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda\, dx}{r^2}\sin\theta$$ 3 variables: x, r, θ

$$r = \sqrt{x^2+R^2}$$

$$\sin\theta = \frac{y}{r} = \frac{R}{\sqrt{x^2+R^2}}$$

$$E = \frac{1}{4\pi\varepsilon_0}\lambda\int\frac{dx}{(x^2+R^2)}\frac{R}{\sqrt{x^2+R^2}}$$

$$E = \frac{\lambda R}{4\pi\varepsilon_0}\int_{-L/2}^{L/2}\frac{dx}{(x^2+R^2)^{3/2}} = (2)\frac{\lambda R}{4\pi\varepsilon_0}\int_0^{L/2}\frac{dx}{(x^2+R^2)^{3/2}}$$ by symmetry

$$\boxed{\int\frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}}}$$

from trigonometric substitution $x = a\tan\theta$

$$E = \frac{2\lambda R}{4\pi\varepsilon_0}\left[\frac{x}{R^2(x^2+R^2)^{1/2}}\right]_0^{L/2}$$

$$= \frac{\lambda R}{4\pi\varepsilon_0}\frac{L}{R^2(L^2/4+R^2)^{1/2}}$$ $q = \lambda L$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R\sqrt{L^2/4+R^2}}}$$

$q = 7.81\times10^{-12}$ C

$R = 0.060$ m $\quad L = 0.145$ m $\rightarrow E = 12.4$ N/C upward or $\boxed{(12.4\,\text{N/C})\,\hat{j}}$

Problem 6

A disk of radius 2.5 cm has a surface charge density of 5.3 $\mu C/m^2$ on its upper face. What is the magnitude of the electric field produced by the disk at a point on its central axis at distance z = 12 cm from the disk?

$R = 2.5cm = 0.025m$

$z = 12cm = 0.12m$

$\sigma = 5.3\times10^{-6}\ C/m^2$

⇒ the electric field a distance z from a disk along its central axis is given by:

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$

$$E = \frac{(5.3\times10^{-6}C/m^2)}{2(8.85\times10^{-12}C^2/Nm^2)}\left[1 - \frac{0.12m}{\sqrt{(0.12m)^2+(0.025m)^2}}\right] = \boxed{6.3\times10^3\ N/C}$$

Problem 7

At what distance along the central perpendicular axis of a uniformly charged plastic disk of radius 0.600 m is the magnitude of the electric field equal to one-half the magnitude of the field at the center of the surface of the disk.

$\vec{E}$ for a charged disk $\rightarrow E = \sigma/2\varepsilon_0\left(1 - z/\sqrt{z^2+R^2}\right)$

⇒ at center of surface of disk, z = 0 so $E = \sigma/2\varepsilon_0$

we want z so that $E = \frac{1}{2}(\sigma/2\varepsilon_0) \rightarrow \sigma/2\varepsilon_0\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right) = \frac{1}{2}(\sigma/2\varepsilon_0)$

$$1 - \frac{z}{\sqrt{z^2+R^2}} = \frac{1}{2} \rightarrow \frac{z}{\sqrt{z^2+R^2}} = \frac{1}{2} \rightarrow 2z = \sqrt{z^2+R^2} \rightarrow 4z^2 = z^2 + R^2$$

$$z^2 = R^2/3 \rightarrow z = R/\sqrt{3} = \frac{0.600m}{\sqrt{3}} = \boxed{0.346m}$$

Problem 8

A charged cloud system produces an electric field in the air near Earth's surface. A particle of charge -2.0×10^{-9} C is acted on by a downward electrostatic force of 3.0×10^{-6} N when placed in this field. (a) What is the magnitude of the electric field? What are the (b) magnitude and (c) direction of the electrostatic force F_{el} exerted on a proton placed in this field? (d) What is the magnitude of the gravitational force F_g on the proton? (e) What is the ratio of F_{el}/F_g in this case?

(a) $\vec{F}_e = q\vec{E} \rightarrow \vec{E} = \vec{F}/q = \frac{(-3.0\times10^{-6}N)\hat{j}}{-2.0\times10^{-9}C} \rightarrow \boxed{\vec{E} = (1.50\times10^3\ N/C)\hat{j}}$

(b) $\vec{F}_e = q\vec{E} = (1.602\times10^{-19}c)(1.50\times10^3 N/C)\hat{j} = \boxed{(2.4\times10^{-16}N)\hat{j}}$

(c) $\boxed{\text{upwards}}$ $(\hat{j})$

(d) $F_g = mg = (1.67\times10^{-27}Kg)(9.80m/s^2) = \boxed{1.64\times10^{-26}N\ \text{downward}}$

(e) $F_e/F_g = \frac{2.4\times10^{-16}N}{1.64\times10^{-26}N} = \boxed{1.5\times10^{10}}$

Problem 9

A 10.0 g block with a charge of +8.00 × 10^{-5} C is placed in electric field $\vec{E} = (3000\hat{i} - 600\hat{j})\, N/C$. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the force on the block? If the block is released from rest at the origin at t = 0, what are its (c) x and (d) y coordinates at t = 3.00 s?

$q_0 = 8.00\times10^{-5}\,C$

$\vec{E} = (3.00\times10^{3}\,N/C)\hat{i} - (6.00\times10^{2}\,N/C)\hat{j}$

(a) & (b) $\vec{F} = q_0\vec{E} = (8.00\times10^{-5}\,C)\left[(3.00\times10^{3}\,N/C)\hat{i} - (6.00\times10^{2}\,N/C)\hat{j}\right]$

$= (0.240N)\hat{i} - (0.048N)\hat{j}$ $\qquad \theta = \tan^{-1}\left(\frac{-0.048N}{0.240N}\right)$

$F = \sqrt{(0.240N)^2 + (-0.048N)^2} = \boxed{0.245N}$ $\qquad \boxed{\theta = -11.3°}$

(c) $x_0 = 0m$ $\quad a_x = \frac{F_x}{m} = \frac{0.240N}{0.010Kg} = \underline{24 m/s^2}$ $\quad x = \overset{0}{x_0} + \overset{0}{v_{0x}t} + \frac{1}{2}a_x t^2$

$v_{0x} = 0 m/s$ $\quad t = 3.00s$ $\quad x = \frac{1}{2}(24 m/s^2)(3.00s)^2$

$\boxed{x = 108\,m}$

(d) $y_0 = 0m$ $\quad a_y = F_y/m = \frac{-0.048N}{0.010Kg} = \underline{-4.8 m/s^2}$

$v_{0y} = 0 m/s$ $\quad t = 3.00s$ $\quad y = \frac{1}{2}a_y t^2 = \boxed{-21.6m}$

Problem 10

An electron with a speed of 5.00 × 10^{8} cm/s enters an electric field of magnitude 1.00 × 10^{3} N/C, traveling along the field lines in the direction that retards its motion. (a) How far will the electron travel in the field before stopping momentarily and (b) how much time will have elapsed? (c) If the region with the electric field is only 8.00 mm long (too short for the electron to stop within it), what fraction of the electron's initial kinetic energy will be lost in that region?

$v_0 = 5.00\times10^{6} m/s$ $\quad \overset{0}{v^2} = v_0^2 + 2a(x - x_0) \longrightarrow x - x_0 = -v_0^2/2a$

$E = 1.00\times10^{3} N/C$

$|a| = F/m = qE/m = \frac{(1.602\times10^{-19}c)(1.00\times10^{3}N/c)}{(9.11\times10^{-31}Kg)}$

$|a| = 1.76\times10^{14} m/s^2 \longrightarrow a = -1.76\times10^{14} m/s^2$

$x - x_0 = -\frac{(5.00\times10^{6} m/s)^2}{(-1.76\times10^{14} m/s^2)} = \boxed{7.12\times10^{-2}\,m}$

(b) $\overset{0}{v} = v_0 + at \longrightarrow t = -v_0/a = \frac{-5.00\times10^{6} m/s}{-1.76\times10^{14} m/s^2} \longrightarrow \boxed{t = 2.85\times10^{-8} s}$

(c) $\frac{\Delta KE}{KE_0} = ?$ $\quad \frac{\Delta(\frac{1}{2}mv^2)}{\frac{1}{2}mv_0^2} = \frac{\frac{1}{2}m(\Delta v)^2}{\frac{1}{2}mv_0^2} = \frac{\Delta v^2}{v_0^2}$ $\quad v^2 - v_0^2 = 2a(\Delta x)$

$\Delta v^2 = 2a(\Delta x)$

$= \frac{2a(\Delta x)}{v_0^2} = \frac{2(-1.76\times10^{14} m/s^2)(8.00\times10^{-3}m)}{(5.00\times10^{6} m/s^2)} = \boxed{-11.3\%}$