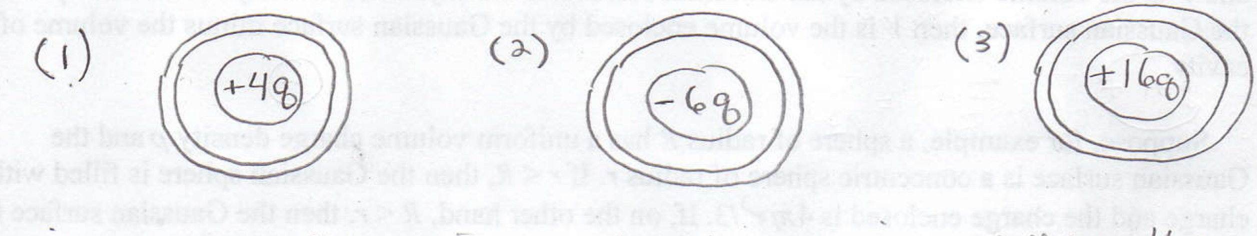
Questions and Example Problems from Chapter 23

Question 1

A small charged ball lies within the hollow of a metallic spherical ball of radius R. For three situations, the net charges on the ball and shell, respectively, are (1) +4q, 0; (2) –6q, +10 q; (3) +16 q, -12q. Rank the situations according to the charge on (a) the inner surface of the shell and (b) the outer surface, most positive first.



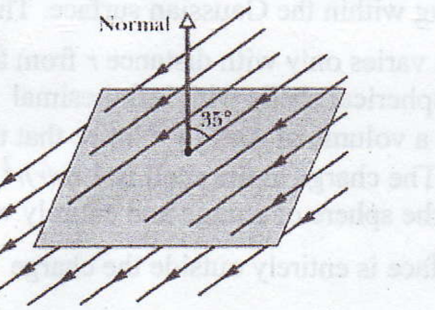
inner shell: $-4q$ | inner shell: $-6q$ | inner shell: $-16q$

outer shell: $+4q$ | outer shell: $+16q$ | outer shell: $+4q$

(a) 1, 2, 3 (b) 1 & 3 tie, 2

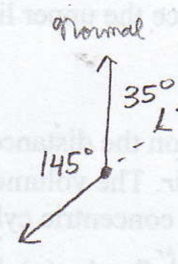
Problem 1

The square surface shown in the figure below measures 3.2 mm on each side. It is immersed in a uniform electric field with magnitude E = 1800 N/C. The field lines make an angle of 35° with a normal to the surface, as shown. Take that normal to be directed "outward," as though the surface were one face of a box. Calculate the electric flux through the surface.



⇒ for a flat surface of area A in a uniform electric field of magnitude E, the electric flux is given by:

$$\Phi = \vec{E}\cdot\vec{A} = EA\cos\theta$$



$E = 1800\ N/C$

$A = (3.2\times10^{-3}\ m)^2$

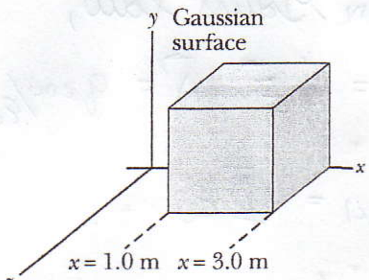
* note: the angle between $\vec{E}$ & $\vec{A}$ is $\theta = 145°$ not 35°

$$\Phi = EA\cos\theta$$
$$= (1800\ N/C)(3.2\times10^{-3}\ m)^2\cos 145°$$

$$\boxed{\Phi = -1.5\times10^{2}\ Nm^2/C^2}$$

Problem 2

An electric field given by $E = 4.0\hat{i} - 3.0(y^2 + 2.0)\hat{j}$ pierces a Gaussian cube of length 2.0 m and positioned as shown in the figure below. (The magnitude of E is in N/C and position x is in m.) What is the electric flux through the (a) top face, (b) bottom face, (c) left face, and (d) back face? (e) What is the net electric flux through the cube?



$\vec{E} = 4.0\hat{i} - 3.0(y^2+2.0)\hat{j}$

note: should be written as

$\vec{E} = (4.0\,N/C)\hat{i} - (3.0\,N/C)\left[y^2(m^{-2}) + 2.0\right]\hat{j}$

(a) for top face $\longrightarrow y = 2.0\,m$

$\vec{E} = 4.0\hat{i} - 3.0\left[(2.0)^2 + 2.0\right]\hat{j}$

$= 4.0\hat{i} - 18\hat{j}$ (or $\vec{E} = (4.0\,N/C)\hat{i} - (18\,N/C)\hat{j}$)

$\Phi = \int \vec{E}\cdot d\vec{A}$ $\quad d\vec{A} = dA\,\hat{j}$ for top face

$= \int (4.0\hat{i} - 18\hat{j})\cdot(dA\,\hat{j}) = \int(-18)\,dA = -18\,N/C \int dA$

$\int dA = 4.0\,m^2$ for area of top face

$\Phi = -18\,N/C\,(4.0\,m^2) = \boxed{72\,Nm^2/C^2}$

(c) for left face, $d\vec{A} = dA(-\hat{i})$ y varies over left face

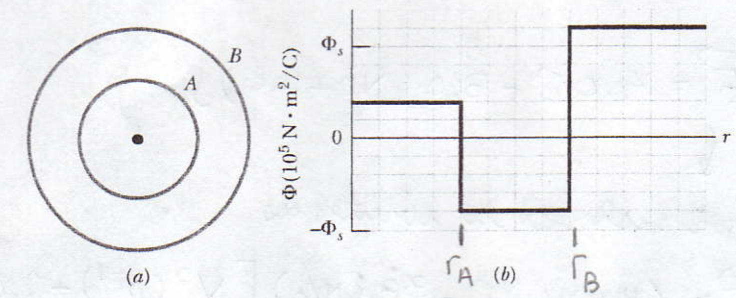
$\Phi = \int \vec{E}\cdot d\vec{A} = \int\left[(4.0)\hat{i} - (3.0)(y^2+2.0)\hat{j}\right]\cdot dA(-\hat{i})$ $\quad \hat{j}\cdot -\hat{i} = 0$

$= \int (4.0\,\hat{i})\cdot dA(-\hat{i}) = -4.0\int dA = -4.0\,N/C\,(4.0\,m^2)$

$\boxed{\Phi = -16\,Nm^2/C^2}$

Problem 3

A charged particle is suspended at the center of two concentric spherical shells that are very thin and made of nonconducting material. Figure a below shows a cross section. Figure b gives the net flux Φ through a Gaussian sphere centered on the particle, as a function of radius r of the sphere. The scale of the vertical axis is set by $\Phi s = 5.0 \times 10^5$ Nm^2/C. (A) What is the charge of the central particle? What are the net charges of (b) shell A and (c) shell B?



from Gauss' law,

$$\Phi = \oint \vec{E} \cdot d\vec{A} = q_{encl}/\varepsilon_0$$

$$q_{encl} = \Phi \varepsilon_0$$

a) for $r < r_A$, $\Phi = 1.0 \times 10^5\ Nm^2/C$ and q_{encl} is just charged particle

$$q_{encl} = \Phi \varepsilon_0 = (2.0 \times 10^5\ Nm^2/C)(8.85 \times 10^{-12}\ C^2/Nm^2) = \boxed{1.77 \times 10^{-6}\ C}$$

b) for $r_A < r < r_B$, $\Phi = -5.0 \times 10^5\ Nm^2/C$ and $q_{encl} = q_{particle} + q_A$

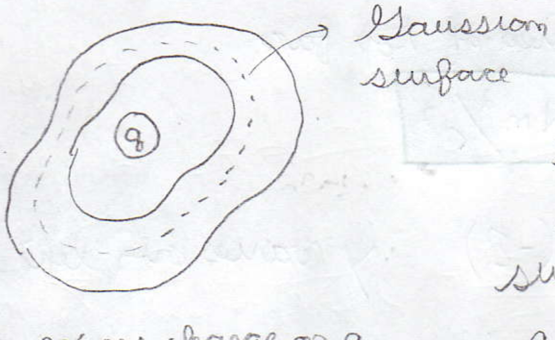
$$q_{encl} = (-5.0 \times 10^5\ Nm^2/C)(8.85 \times 10^{-12}\ C^2/Nm^2) = \boxed{-4.43 \times 10^{-6} C}$$

$$q_A = q_{encl} - q_{particle} = -4.43 \times 10^{-6} C - 1.77 \times 10^{-6} C = -6.20 \times 10^{-6} C$$

(c) similarly, $\boxed{q_B = 9.74 \times 10^{-6} C}$

Problem 4

An isolated conductor of arbitrary shape has a net charge of $+10 \times 10^{-6}$ C. Inside the conductor is a cavity within which is a point charge $q = +3.0 \times 10^{-6}$ C. What is the charge (a) on the cavity wall and (b) on the outer surface of the conductor?



(a) We choose a Gaussian surface within the conductor that surround the cavity. Since $\vec{E} = 0$ inside a conductor, the electric field everywhere on the Gaussian surface is zero so $\oint \vec{E} \cdot d\vec{A} = 0$. Therefore

$q_{encl} = 0$

* excess charge on a conductor is always on the surface

$$q_{encl} = q + q_{cavity\ wall} = 0 \qquad q_{cavity\ wall} = -q$$

$$\boxed{q_{cavity\ wall} = -3.0 \times 10^{-6} C}$$

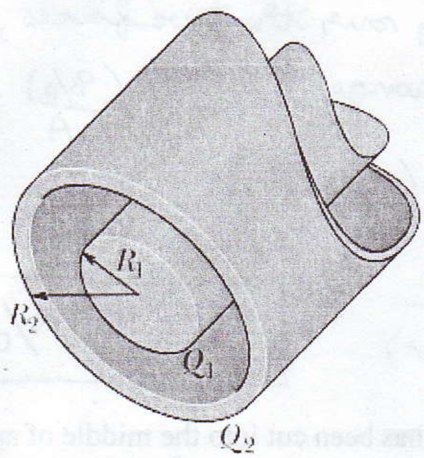
(b) The net charge Q on the conductor is charge on cavity wall plus charge on outer surface.

$$Q = q_{cavity\ wall} + q_{outer\ surface} \rightarrow q_{outer\ surface} = Q - q_{cavity\ wall} = \boxed{13 \times 10^{-6} C}$$

$R_2 = 13.0\text{ mm}$

Problem 5

The figure below is a section of a conducting rod of radius $R_1 = 1.30$ mm and length $L = 11.00$ m inside a thin-walled coaxial conducting cylindrical shell of radius $R_2 = 10.0R_1$ and the (same) length L. The net charge on the rod is $Q_1 = +3.40 \times 10^{-12}$ C; that on the shell is $Q_2 = -2.00Q_1$. What are the (a) magnitude E and (b) direction (radially inward or outward) of the electric field at radial distance $r = 2.00R_2$? What are (c) E and (d) the direction at $r = 5.00R_1$? What is the charge on the (e) interior and (f) exterior surface of the shell?



$$\oint \vec{E}\cdot d\vec{A} = q_{encl}/\varepsilon_0$$

for a cylinder: $\oint \vec{E}\cdot d\vec{A} = E(2\pi r L)$

$$E(2\pi r L) = q_{encl}/\varepsilon_0$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{q_{encl}}{rL} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$

(a) for $r = 2.00R_2$, $q_{encl} = Q_1 + Q_2 = 3.40\times10^{-12}\text{ C} + (-6.80\times10^{-12}\text{ C})$

$$= -3.40\times10^{-12}\text{ C}$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{q_{encl}}{rL} = \frac{1}{2\pi(8.85\times10^{-12}\text{ C}^2/\text{Nm}^2)}\frac{3.40\times10^{-12}\text{ C}}{(26\times10^{-3}\text{ m})(11.0\text{ m})}$$

$E = 0.214$ N/C (b) **radially inward** (since $q_{encl} < 0$)

(c) for $r = 5.00R_1$, $q_{encl} = Q_1 = 3.40\times10^{-12}$ C

$$E = \frac{1}{2\pi\varepsilon_0}\frac{q_{encl}}{rL} = \frac{1}{2\pi(8.85\times10^{-12}\text{ C}^2/\text{Nm}^2)}\frac{3.40\times10^{-12}\text{ C}}{(6.50\times10^{-3}\text{ m})(11.0\text{ m})}$$

$E = 0.855$ N/C (d) **radially outward** (since $q_{encl} > 0$)

(e) $Q_{inner} = -Q_1 =$ **-3.40×10^{-12} C**

(f) $Q_{outer} = Q_2 - Q_{inner} =$ **-3.40×10^{-12} C**

Problem 6

A square metal plate of edge length 8.0 cm and negligible thickness has a total charge of 6.0×10^{-6} C. (a) Estimate the magnitude E of the electric field just off the center of the plate (at, say, a distance of 0.50 mm) by assuming that the charge is spread uniformly over the two faces of the plate.

(a) the electric field just outside a conducting surface is given by $E = \sigma/\varepsilon_0$ where σ = surface charge density at the surface.

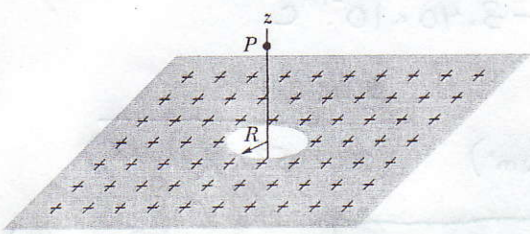
⇒ since the charge is spread uniformly over the two faces, the charge at the surface is ½ the total charge so $\sigma = \frac{(q/2)}{A} = \frac{q}{2A}$

$$E = \sigma/\varepsilon_0 = \frac{(q/2A)}{\varepsilon_0} = \frac{q}{2A\varepsilon_0} \qquad (A = x^2)$$

$$E = \frac{6.0\times10^{-6}\,C}{2(8.0\times10^{-2}\,m)^2(8.85\times10^{-12}\,C^2/Nm^2)} = \boxed{5.3\times10^{7}\,N/C}$$

Problem 7

In the figure below, a small circular hole of radius R = 1.80 cm has been cut into the middle of an infinite flat, nonconducting surface that has uniform charge density $\sigma = 4.50$ pC/m^2. A z axis, with its origin at the hole's center, is perpendicular to the surface. In unit vector notation, what is the electric field at point P at z = 2.56 cm?



⇒ we can get $\vec{E}$ by finding the superposition of $\vec{E}$ from an infinite sheet of charge density σ and $\vec{E}$ from a disk of radius R and charge density $-\sigma$

infinite non-conducting sheet ⇒ $E = \sigma/2\varepsilon_0$

disk ⇒ $E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$

$$\vec{E}_{net} = \frac{\sigma}{2\varepsilon_0} + \frac{-\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right) = \frac{\sigma z}{2\varepsilon_0\sqrt{z^2+R^2}}$$ straight upwards

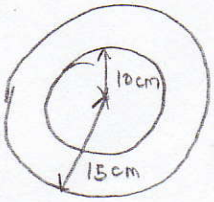
$$\vec{E} = \frac{(4.50\times10^{-12}\,C/m^2)(0.0256\,m)}{2(8.85\times10^{-12}\,C^2/Nm^2)\sqrt{(0.0256m)^2+(0.018m)^2}} \quad (\hat{k})$$

$$\boxed{\vec{E} = (0.208\,N/C)\,\hat{k}}$$

Problem 8

Two charged concentric spheres have radii of 10.0 cm and 15.0 cm. The charge on the inner sphere is 4.00×10^{-8} C, and that on the outer sphere is 2.00×10^{-8} C. Find the electric field (a) at r = 12.0 cm and (b) at r = 20.0 cm.

$$\oint \vec{E} \cdot d\vec{A} = q_{encl}/\varepsilon_0 \longrightarrow \oint E\, dA = q_{encl}/\varepsilon_0$$



$$E(4\pi r^2) = q_{encl}/\varepsilon_0 \longrightarrow E = \frac{1}{4\pi\varepsilon_0} \frac{q_{encl}}{r^2}$$

(a) for $r = 12.0\,\text{cm}$, $q_{encl} = 4.00 \times 10^{-8}\,\text{C}$

$$E = \frac{(8.99 \times 10^9\,\text{Nm}^2/\text{C}^2)(4.00 \times 10^{-8}\,\text{C})}{(0.12\,\text{m})^2} = \boxed{2.50 \times 10^4\,\text{N/C}}$$

(b) for $r = 20.0\,\text{cm}$, $q_{encl} = 4.00 \times 10^{-8}\,\text{C} + 2.000 \times 10^{-8}\,\text{C} = 6.00 \times 10^{-8}\,\text{C}$

$$E = \frac{(8.99 \times 10^9\,\text{Nm}^2/\text{C}^2)(4.00 \times 10^{-8}\,\text{C})}{(0.20\,\text{m})^2} = \boxed{1.35 \times 10^4\,\text{N/C}}$$

Problem 9

A solid nonconducting sphere of radius $R = 5.60$ cm has a nonuniform charge distribution of volume charge density $\rho = (14.1\ \text{pC/m}^3)\ r/R$, where r is the radial distance from the sphere's center. (a) What is the sphere's total charge? What is the magnitude E of the electric field at (b) r = 0, (c) r = R/2.00, and (d) r = R?

$$\rho = Ar/R \qquad A = 14.1 \times 10^{-12}\,\text{C/m}^2 \qquad R = 5.60\,\text{cm}$$

(a) $$q = \int \rho(r)\, dV = \int_0^R (Ar/R)\, 4\pi r^2 dr = \frac{4\pi A}{R} \int_0^R r^3 dr$$

$$q = \frac{4\pi A}{R}\left(\frac{R^4}{4}\right) = \pi A R^3 = \pi(14.1 \times 10^{-12}\,\text{C/m}^2)(5.60 \times 10^{-2}\,\text{m})^3$$

$$\boxed{q = 7.78 \times 10^{-15}\,\text{C}}$$

(b) at $r = 0$, $q_{encl} = 0$ so $\boxed{E = 0}$

(c) $$\oint \vec{E} \cdot d\vec{A} = q_{encl}/\varepsilon_0 \rightarrow E(4\pi r^2) = \frac{q_{encl}}{\varepsilon_0} \qquad q_{encl} = \frac{4\pi A}{R} \int_0^r r^3 dr$$

$$E(4\pi r^2) = \left(\frac{4\pi A}{R}\right)\left(r^4/4\right) \rightarrow E = \frac{1}{4\pi\varepsilon_0} \frac{\pi A r^2}{R}$$

for $r = R/2 = 2.80\,\text{cm} \rightarrow \boxed{E = 5.58 \times 10^{-3}\,\text{N/C}}$

(d) for $r = R$, $q_{encl} = 7.78 \times 10^{-15}\,\text{C} \rightarrow \boxed{E = 2.23 \times 10^{-2}\,\text{N/C}}$