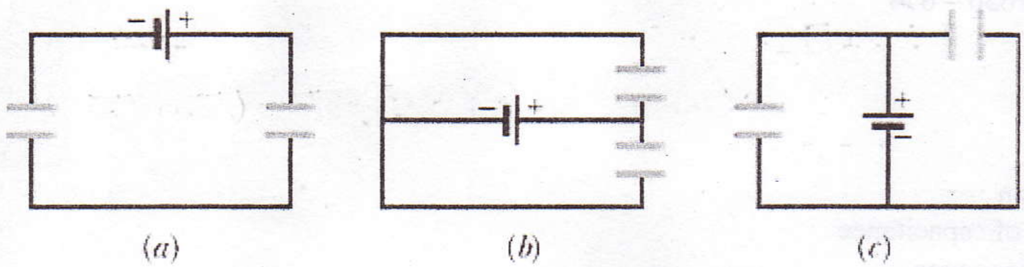
You should know how to compute the energy stored in a capacitor, given either its charge or potential difference. You should also know how to compute the energy density at points between the plates or, in general, at any point within a given electric field.

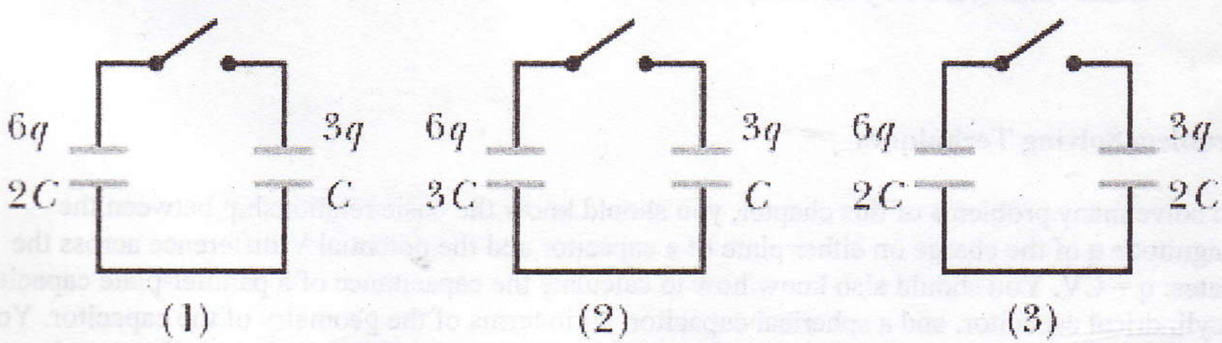
Questions and Example Problems from Chapter 25

Question 1
For each circuit in the figure below, are the capacitors connected in series, in parallel, or in neither mode?



series | parallel | parallel

Question 2
The figure below shows three circuits, each consisting of a switch and two capacitors, initially charged as indicated. After the switches have been closed, in which circuit (if any) will the charge on the left-hand capacitor (a) increase, (b) decrease, and (c) remain the same?



⇒ capacitors are in parallel so V must be the same ($q = CV \to V = q/C$ must be the same for each)

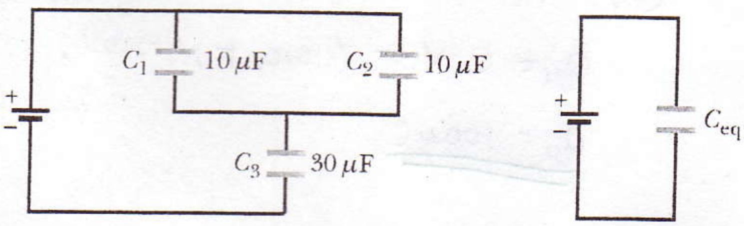
(a) circuit 2

(b) circuit 3

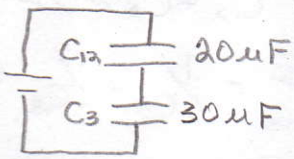
(c) circuit 1

Question 3

Figure a shows a circuit with three capacitors, and Fig. b shows the circuit with their equivalent capacitor, of capacitance C_{eq}, which has a charge of 60 μC. Without a calculator (and, if possible, without written calculation), find (a) the charge on and (b) the voltage across capacitor 3 and then (c) the charge on and (d) the voltage across capacitor 1.



(a) (b)



$q_{12} = q_3 = 60\mu C$

(a) $q_3 = 60\mu C$

(b) $V_3 = q_3/C_3 = \frac{60\mu C}{30\mu F} = 2V$

(d) $V_{12} = q_{12}/C_{12} = \frac{60\mu C}{20\mu F} = 3V = V_1 = V_2$

(c) $q_1 = C_1V_1 = (10\mu F)(3V) = 30\mu C$

Problem 1

A parallel-plate capacitor has circular plates of 8.20 cm radius and 1.30 mm separation. (a) Calculate the capacitance. (b) Find the charge for a potential difference of 120 V.

$r = 8.20cm = 8.20\times10^{-2}m$

$A = \pi r^2 = \pi(8.20\times10^{-2}m)^2$

$= 2.11\times10^{-2}m^2$

$d = 1.30\times10^{-3}m$

(a) $C = \varepsilon_0 A/d$

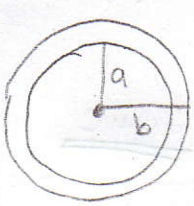
$$C = \frac{(8.854\times10^{-12}C^2/Nm^2)(2.11\times10^{-2}m^2)}{(1.30\times10^{-3}m)}$$

$C = 1.44\times10^{-10}F = 14.4nF$

(b) $q = CV = (1.44\times10^{-10}F)(120V) = 1.73\times10^{-8}C$

Problem 2

Suppose that the two spherical shells of a spherical capacitor have approximately equal radii. Under these conditions, the device approximates a parallel-plate capacitor with b - a = d. Show that Eq. 25-17 ($C = 4\pi\varepsilon_0\, ab/(b-a)$) does indeed reduce to Eq. 25-9 ($C = \varepsilon_0 A/d$) in this case.



area of inner sphere $= 4\pi a^2$

area of outer sphere $= 4\pi b^2$

since $a \approx b$, the area of either sphere can be written as

$A \approx 4\pi ab$

$$C = \frac{4\pi\varepsilon_0 ab}{b-a} = \frac{\varepsilon_0(4\pi ab)}{b-a}$$

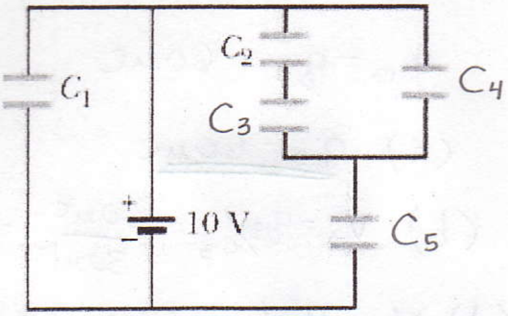
$4\pi ab = A$

$b - a = d$

$C = \varepsilon_0 A/d$

Problem 3

In the figure below, the battery has a potential difference of 10.0 V and the five capacitors each have a capacitance of 10.0 μF. What is the charge on (a) capacitor 1 and (b) capacitor 2?

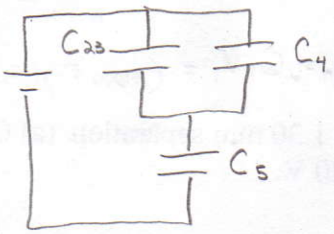


(a) $V_1 = 10V$

$q_1 = C_1 V_1 = (10.0\mu F)(10.0V)$

$q_1 = 100\mu C$

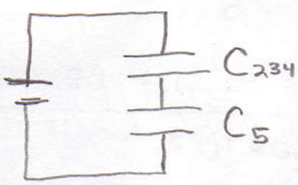
(b) $C_2 + C_3$ in series



$\frac{1}{C_{23}} = 1/C_2 + 1/C_3 = \frac{1}{10\mu F} + \frac{1}{10\mu F} = \frac{1}{5\mu F}$

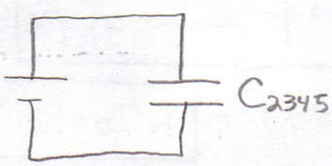
$1/C_{23} = 0.20\mu F^{-1} \longrightarrow C_{23} = 5\mu F$

$C_{23} + C_4$ in parallel $\longrightarrow C_{234} = C_{23} + C_4 = 5\mu F + 10\mu F$



$C_{234} = 15\mu F$

$C_{234} + C_5$ in series $\longrightarrow \frac{1}{C_{2345}} = \frac{1}{C_{234}} + \frac{1}{C_5}$



$\frac{1}{C_{2345}} = \frac{1}{15\mu F} + \frac{1}{10\mu F} \longrightarrow C_{2345} = 6\mu F$

$q_{2345} = C_{2345} V_{2345} = (6\mu F)(10V) = 60\mu C \longrightarrow q_{234} = q_5 = 60\mu C$

$V_{234} = q_{234}/C_{234} = \frac{60\mu C}{15\mu F} = 4V \longrightarrow V_{23} = V_4 = 4V$ $(q_4 = C_4 V_4 = 40\mu C)$

$q_{23} = C_{23} V_{23} = (5\mu F)(4.0V) = 20\mu C \longrightarrow q_2 = q_3 = 20\mu C$

$V_5 = q_5/C_5 = \frac{60\mu C}{10\mu F} = 6V$

$V_2 = q_2/C_2 = 20\mu C/10\mu F = 2V$

$V_3 = q_3/C_3 = 20\mu C/10\mu F = 2V$

	q	V
C_1	100μC	10V
C_2	20μC	2V
C_3	20μC	2V
C_4	40μC	4V
C_5	60μC	6V

Problem 4

A 100 pF capacitor is charged to a potential difference of 50 V, and the charging battery is disconnected. The capacitor is then connected in parallel with a second (initially uncharged) capacitor. If the potential difference across the first capacitor drops to 35 V, what is the capacitance of this second capacitor?

$C_1 = 100\,pF$, $V_0 = 50\,V$ } initially, the first capacitor has charge $q = C_1 V_0$

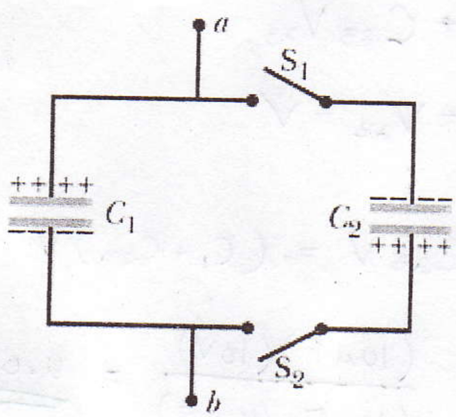
⇒ after the first capacitor is connected in parallel to a second capacitor, its voltage drops to 35 V and it now has charge $q_1 = C_1 V$

↓ since charge is conserved, the second capacitor must have charge $q_2 = q - q_1 = C_1 V_0 - C_1 V = C(V_0 - V)$; its voltage is also 35 V since the capacitors are in parallel

$$C_2 = q_2/V = C_1(V_0 - V)/V = \frac{(100\,pF)(50V - 35V)}{35V} = \boxed{43\,pF}$$

Problem 5

In the figure below, the capacitances are $C_1 = 1.0\ \mu F$ and $C_2 = 3.0\ \mu F$ and both capacitors are charged to a potential difference of V = 100 V but with opposite polarity as shown. Switches S_1 and S_2 are now closed. (a) What is now the potential difference between points a and b? What are now the charges on capacitors (b) 1 and (c) 2?



⇒ after the switch is closed, the capacitors are in parallel so the voltage across each is the same

↓ $V_1 = V_2 = V_{12} = q_{12}/C_{12}$ q_{12} → total charge, C_{12} → C_{eq}

$q_1 = C_1 V_1 = (1.0\,\mu F)(100V) = 100\,\mu C$

$q_2 = C_2 V_2 = (3.0\,\mu F)(100V) = 300\,\mu C$

* $q_{12} = 200\,\mu C$ and not $400\,\mu C$ because the polarity is opposite

$C_1 + C_2$ are in parallel → $C_{12} = C_1 + C_2 = 1.0\,\mu F + 3.0\,\mu F = 4.0\,\mu F$

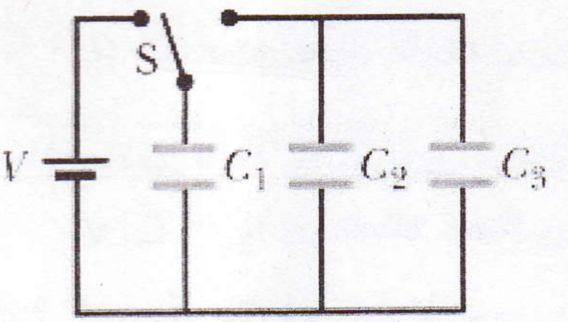
$V_{12} = q_{12}/C_{12} = \frac{200\,\mu C}{4.0\,\mu F} = \boxed{50\,V}$ $V_1 = V_2 = 50V$

(b) $q_1 = C_1 V_1 = (1.0\,\mu F)(50V) = \boxed{50\,\mu C}$

$q_2 = C_2 V_2 = (3.0\,\mu F)(50V) = \boxed{150\,\mu C}$

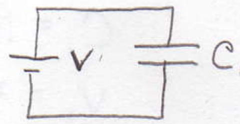
Problem 6

In the figure below, V = 10 V, $C_1 = 10\ \mu F$, and $C_2 = C_3 = 20\ \mu F$. Switch S is first thrown to the left side until capacitor 1 reaches equilibrium. Then the switch is thrown to the right. When equilibrium is again reached, how much charge is on capacitor 1.



switch to left:

initial charge on C_1

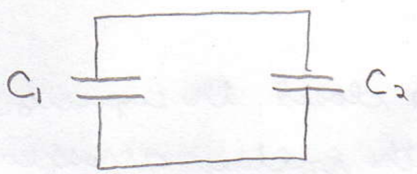


$$q = C_1 V_0 = (10\mu F)(10V) = 100\mu C$$

C_2 & C_3 are in parallel → $C_{23} = C_2 + C_3 = 20\mu F + 20\mu F$

$$C_{23} = 40\mu F$$

switch to right:



charge is conserved: $q = q_1 + q_{23}$

$$C_1 V_0 = C_1 V_1 + C_{23} V_{23}$$

$$V_1 = V_{23} = V$$

$$C_1 V_0 = C_1 V + C_{23} V = (C_1 + C_{23}) V$$

$$V = \frac{C_1 V_0}{(C_1 + C_{23})} = \frac{(10\mu F)(10V)}{(10\mu F + 40\mu F)} = 2.0V$$

$V_1 = 2.0V$

$V_{23} = 2.0V$

$$q_1 = C_1 V_1 = (10\mu F)(2.0V) = \boxed{20\mu C}$$

$$q_{23} = C_{23} V_{23} = (40\mu F)(2.0V) = 80\mu C$$

Problem 7

The parallel-plates in a capacitor, with a plate area of 8.50 cm^2 and an air-filled separation of 3.00 mm, are charged by a 6.00 V battery. They are then disconnected from the battery and pulled apart (without discharge) to a separation of 8.00 mm. Neglecting fringing, find (a) the potential difference between the plates (b) the initial stored energy, (c) the final stored energy and (d) the work required to separate the plates.

$A = 8.50\,cm^2 = 8.50\times10^{-4}\,m^2$

$d = 3.00\,mm = 3.00\times10^{-3}\,m$

$V = 6.00\,V$

$C = \varepsilon_0 A/d$ once capacitor is disconnected from the battery, the charge remains the same

originally $\rightarrow$ $C = \dfrac{(8.85\times10^{-12}\,C^2/Nm^2)(8.50\times10^{-4}\,m^2)}{(3.00\times10^{-3}\,m)}$

$C = 2.51\times10^{-12}\,F$ $\quad V = 6.00\,V \quad q = CV = (2.51\times10^{-12}\,F)(6.00\,V) = 1.51\times10^{-11}\,C$

now charge remains same

$C = \dfrac{(8.85\times10^{-12}\,C^2/Nm^2)(8.50\times10^{-4}\,m^2)}{(8.00\times10^{-3}\,m)} = 9.40\times10^{-13}\,F$

(a) $V = q/C = \dfrac{1.51\times10^{-11}\,C}{9.40\times10^{-13}\,F}$

$V = 16\,V$

(b) $U = \frac{1}{2} C_i V_i^2 = \frac{1}{2}(2.51\times10^{-12}\,F)(6.00\,V)^2 = 4.51\times10^{-11}\,J$

(c) $U = \frac{1}{2} C_f V_f^2 = 1.20\times10^{-10}\,J$

(d) $W = \Delta U \approx 7.52\times10^{-11}\,J$

Problem 8

A parallel-plate air-filled capacitor has a capacitance of 50 pF. (a) If each of its plates has an area of 0.35 m^2, what is the separation? (b) If the region between the plates is now filled with material having $\kappa = 5.6$, what is the capacitance?

$C = 50\,pF$

$A = 0.35\,m^2$

$K = 5.6$

(a) $C = \varepsilon_0 A/d \longrightarrow d = \varepsilon_0 A/C$

$d = \dfrac{(8.85\times10^{-12}\,F/m)(0.35\,m^2)}{(50\times10^{-12}\,F)}$

$d = 6.2\times10^{-3}\,m$

(b) with a dielectric between the plates, the new capacitance is given by:

$C_{new} = (K\varepsilon_0)A/d = K(\varepsilon_0 A/d) = K C_{old}$

$C_{new} = (5.6)(50\,pF) = 280\,pF$

Problem 9

The space between two concentric conducting spherical shells of radii a = 1.70 cm and b = 1.20 cm is filled with a substance of dielectric constant κ = 23.5. A potential difference V = 73.0 V is applied across the inner and outer shells. Determine (a) the capacitance of the device, (b) the free charge q on the inner shell, and (c) the charge q' induced along the surface of the inner shell.

$a = 1.70 \times 10^{-2}\,m$

$b = 1.20 \times 10^{-2}\,m$

$K = 23.5$

$V = 73.0\,V$

(a) $C_0 = 4\pi\varepsilon_0\left(\frac{ab}{a-b}\right) \rightarrow C = KC_0 = 4\pi K\varepsilon_0\left(\frac{ab}{a-b}\right)$

$C = 4\pi(23.5)(8.85\times10^{-12}\,C^2/Nm^2)\left[\frac{(1.70\times10^{-2}\,m)(1.20\times10^{-2}\,m)}{0.50\times10^{-2}\,m}\right]$

$C = 1.07\times10^{-8}\,F = \boxed{0.107\,nF}$

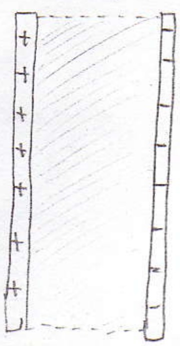
(b) free charge $q = CV = (1.07\times10^{-8}\,F)(73.0\,V) = \boxed{7.79\,nC}$

(c) $q - q' = q/K \rightarrow q' = q - q/K = q(1 - 1/K)$

$q' = (7.79\,nC)\left(1 - \frac{1}{23.5}\right) = \boxed{7.46\,nC}$

Problem 10

Two parallel plates of area 100 cm² are given charges of equal magnitudes 8.9 × 10⁻⁷ C but opposite signs. The electric field within the dielectric material filling the space between the plates is 1.4 × 10⁶ V/m. (a) Calculate the dielectric constant of the material. (b) Determine the magnitude of the charge induced on each dielectric surface.



$q = 8.9\times10^{-7}\,C$

$E = 1.4\times10^{6}\,V/m$

$A = 100\,cm^2\left(\frac{1\,m}{100\,cm}\right)^2 = 10^{-2}\,m^2$

with a dielectric, $E = E_0/K = \frac{q}{K\varepsilon_0 A}$

(a) $K = \frac{q}{\varepsilon_0 E A} \longrightarrow K = \frac{(8.9\times10^{-7}\,C)}{(8.85\times10^{-12}\,C^2/Nm^2)(1.4\times10^{6}\,V/m)(10^{-2}\,m^2)}$

$\boxed{K = 7.2}$

(b) $q - q' = q/K$ where q' = induced charge, q = charge on plates

$q' = q - q/K = q(1 - 1/K)$

$= (8.9\times10^{-7}\,C)(1 - 1/7.2) \longrightarrow \boxed{q' = 7.7\times10^{-7}\,C}$