Answer Key

You should understand that resistivity is determined to a large extent by the mean time between collisions and that this quantity is temperature dependent because the average velocity of the electrons is temperature dependent. The relationship between the resistivity ρ and mean time t between collision is $\rho = m/e^2 n\tau$, where m is the mass of a charge carrier and n is the number of charge carriers per unit volume in the current.

Charge carriers lose kinetic energy in collisions with the atoms of a resistor. You should know how to compute the rate of energy dissipation, given the resistance and either the current in the resistor or the potential difference along the length of the resistor. Use $P = i^2R$ or $P = V^2/R$, where i is the current in the resistor and V is the potential difference across it.

## Questions and Example Problems from Chapter 26

**Problem 1**

During the 4.0 min a 5.0 A current is set up in a wire, how many (a) coulombs and (b) electrons pass through any cross section across the wire's width?

$i = 5.0\,A$

$\Delta t = 4.0\,min = 240\,s$

(a) $i = \Delta q/\Delta t \rightarrow \Delta q = i\Delta t$

$\Delta q = (5.0A)(240s) = \boxed{1.2\times10^{3}\,C}$

(b) $q = ne \rightarrow n = q/e = \dfrac{1.2\times10^{3}C}{1.602\times10^{-19}C} \rightarrow \boxed{n = 7.5\times10^{21}}$

**Problem 2**

The magnitude J of the current density in a certain wire with a circular cross section of radius R = 2.00 mm is given by $J = (3.00 \times 10^8)r^2$, with J in amperes per square meter and radial distance r in meters. What is the current through the outer section bounded by r = 0.900R and r = R?

$i = \int \vec{J}\cdot d\vec{A} \quad J = Br^2 \quad B = (3.00\times10^{8}\,A/m^4) \quad dA = 2\pi r\,dr$

$i = \int (Br^2)(2\pi r\,dr)\cos 0^\circ = 2\pi B\int_{0.900R}^{R} r^3\,dr = \left.\dfrac{2\pi Br^4}{4}\right|_{0.900R}^{R}$

$i = \pi B/2\left[R^4 - (0.900R)^4\right] = \dfrac{\pi BR^4}{2}\left[1 - (0.900)^4\right] = \dfrac{\pi(3.00\times10^{8}A/m^4)(2.0\times10^{-3}m)^4(1-0.900^4)}{2} = \boxed{2.59\,mA}$

**Problem 3**

A fuse in an electric circuit is a wire that is designed to melt, and thereby open the circuit, if the current exceeds a predetermined value. Suppose that the material to be used in a fuse melts when the current density rises to 440 A/cm². What diameter of cylindrical wire should be used to make a fuse that will limit the current to 0.50 A?

$J = 440\,A/cm^2\left(\dfrac{100cm}{1m}\right)^2 = 440\times10^{4}\,A/m^2$

$i = 0.50\,A$

$J = i/A = i/\pi r^2$

$r = \sqrt{\dfrac{i}{\pi J}} = \sqrt{\dfrac{(0.50A)}{\pi(440\times10^{4}A/m^2)}}$

$r = 1.9\times10^{-4}\,m$

$\text{diameter} = 2r = \boxed{3.8\times10^{-4}\,m}$

**Problem 4**

A small but measurable current of $1.2 \times 10^{-10}$ A exists in a copper wire whose diameter is 2.5 mm. The number of charge carriers per unit volume is $8.48 \times 10^{28}$ $m^{-3}$. Assuming the current is uniform, calculate the (a) current density and (b) electron drift speed.

$i = 1.2\times10^{-10}\,A$

$r = 1.25\,mm = 1.25\times10^{-3}\,m$

$n = 8.48\times10^{28}\,m^{-3}$

$J = ?$

$V_d = ?$

(a) $J = i/A$ if current is uniform

$$J = i/(\pi r^2) = \frac{1.2\times10^{-10}\,A}{\pi(1.25\times10^{-3}\,m)^2} = \boxed{2.44\times10^{-5}\,A/m^2}$$

(b) $$V_d = \frac{i}{nAe} = \frac{J}{ne} = \frac{(2.44\times10^{-5}\,A/m^2)}{(8.48\times10^{28}\,m^{-3})(1.602\times10^{-19}\,C)}$$

$$\boxed{V_d = 1.80\times10^{-15}\,m}$$

**Problem 5**

A wire of Nichrome (a nickel–chromium–iron alloy commonly used in heating elements) is 1.0 m long and 1.0 $mm^2$ in cross-sectional area. It carries a current of 4.0 A when a 2.0 V potential difference is applied between its ends. Calculate the conductivity σ of Nichrome.

$L = 1.0\,m$

$A = 1.0\,mm^3$

$= 1.0\times10^{-6}\,m^3$

$i = 4.0\,A$

$V = 2.0\,V$

$$\left.\begin{array}{l} R = \rho L/A \\ R = V/i \end{array}\right\} \rho L/A = V/i \rightarrow \rho = \frac{AV}{iL}$$

$$\sigma = 1/\rho = \frac{iL}{AV} \rightarrow \sigma = \frac{(4.0A)(1.0m)}{(1.0\times10^{-6}m^3)(2.0V)}$$

$$\boxed{\sigma = 2.0\times10^{6}\,\Omega\cdot m}$$

**Problem 6**

A common flashlight bulb is rated at 0.30 A and 2.9 V (the values of the current and voltage under operating conditions). If the resistance of the bulb filament at room temperature (20°C) is 1.1 Ω, what is the temperature of the filament when the bulb is on? The filament is made of tungsten.

$R_0 = 1.1\,\Omega$

when on $\rightarrow R = V/i = \frac{2.9V}{0.3A} = 9.67\,\Omega$

tungsten $\alpha = 4.5\times10^{-3}/C°$

$R - R_0 = R_0\alpha(T - T_0)$

$$T - T_0 = \frac{R - R_0}{R_0\alpha}$$

$$T = T_0 + \frac{R - R_0}{R_0\alpha}$$

$$T = 20°C + \frac{(9.67\Omega - 1.1\Omega)}{(1.1\Omega)(4.5\times10^{-3}/C°)}$$

$$\boxed{T = 1751°C}$$

**Problem 7**

An unknown resistor is connected between the terminals of a 3.00 V battery. Energy is dissipated in the resistor at the rate of 0.540 W. The same resistor is then connected between the terminals of a 1.50 V battery. At what rate is energy now dissipated?

$V = 3.00\,V$ $\quad P = i^2 R = V^2/R \longrightarrow R = \frac{V^2}{P} = \frac{(3.00\,V)^2}{0.540\,W}$

$P = 0.540\,W$ $\quad \boxed{R = 16.7\,\Omega}$

$P = ?$ when $V = 1.50\,V$

$\rightarrow P = V^2/R = \frac{(1.50\,V)^2}{16.7\,\Omega} = \boxed{0.135\,W}$

**Problem 8**

A 1250 W radiant heater is constructed to operate at 115 V. (a) What will be the current in the heater? (b) What is the resistance of the heating coil? (c) How much thermal energy is produced in 1.0 h by the heater?

$P = 1250\,W$ $\quad$ (a) $P = iV \longrightarrow i = P/V = \frac{1250\,W}{115\,V} = \boxed{10.9\,A}$

$V = 115\,V$

(b) can use $R = V/i$ or from $P = V^2/R \rightarrow R = V^2/P$

$R = \frac{115\,V}{10.9\,A} = \boxed{10.6\,\Omega}$ $\quad R = \frac{(115\,V)^2}{1250\,W} = \boxed{10.6\,\Omega}$

(c) $P = dE/dt$ $\quad E = Pt = (1250\,W)(3600\,s)$

$= E/t \longrightarrow \boxed{E = 4.5 \times 10^6\,J}$

**Problem 9**

A 100 W light bulb is plugged into a standard 120 V outlet. (a) How much does it cost per 31-day month to leave the light turned on continuously? Assume electric energy costs 6¢/kW·h. (b) What is the resistance of the bulb? (c) What is the current in the bulb?

⇒ we need to figure out how many KW·h from 100 W running for 31 days

$(100\,W)\left(\frac{1\,KW}{10^3\,W}\right)(31\,days)(24\,hrs/day) = 74.4\,KW\cdot h$

(a) $cost = (74.4\,KW\cdot h)(6¢/KW\cdot hr) = 446.4¢ = \boxed{\$4.46}$

(b) $P = V^2/R \longrightarrow R = V^2/P = (120\,V)^2/(100\,W) = \boxed{144\,\Omega}$

(c) $P = iV \longrightarrow i = P/V = \frac{100\,W}{120\,V} = \boxed{0.833\,A}$