

Some problems deal with a charged particle that moves around a circular orbit, under the influence of a uniform magnetic field. You should know the relationship between the particle speed, the orbit radius, and the magnetic field. The period of the motion is given by  $T = 2\pi m/qB$ , where  $m$  is the mass of the particle,  $q$  is its charge, and  $B$  is the magnitude of the magnetic field. The radius of the orbit is given by  $r = mv/qB$ , where  $v$  is the particle speed.

You should know how to calculate the force of a magnetic field on a current-carrying wire and the torque of a uniform magnetic field on a current-carrying loop. Often the easiest way to compute the torque is by using  $\vec{\tau} = \vec{\mu} \times \vec{B}$ . You will need to know how to compute the dipole moment  $\vec{\mu}$ , both magnitude and direction, of a current loop.

## Questions and Example Problems from Chapter 28

### Question 1

For four situations, here is the velocity  $\vec{v}$  of a proton at a certain instant as it moves through a uniform magnetic field  $\vec{B}$ :

(a)  $\vec{v} = 2\hat{i} - 3\hat{j}$  and  $\vec{B} = 4\hat{k}$       (b)  $\vec{v} = 3\hat{i} + 2\hat{j}$  and  $\vec{B} = -4\hat{k}$

(c)  $\vec{v} = 3\hat{j} - 2\hat{k}$  and  $\vec{B} = 4\hat{i}$       (d)  $\vec{v} = 20\hat{i}$  and  $\vec{B} = -4\hat{i}$

$$a = b = c, d \text{ (zero)}$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

Without written calculation, rank the situations according to the magnitude of the magnetic force on the proton, greatest first.

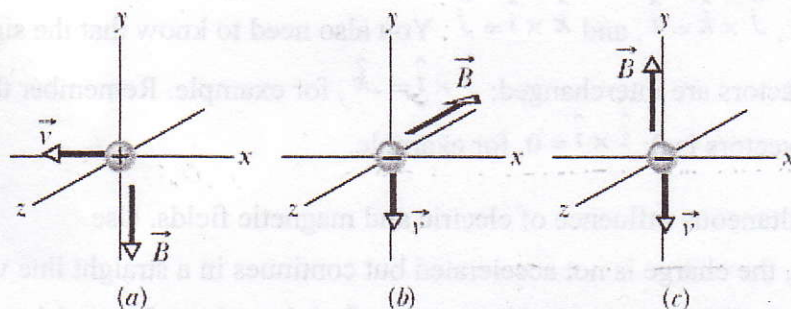
$$(a) \quad \underbrace{8(\hat{i} \times \hat{k})}_{-\hat{j}} - \underbrace{12(\hat{j} \times \hat{k})}_{\hat{i}} = \underline{-8\hat{j} - 12\hat{i}}$$

$$(b) \quad \underbrace{-12(\hat{i} \times \hat{k})}_{-\hat{j}} - \underbrace{8(\hat{j} \times \hat{k})}_{\hat{i}} = \underline{12\hat{j} - 8\hat{i}}$$

$$(c) \quad \underbrace{12(\hat{j} \times \hat{i})}_{-\hat{k}} - \underbrace{8(\hat{k} \times \hat{i})}_{\hat{j}} = \underline{-12\hat{k} - 8\hat{j}}$$

### Question 2

The figure shows three situations in which a charged particle with velocity  $\vec{v}$  travels through a uniform magnetic field  $\vec{B}$ . In each situation, what is the direction of the magnetic force  $\vec{F}_B$  on the particle?



$$+z$$

$$-x$$

(charge is -)

$$\vec{F}_B = 0$$



### Problem 1

An electron in a TV camera tube is moving at  $7.20 \times 10^6$  m/s in a magnetic field of strength 83.0 mT. (a) Without knowing the direction of the field, what can you say about the greatest and least magnitudes of the force acting on the electron due to the field? (b) At one point the electron has an acceleration of magnitude  $4.90 \times 10^{14}$  m/s<sup>2</sup>. What is the angle between the electron's velocity and the magnetic field?

$$v = 7.20 \times 10^6 \text{ m/s}$$

$$B = 83.0 \times 10^{-3} \text{ T}$$

$$q = -1.602 \times 10^{-19} \text{ C}$$

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$(a) F_B = |q|vB \sin \theta \rightarrow \text{max when } \sin \theta = 1$$

$$\rightarrow \text{min when } \sin \theta = 0$$

$$\text{max } F_B = |q|vB = (1.602 \times 10^{-19} \text{ C})(7.20 \times 10^6 \text{ m/s})(83.0 \times 10^{-3} \text{ T})$$

$$= 9.56 \times 10^{-14} \text{ N}$$

$$\text{min } F_B = 0 \text{ N}$$

$$(b) a = \frac{F_B}{m} = \frac{|q|vB \sin \theta}{m} \rightarrow \sin \theta = \frac{ma}{|q|vB} = 4.67 \times 10^{-3}$$

$$\theta = 0.267^\circ$$

### Problem 2

An electron that has velocity  $\vec{v} = (2.0 \times 10^6 \text{ m/s})\hat{i} + (3.0 \times 10^6 \text{ m/s})\hat{j}$  moves through the magnetic field  $\vec{B} = (0.300 \text{ T})\hat{i} - (0.15 \text{ T})\hat{j}$ . (a) Find the force on the electron. (b) Repeat your calculation for a proton having the same velocity.

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & 0 \\ B_x & B_y & 0 \end{vmatrix}$$

$$\text{or } \vec{v} \times \vec{B} = (v_x \hat{i} + v_y \hat{j}) \times (B_x \hat{i} + B_y \hat{j})$$

$$= v_x B_x (\hat{i} \times \hat{i}) + v_x B_y (\hat{i} \times \hat{j}) + v_y B_x (\hat{j} \times \hat{i}) + v_y B_y (\hat{j} \times \hat{j})$$

$$= \hat{k} [v_x B_y - v_y B_x]$$

$$= \hat{i} \begin{vmatrix} v_y & 0 \\ B_y & 0 \end{vmatrix} - \hat{j} \begin{vmatrix} v_x & 0 \\ B_x & 0 \end{vmatrix} + \hat{k} \begin{vmatrix} v_x & v_y \\ B_x & B_y \end{vmatrix}$$

$$= \hat{i}(0) - \hat{j}(0) + \hat{k} [v_x B_y - v_y B_x]$$

$$v_x = 2.0 \times 10^6 \text{ m/s} \quad v_y = 3.0 \times 10^6 \text{ m/s}$$

$$B_x = 0.300 \text{ T} \quad B_y = -0.15 \text{ T}$$

$$\vec{v} \times \vec{B} = \hat{k} [(2.0 \times 10^6 \text{ m/s})(-0.15 \text{ T}) - (3.0 \times 10^6 \text{ m/s})(0.30 \text{ T})]$$

$$= (-1.2 \times 10^6 \text{ T m/s}) \hat{k}$$

$$F = q \vec{v} \times \vec{B} = (-1.602 \times 10^{-19} \text{ C})(-1.2 \times 10^6 \text{ T m/s}) \hat{k} = (1.92 \times 10^{-13} \text{ N}) \hat{k}$$

$$(b) \text{ for a proton, } q = +1.602 \times 10^{-19} \text{ C so } F = -(1.92 \times 10^{-13} \text{ N}) \hat{k}$$



### Problem 3

A proton travels through uniform magnetic and electric fields. The magnetic field is  $\vec{B} = -2.5\hat{i}$  mT. At one instant the velocity of the proton is  $\vec{v} = 2000\hat{j}$  m/s. At that instant and in unit vector notation, what is the net force acting on the proton if the electric field is (a)  $4.00\hat{k}$  V/m, (b)  $-4.00\hat{k}$  V/m, and (c)  $4.00\hat{i}$  V/m?

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{v} = (2000 \text{ m/s})\hat{j}$$

$$\vec{B} = (-2.5 \times 10^{-3} \text{ T})\hat{i}$$

$$\begin{aligned} \text{(a)} \quad \vec{F} &= (1.602 \times 10^{-19} \text{ C}) \left[ (4.00 \text{ V/m})\hat{k} + (2000 \text{ m/s})(-2.5 \times 10^{-3} \text{ T})(\hat{j} \times \hat{i}) \right] \\ &= (6.4 \times 10^{-19} \text{ N})\hat{k} + (8.0 \times 10^{-19} \text{ N})\hat{k} \end{aligned}$$

$\hat{j} \times \hat{i} = -\hat{k}$   
 $\sim 5.0 \text{ Tm/s}$

$$\vec{F} = (1.4 \times 10^{-18} \text{ N})\hat{k}$$

b + c  $\rightarrow \vec{F}_B$  doesn't change but  $\vec{F}_E$  does

$$\text{(b)} \quad \vec{F} = (-6.4 \times 10^{-19} \text{ N})\hat{k} + (8.0 \times 10^{-19} \text{ N})\hat{k} = (1.6 \times 10^{-19} \text{ N})\hat{k}$$

$$\text{(c)} \quad \vec{F} = (6.4 \times 10^{-19} \text{ N})\hat{i} + (8.0 \times 10^{-19} \text{ N})\hat{k}$$

### Problem 4

An electron with kinetic energy 1.20 keV circles in a plane perpendicular to a uniform magnetic field. The orbit radius is 25.0 cm. Find (a) the speed of the electron, (b) the magnetic field, (c) the frequency of circling, and (d) the period of the motion.

$$\begin{aligned} KE &= 1.20 \times 10^3 \text{ eV} \left( \frac{1.602 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right) \\ &= 1.92 \times 10^{-16} \text{ J} \end{aligned}$$

$$r = 25.0 \text{ cm} = 0.250 \text{ m}$$

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$\text{(a)} \quad KE = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2KE}{m}}$$

$$v = \sqrt{\frac{2(1.92 \times 10^{-16} \text{ J})}{9.11 \times 10^{-31} \text{ kg}}} = 2.05 \times 10^7 \text{ m/s}$$

$$\text{(b)} \quad r = \frac{mv}{qB} \rightarrow B = \frac{mv}{qr} = \frac{(9.11 \times 10^{-31} \text{ kg})(2.05 \times 10^7 \text{ m/s})}{(1.602 \times 10^{-19} \text{ C})(0.250 \text{ m})}$$

$$\text{(c)} \quad f = \frac{qB}{2\pi m}$$

$$B = 4.67 \times 10^{-4} \text{ T}$$

$$f = \frac{(1.602 \times 10^{-19} \text{ C})(4.67 \times 10^{-4} \text{ T})}{2\pi(9.11 \times 10^{-31} \text{ kg})} \rightarrow f = 1.31 \times 10^7 \text{ Hz}$$

$$\text{(d)} \quad T = \frac{1}{f} = \frac{1}{1.31 \times 10^7 \text{ Hz}} \rightarrow T = 7.66 \times 10^{-8} \text{ s}$$



### Problem 5


An electron has a velocity of  $(32\hat{i} + 40\hat{j})$  km/s as it enters a uniform magnetic field of magnitude  $B = 60\hat{i} \mu\text{T}$ . What are (a) the radius of the helical path taken by the electron and (b) the pitch of the path? (c) To an observer looking into the magnetic field region from the entrance point of the electron, does the electron spiral clockwise or counterclockwise as it moves?

$$\begin{aligned} \vec{V} &= (32 \text{ km/s})\hat{i} + (40 \text{ km/s})\hat{j} \\ \vec{B} &= (60 \times 10^{-6} \text{ T})\hat{i} \end{aligned} \quad \left. \begin{aligned} V_{\perp} &= 40 \text{ km/s} = 4.0 \times 10^4 \text{ m/s} \\ V_{\parallel} &= 32 \text{ km/s} = 3.2 \times 10^4 \text{ m/s} \end{aligned} \right\}$$

$$(a) \quad r = \frac{m V_{\perp}}{q B} = \frac{(9.11 \times 10^{-31} \text{ Kg})(4.0 \times 10^4 \text{ m/s})}{(1.602 \times 10^{-19} \text{ C})(60 \times 10^{-6} \text{ T})} \rightarrow \boxed{r = 3.8 \times 10^{-3} \text{ m}}$$

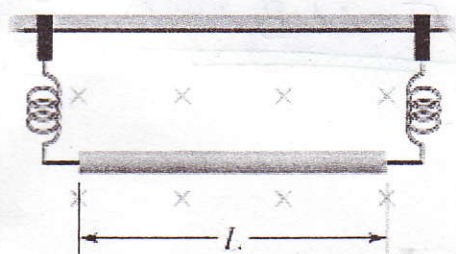
$$(b) \quad P = V_{\parallel} T = V_{\parallel} \left( \frac{2\pi m}{q B} \right) = \frac{(3.2 \times 10^4 \text{ m/s})(2\pi)(9.11 \times 10^{-31} \text{ Kg})}{(1.602 \times 10^{-19} \text{ C})(60 \times 10^{-6} \text{ T})}$$

$$\boxed{P = 1.9 \times 10^{-2} \text{ m}}$$

(c)  force on electron is initially out of the page  
clockwise

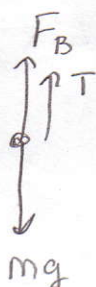
### Problem 6

A 13.0 g wire of length  $L = 62.0$  cm is suspended by a pair of flexible leads in a uniform magnetic field of magnitude 0.440 T (see the figure below). What are the (a) magnitude and (b) direction (left or right) of the current required to remove the tension in the supporting leads?



current must be to the right to produce an upward force

$$\vec{F} = i \vec{L} \times \vec{B}$$



$$\sum F_y = m a_y = 0$$

$$F_B + T - mg = 0 \quad \text{if } T = 0 \rightarrow F_B = mg$$

$$i L B \sin 90^\circ = mg$$

$$i = \frac{mg}{LB} = \frac{(0.013 \text{ Kg})(9.8 \text{ m/s}^2)}{(0.62 \text{ m})(0.440 \text{ T})} \rightarrow$$

$$\boxed{i = 0.467 \text{ A to the right}}$$



### Problem 7

A wire 50.0 cm long lying along the x axis carries a current of 0.500 A in the positive x direction, through a magnetic field  $\vec{B} = (3.00 \text{ mT})\hat{j} + (10.0 \text{ mT})\hat{k}$ . In unit vector notation, what is the magnetic force on the wire?

$$L = 0.50 \text{ m} \rightarrow \vec{L} = (0.50 \text{ m})\hat{i} \text{ since current}$$

$$i = 0.50 \text{ A}$$

$$\vec{B} = (0.0030 \text{ T})\hat{j} + (0.010 \text{ T})\hat{k}$$

$$\vec{F} = i \vec{L} \times \vec{B}$$

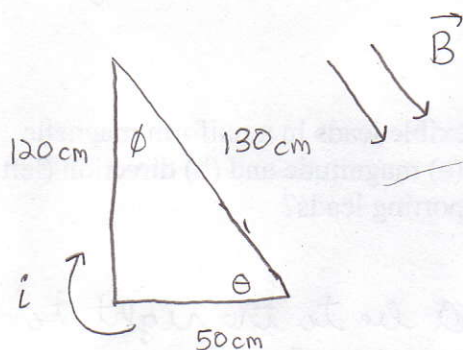
$$= (0.50 \text{ A})(0.50 \text{ m})\hat{i} \times [(0.0030 \text{ T})\hat{j} + (0.010 \text{ T})\hat{k}]$$

$$\vec{F} = (0.50 \text{ A})(0.50 \text{ m})(0.0030 \text{ T})(\hat{i} \times \hat{j}) + (0.50 \text{ A})(0.50 \text{ m})(0.010 \text{ T})(\hat{i} \times \hat{k})$$

$$\vec{F} = (-2.5 \times 10^{-3} \text{ T})\hat{j} + (7.5 \times 10^{-4} \text{ T})\hat{k}$$

### Problem 8

A single-turn current loop, carrying a current of 4.00 A, is in the shape of a right triangle with sides 50.0, 120, and 130 cm. The loop is in a uniform magnetic field of magnitude 75.0 mT whose direction is parallel to the current in the 130 cm side of the loop. What is the magnitude of the magnetic force on (a) the 130 cm side, (b) the 50.0 cm side, and (c) the 120 cm side? (d) What is the magnitude of the net force on the loop?



assume current is clockwise

$$\theta = \sin^{-1}\left(\frac{120 \text{ cm}}{130 \text{ cm}}\right) = 67.4^\circ$$

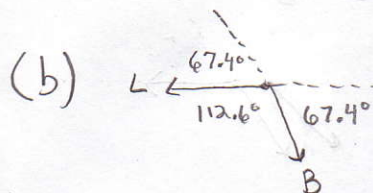
$$\phi = 90 - \theta = 22.6^\circ$$

$$\vec{F} = i \vec{L} \times \vec{B}$$

$$F = i L B \sin \theta$$

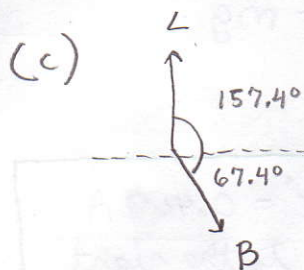
$\theta$  is angle between  $\vec{L}$  and  $\vec{B}$

$$(a) F = i L B \sin 0^\circ = \boxed{0 \text{ N}}$$



$$\vec{F} = (4.00 \text{ A})(0.50 \text{ m})(75 \times 10^{-3} \text{ T}) \sin 112.6^\circ$$

$$\vec{F} = 0.138 \text{ N out of page}$$



$$\vec{F} = (4.00 \text{ A})(1.20 \text{ m})(75 \times 10^{-3} \text{ T}) \sin 157.4^\circ$$

$$\vec{F} = 0.138 \text{ N into page}$$

$$(d) \vec{F}_{\text{net}} = 0 \text{ N} + (0.138 \text{ N})\hat{k} - (0.138 \text{ N})\hat{k} = \boxed{0 \text{ N}}$$



### Problem 9

A circular wire loop whose radius is 15.0 cm carries a current of 2.60 A. It is placed so that the normal to its plane makes an angle of  $41.0^\circ$  with a uniform magnetic field of 12.0 T. (a) Calculate the magnetic dipole moment of the loop. (b) What torque acts on the loop?

$$N = 1$$

$$r = 0.15 \text{ m}$$

$$i = 2.60 \text{ A}$$

$$\theta = 41.0^\circ$$

$$B = 12.0 \text{ T}$$

$$(a) \mu = N i A = N i (\pi r^2)$$

$$\mu = (1)(2.60 \text{ A}) \pi (0.15 \text{ m})^2 \rightarrow \boxed{\mu = 0.184 \text{ A m}^2}$$

$$(b) \vec{\tau} = \vec{\mu} \times \vec{B} \rightarrow \tau = \mu B \sin \theta$$

$$\tau = (0.184 \text{ A m}^2)(12.0 \text{ T}) \sin 41.0^\circ \rightarrow \boxed{\tau = 1.45 \text{ N m}}$$

### Problem 10

A circular coil of 160 turns has a radius of 1.90 cm. (a) Calculate the current that results in a magnetic dipole moment of  $2.30 \text{ A m}^2$ . (b) Find the maximum torque that the coil, carrying this current, can experience in a uniform 35.0 mT magnetic field.

$$N = 160$$

$$r = 0.0190 \text{ m}$$

$$\mu = 2.30 \text{ A m}^2$$

$$B = 35.0 \times 10^{-3} \text{ T}$$

$$(a) \mu = N i A = N i (\pi r^2)$$

$$i = \frac{\mu}{N \pi r^2} = \frac{2.30 \text{ A m}^2}{(160) \pi (0.0190 \text{ m})^2}$$

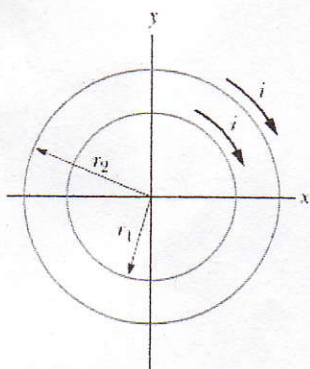
$$\boxed{i = 12.7 \text{ A}}$$

$$(b) \vec{\tau} = \vec{\mu} \times \vec{B} \rightarrow \tau = \mu B \sin \theta \quad \max \tau = \mu B$$

$$\tau_{\max} = (2.30 \text{ A m}^2)(35 \times 10^{-3} \text{ T}) = \boxed{8.05 \times 10^{-2} \text{ N m}}$$

### Problem 11

Two concentric, circular wire loops, of radii  $r_1 = 20.0 \text{ cm}$  and  $r_2 = 30.0 \text{ cm}$ , are located in the xy plane; each carries a clockwise current of 7.00 A (see the figure below). (a) Find the net magnetic dipole moment of this system. (b) Repeat for reversed current in the inner loop.



$$\mu = N i A = N i (\pi r^2) \text{ where } \vec{\mu} \text{ is in direction of normal}$$

(a) dipole moment from both wires point into page

$$\mu_{\text{net}} = N i \pi r_1^2 + N i \pi r_2^2 = i \pi (r_1^2 + r_2^2)$$

$$\mu_{\text{net}} = (7.00 \text{ A}) \pi [(0.20 \text{ m})^2 + (0.30 \text{ m})^2] = \boxed{2.86 \text{ A m}^2 \text{ (into page)}}$$

(b) with current on the inner loop reversed, dipole moments point in opposite directions

$$\mu_{\text{net}} = i \pi r_2^2 - i \pi r_1^2 = i \pi (r_2^2 - r_1^2)$$

$$= (7.00 \text{ A}) \pi [(0.30 \text{ m})^2 - (0.20 \text{ m})^2] = \boxed{1.10 \text{ A m}^2 \text{ (into page)}}$$