Answer Key

In many cases you may think of an electrical circuit as composed of finite straight-line and circular-arc segments, each of which produces a magnetic field. You can then calculate the field produced by each segment and vectorially sum the individual fields to find the total field. Use the result given in Problem 11 if you need to find the field on the perpendicular bisector of a finite straight wire. To find the field at some other point you sill need to integrate the Biot-Savart law. Use the procedure given in Section 1 for an infinite wire but replace the limits of integration with finite values. Use $B = \mu_0 i\phi/4\pi R$ for the field of a circular arc at its center of curvature.

Some problems ask you to use what you learned in the last chapter to calculate the force that one wire exerts on another.

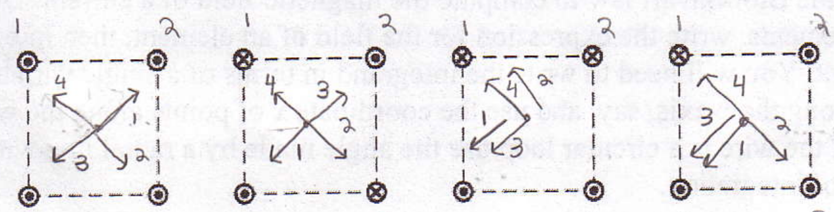
Ampere's law problems take three forms. The most straightforward give the currents and ask you to find the value of $\oint \vec{B}\cdot d\vec{s}$ around a given path. You must pay attention to the directions of the currents as they pierce the plane of the path and carefully observe which currents are encircled by the path and which are not. Other problems ask you to use Ampere's law to calculate the magnetic field. Carefully choose the Amperian path you will use and pay attention to the evaluation of the Ampere's law integral in terms of the unknown field. Still other problems give you the magnetic field as a function of position and ask for the current through a given region. Carry out the line integral of the tangential component of the field around the boundary of the region and equate the result to $\mu_0 i$, then solve for i.

You should know how to compute the magnetic fields of some special current configurations (in addition to a long straight wire): a solenoid, a toroid, and a magnetic dipole. The field of an ideal solenoid is given by $B = \mu_0 ni$ inside the solenoid and by $B = 0$ outside. The field inside is along the cylinder axis. The field of a toroid is given by $B = \mu_0 iN/2\pi r$ at a point inside, a distance r from the center. The field inside the hole and outside the toroid is zero. The field lines inside are circles that at concentric with the toroid. The field of a magnetic dipole is given by $\vec{B} = \mu_0\vec{\mu}/2\pi z^3$ at a point on the axis defined by the direction of the dipole moment, a distance z from the dipole. You should recall from the last chapter how to find the dipole moment of a current loop (both magnitude and direction) and how to compute the torque of a uniform magnetic field on a dipole.

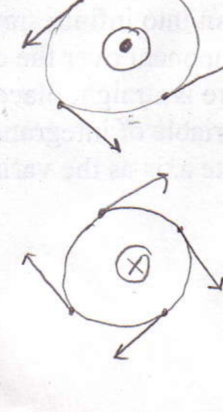
Questions and Example Problems from Chapter 29

Question 1

The figure below shows four arrangements in which long parallel wires carry equal currents directly into or out of the page at the corners of identical squares. Rank the arrangements according to the magnitude of the net magnetic field at the center of the square, greatest first.



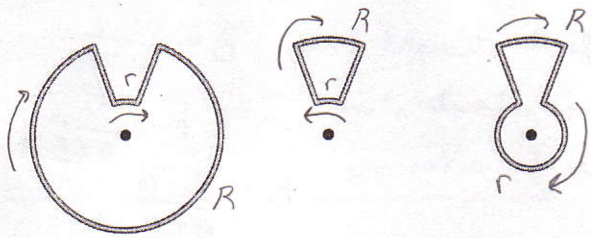
(a) (b) (c) (d)



$\boxed{\text{c, d, a\&b tie (zero)}}$

Question 2

The Figure below shows three circuits, each consisting of two radial lengths and two concentric arcs, one of radius r and the other of radius R > r. The circuits have the same current through them and the same angle between the two radial lengths. Rank the circuits according to the magnitude of the net magnetic field at the center, greatest first.



(a) (b) (c)

$B = \frac{\mu_0 i \phi}{4\pi R}$ center of circular arc

RHR → curl fingers in direction of conventional current, thumb points in direction of $\vec{B}$ at center of arc

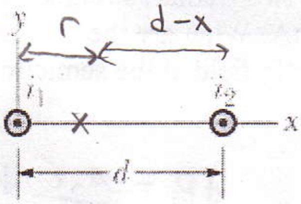
r → into | r → out of page | r → into page

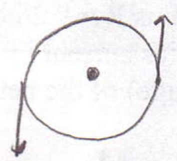
R → into | R → into page | R → into page

$\boxed{c, a, b}$

Problem 1

In the figure below, two long straight wires at separation d = 16.0 cm carry current i_1 = 3.61 mA and $i_2 = 3.00 i_1$ out of the page. (a) At what point on the x axis shown is the net magnetic field due to the currents equal to zero? (b) If the two currents are doubled, is the point of zero magnetic field shift toward wire 1, shifted toward wire 2, or unchanged?





$\vec{B}_{net}$ can only equal zero between the two wires; $\vec{B}_1$ points upwards and $\vec{B}_2$ points downward

$$\vec{B}_{net} = B_1 - B_2 = 0 \longrightarrow B_1 = B_2$$

$$\frac{\mu_0 i_1}{2\pi r_1} = \frac{\mu_0 i_2}{2\pi r_2}$$

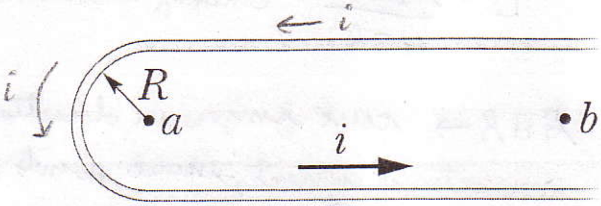
$$\frac{\mu_0 i_1}{2\pi r} = \frac{\mu_0 (3.00 i_1)}{2\pi (d-r)} \longrightarrow \frac{1}{r} = \frac{3}{d-r} \longrightarrow 3r = d - r$$

$$r = d/4 = \frac{16.0\text{cm}}{4} = 4.0\text{ cm} \qquad \boxed{\vec{B} = 0 \text{ at } r = 4.00\text{cm}}$$

(b) as long as $i_2 = 3.00 i_1$, the location of $B_{net} = 0$ doesn't change

Problem 2

In the figure below, a current i = 10 A is set up in a long hairpin conductor formed by bending a wire into a semicircle of radius R = 5.0 mm. Point b is midway between the straight sections and so distant from the semicircle that each section can be approximated as being an infinite wire. What are the (a) magnitude and (b) direction (into or out of the page) of B at point a and the (c) magnitude and (d) direction of B at point b.



infinite straight wire $B = \frac{\mu_0 i}{2\pi R}$

semi-infinite straight wire $B = \frac{\mu_0 i}{4\pi R}$

center of circular arc $B = \frac{\mu_0 i \phi}{4\pi R}$

(a) + (b) $\vec{B}$ from the two wire and the circular arc

point out of page : $\vec{B}_{net} = 2\left(\frac{\mu_0 i}{4\pi R}\right) + \frac{\mu_0 i \phi}{4\pi R} = \frac{\mu_0 i}{2\pi R} + \frac{\mu_0 i \pi}{4\pi R}$

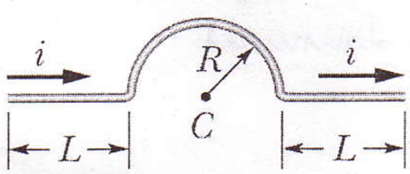
$\vec{B}_{net} = \frac{\mu_0 i}{2R}\left[\frac{1}{\pi} + \frac{1}{2}\right] = \frac{(4\pi \times 10^{-7}\,Tm/A)(10A)}{2(5.00\times10^{-3}m)}\left[\frac{1}{\pi}+\frac{1}{2}\right] = \boxed{1.0\times10^{-3}\,T \text{ out of page}}$

(c) + (d) we can treat wires as infinite

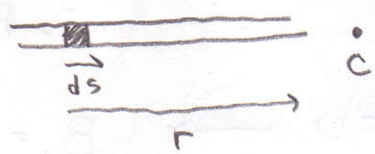
$\vec{B}_{net} = 2\left(\frac{\mu_0 i}{2\pi R}\right) = \frac{\mu_0 i}{\pi R} = \frac{(4\pi\times10^{-7}\,Tm/A)(10A)}{\pi(5.00\times10^{-3}m)} = \boxed{8.0\times10^{-4}\,T \text{ out of page}}$

Problem 3

In the figure below, a wire forms a semicircle of radius R = 9.26 cm and two (radial) straight segments each of length L = 13.1 cm. The wire carries current i = 34.8 mA. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at the semicircle's center of curvature C?



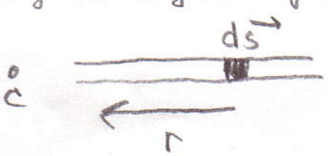
left straight segment :



$dB = \frac{\mu_0 i}{4\pi}\frac{ds \sin 0°}{r^2} = 0$

$B = 0$

right straight segment :



$dB = \frac{\mu_0}{4\pi}\frac{i\,ds \sin 180°}{r^2} = 0 \longrightarrow B = 0$

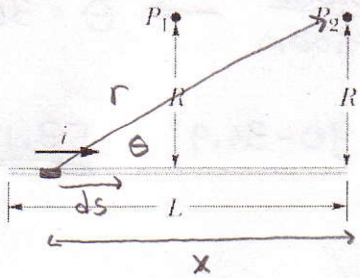
$\Rightarrow$ at the center of a circular arc $\longrightarrow B = \frac{\mu_0 i \phi}{4\pi R}$ $\phi = \pi$ radians

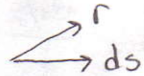
$B = \frac{\mu_0 i \pi}{4\pi R} = \frac{\mu_0 i}{4R} = \frac{(4\pi\times10^{-7}\,Tm/A)(34.8\times10^{-3}A)}{4(0.0926m)} = 1.18\times10^{-7}\,T$

$\boxed{\vec{B} = 1.18\times10^{-7}\,T \text{ into the page}}$

Problem 4

In the figure below, point P_2 is at perpendicular distance R = 25.1 cm from one end of straight wire of length L = 13.6 cm carrying current i= 0.693 A. (Note that the wire is *not* long.) What is the magnitude of the magnetic field at P_2?





$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\, d\vec{s} \times \vec{r}}{r^3}$$

$\downarrow$ $d\vec{B}$ from all segments of wire point out of page

$$dB = \frac{\mu_0}{4\pi} \frac{i\, ds \sin\theta}{r^2} \longrightarrow B = \frac{\mu_0 i}{4\pi} \int \frac{ds \sin\theta}{r^2}$$

$ds \to dx$

$\sin\theta = \frac{R}{(R^2+x^2)^{1/2}}$

$r^2 = R^2 + x^2$

$$B = \frac{\mu_0 i}{4\pi} \int \frac{dx\, R}{(R^2+x^2)^{3/2}} \longrightarrow B = \frac{\mu_0 i R}{4\pi} \int_{-L}^{0} \frac{dx}{(x^2+R^2)^{3/2}}$$

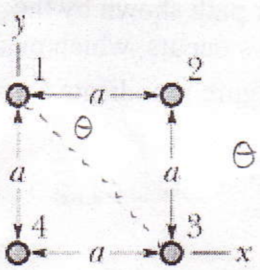
$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}} \qquad B = \frac{\mu_0 i R}{4\pi} \frac{x}{R^2(x^2+R^2)^{1/2}} \Big|_{-L}^{0}$$

$$B = \frac{\mu_0 i}{4\pi R} \frac{L}{(L^2+R^2)^{1/2}}$$

$\left.\begin{array}{l} L = 0.136\text{m} \\ R = 0.251\text{m} \\ i = 0.693 \end{array}\right\}$ $\boxed{\vec{B} = 1.32\times10^{-7}\,T \text{ out of page}}$

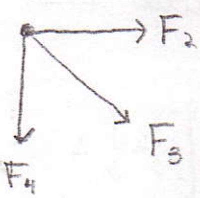
Problem 5

In the figure below, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length a = 8.50 cm. Each wire carries 15.0 A, and all the currents are out of the page. In unit-vector notation, what is the net magnetic field force *per meter of wire length* on wire 1?



$\Rightarrow$ the force between 2 parallel currents is given by $F = \frac{\mu_0 L i_1 i_2}{2\pi d}$; parallel currents attract, antiparallel currents repel

$\theta = 45°$



$$\sum F_x = F_2 + F_3 \cos\theta$$

$$\sum F_y = -F_4 - F_3 \sin\theta$$

$i_1 = i_2 = i_3 = i_4 = i$

$$\sum F_x = \frac{\mu_0 L i^2}{2\pi a} + \frac{\mu_0 L i^2}{2\pi(\sqrt{2}a)}\underbrace{\left(\frac{\sqrt{2}}{2}\right)}_{=\cos 45°} = \frac{\mu_0 L i^2}{2\pi a}\left(1+\tfrac{1}{2}\right) = \frac{3\mu_0 L i^2}{4\pi a}$$

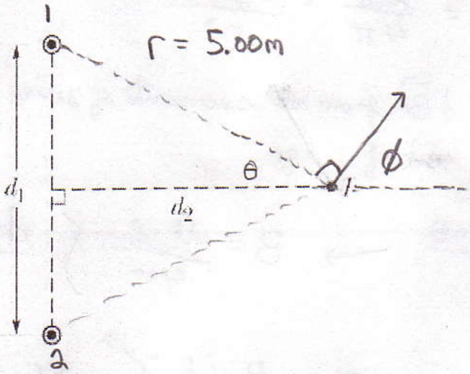
$$\sum F_x / L = \frac{3\mu_0 i^2}{4\pi a} = \frac{3(4\pi\times10^{-7}\,\text{Tm/A})(15.0\text{A})^2}{4\pi(0.085\text{m})} = \underline{7.94\times10^{-4}\,\text{N/m}}$$

* note: $\frac{\sum F_y}{L}$ has the same magnitude so:

$$\boxed{\frac{\vec{F}_1}{L} = (7.94\times10^{-4}\,\text{N/m})\hat{i} - (7.94\times10^{-4}\,\text{N/m})\hat{j}}$$

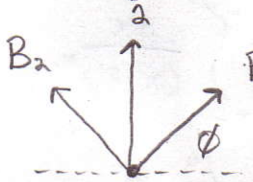
Problem 6

The figure below shows two very long straight wires (in cross section) that each carry a current of 4.00 A directly out of the page. Distance $d_1 = 6.00$ m and distance $d_2 = 4.00$ m. What is the magnitude of the net magnetic field at point P, which lies on a perpendicular bisector of the wires?



$$\tan\theta = \frac{d_1/2}{d_2} = \frac{3.00\text{m}}{4.00\text{m}} \longrightarrow \theta = 36.9^\circ$$

$$\phi = 90 - \theta = 90 - 36.9^\circ = 53.1^\circ$$



$$B_{\text{long wire}} = \frac{\mu_0 i}{2\pi r}$$

$$B_{net} = 2\left[\frac{\mu_0 i}{2\pi r}\right]\sin\phi = \frac{\mu_0 i}{\pi r}\sin\phi$$

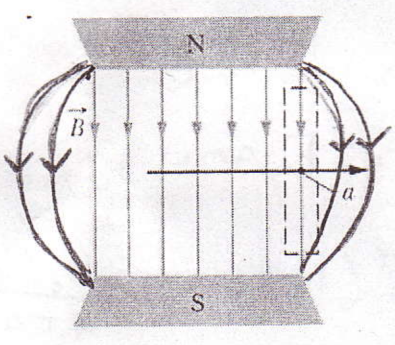
$$\vec{B}_{net} = B_1 \sin\phi + B_2 \sin\phi$$
$$= 2B\sin\phi$$

$$B_{net} = \frac{(4\pi\times10^{-7}\,\text{Tm/A})(4.00\text{A})}{\pi\,(5.00\text{m})^2}\sin 53.1^\circ$$

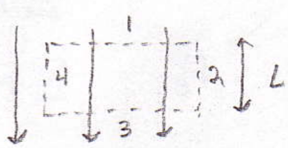
$$\boxed{\vec{B}_{net} = 5.12\times10^{-8}\,\text{T upward}}$$

Problem 7

Show that a uniform magnetic field cannot drop abruptly to zero (as is suggested by the lack of field lines to the right of point a in the figure below) as one moves perpendicular to $\vec{B}$, say along the horizontal arrow in the figure. (Hint: Apply Ampere's law to the rectangular path shown by the dashed lines.) In actual magnets "fringing" of the magnetic field lines always occurs, which means that $\vec{B}$ approaches zero in a gradual manner. Modify the field lines in the figure to indicate a more realistic situation.



Ampere's law $\longrightarrow \oint \vec{B}\cdot d\vec{s} = \mu_0 i_{encl}$



$$\oint \vec{B}\cdot d\vec{s} = \underbrace{\int \vec{B}_1\cdot d\vec{s}_1}_{\substack{=0 \\ (B_1 \perp ds_1)}} + \underbrace{\int \vec{B}_2\cdot d\vec{s}_2}_{\substack{=0 \\ (B_2 = 0)}} + \underbrace{\int \vec{B}_3\cdot d\vec{s}_3}_{\substack{=0 \\ (B_3 \perp ds_3)}} + \int \vec{B}_4\cdot d\vec{s}_4$$

$$\oint \vec{B}\cdot d\vec{s} = \int \vec{B}_4\cdot d\vec{s}_4$$
$$= \int B_4\, ds_4 \cos 0^\circ = B_4 \int ds_4 = BL$$

$\oint \vec{B}\cdot d\vec{s} = BL = \mu_0 i_{encl} \longrightarrow$ $i_{encl} = 0$ so Ampere's law is contradicted

Problem 8

A solenoid 1.30 m long and 2.60 cm in diameter carries a current of 18.0 A. The magnetic field inside the solenoid is 23.0 mT. Find the length of the wire forming the solenoid.

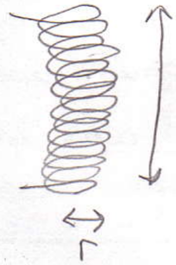
$L = 1.30\,m$

$r = d/2 = 1.30\,cm$

$= 1.30\times10^{-2}\,m$

$i = 18.0\,A$

$B = 23.0\times10^{-3}\,T$



for an ideal solenoid:

$B = \mu_0 i n$

$n = \frac{\#\ turns}{length} = \frac{N}{L} = ?$

$$n = \frac{B}{\mu_0 i} = \frac{23.0\times10^{-3}\,T}{(4\pi\times10^{-7}\,Tm/A)(18.0A)}$$

$n = 1.02\times10^{3}$ turns/m

total length wire = # turns (loops) × length per loop

$= (nL)\times(2\pi r)$

$= (1.02\times10^{3}\,turns/m)(1.30m)(2\pi)(1.30\times10^{-2}\,m)$

$= \boxed{108\,m}$

Problem 9

A long solenoid has 100 turns/cm and carries current *i*. An electron moves within the solenoid in a circle of radius 2.30 cm perpendicular to the solenoid axis. The speed of the electron is 0.0460c (c = speed of light). Find the current *i* in the solenoid.

from Ch.28 $\longrightarrow$ $r = \frac{mv}{qB}$

for a solenoid $\longrightarrow$ $B = \mu_0 i n$

$$r = \frac{mv}{q(\mu_0 i n)}$$

$$i = \frac{mv}{q\mu_0 n r}$$

$m = 9.11\times10^{-31}\,Kg$

$v = 0.0460(3.0\times10^{8}\,m/s)$

$= 1.38\times10^{7}\,m/s$

$q = 1.602\times10^{-19}\,C$

$\mu_0 = 4\pi\times10^{-7}\,Tm/A$

$n = 100\,\frac{turns}{cm}\times\frac{10^{2}\,cm}{1m} = 10^{4}$ turns/m

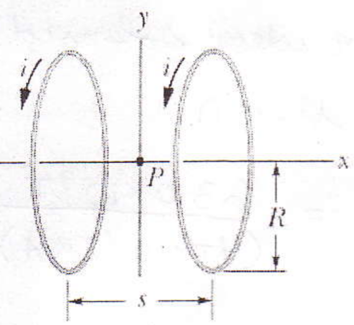
$r = 0.0230\,m$

$$i = \frac{(9.11\times10^{-31}\,Kg)(1.38\times10^{7}\,m/s)}{(1.602\times10^{-19}\,C)(4\pi\times10^{-7}\,Tm/A)(10^{4}\,\frac{turns}{m})(0.0230\,m)}$$

$$\boxed{i = 0.272\,A}$$

Problem 10

The figure below shows an arrangement known as a Helmholtz coil. It consists of two circular coaxial coils, each of 200 turns and radius R = 25.0 cm, separated by a distance s = R. The two coils carry equal currents i = 12.2 mA in the same direction. Find the magnitude of the net magnetic field at P, midway between the coils.



⇒ the magnetic field a distance z along the central axis from a coil of N turns is given by: $B(z) = \frac{N \mu_0 i R^2}{2(R^2+z^2)^{3/2}}$

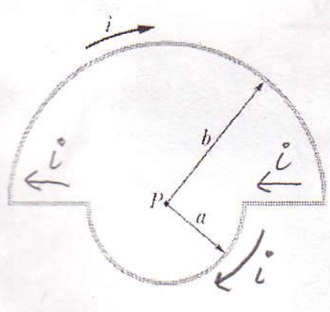
⇒ at point P a distance $z = R/2$ from each coil, the magnetic field from each coil points in the +x-direction

$$B_{net} = 2\left[\frac{N\mu_0 i R^2}{2[R^2+(R/2)^2]^{3/2}}\right] = \frac{N\mu_0 i R^2}{(5R^2/4)^{3/2}}$$

$$B = \frac{(200)(4\pi\times10^{-7}\,Tm/A)(12.2\times10^{-3}A)(0.25m)^2}{[5(0.25m)^2/4]^{3/2}} = \boxed{(8.78\times10^{-6}T)\hat{i}}$$

Problem 11

In the figure below, current *i* = 56.2 mA is set up in a loop having two radial lengths and two semicircles of radii a = 5.72 cm and b = 9.36 cm with a common center P. What are the (a) magnitude and (b) direction (into or out of the page) of the magnetic field at P and the (c) magnitude and (d) direction of the loop's magnetic dipole moment?



(a) $\vec{B} = 0$ from straight segments at P

$B = \frac{\mu_0 i \phi}{4\pi R}$ at center of circular arc

$\vec{B}$ from both arcs point into page

$$B_{net} = \frac{\mu_0 i \pi}{4\pi b} + \frac{\mu_0 i \pi}{4\pi a} = \frac{\mu_0 i}{4}\left(\frac{1}{b}+\frac{1}{a}\right)$$

$$B_{net} = \frac{(4\pi\times10^{-7}Tm/A)(56.2\times10^{-3}A)}{4}\left[\frac{1}{9.36\times10^{-2}m} + \frac{1}{5.72\times10^{-2}m}\right]$$

$$\boxed{B_{net} = 4.97\times10^{-7}T}$$

(b) $\vec{B}$ points into the page