Chapter 22

Electric Fields

22.2 The Electric Field:

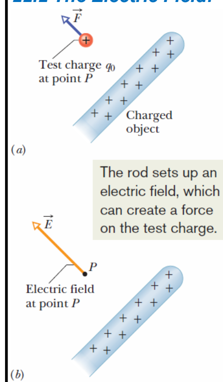


Fig. 22-1 (a) A positive test charge q_0 placed at point P near a charged object. An electrostatic force $\vec{F}$ acts on the test charge. (b) The electric field $\vec{E}$ at point P produced by the charged object.

The Electric Field is a *vector field*.

The electric field, ***E***, consists of a distribution of vectors, one for each point in the region around a charged object, such as a charged rod.

We can define the electric field at some point near the charged object, such as point *P* in Fig. 22-1a, as follows:

•A positive test charge q_0, placed at the point will experience an electrostatic force, **F**.

•The electric field at point *P* due to the charged object is defined as the electric field, **E**, at that point:

$$\vec{E} = \frac{\vec{F}}{q_0} \quad \text{(electric field).}$$

The SI unit for the electric field is the newton per coulomb (N/C).

22.2 The Electric Field:

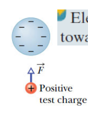
Table 22-1

Some Electric Fields

Field Location or Situation	Value (N/C)
At the surface of a uranium nucleus	3×10^{21}
Within a hydrogen atom, at a radius of 5.29×10^{-11} m	5×10^{11}
Electric breakdown occurs in air	3×10^{6}
Near the charged drum of a photocopier	10^{5}
Near a charged comb	10^{3}
In the lower atmosphere	10^{2}
Inside the copper wire of household circuits	10^{-2}

22.3 Electric Field Lines:

Electric field lines extend away from positive charge (where they originate) and toward negative charge (where they terminate).



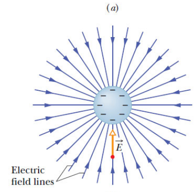


Fig. 22-2 (a) The electrostatic force $\vec{F}$ acting on a positive test charge near a sphere of uniform negative charge. (b) The electric field vector $\vec{E}$ at the location of the test charge, and the electric field lines in the space near the sphere. The field lines extend *toward* the negatively charged sphere. (They originate on distant positive charges.)

- At any point, the direction of a straight field line or the direction of the tangent to a curved field line gives the direction of at that point.
- The field lines are drawn so that the number of lines per unit area, measured in a plane that is perpendicular to the lines, is proportional to the magnitude of ***E***.

  Thus, ***E*** is large where field lines are close together and small where they are far apart.

22.3 Electric Field Lines:

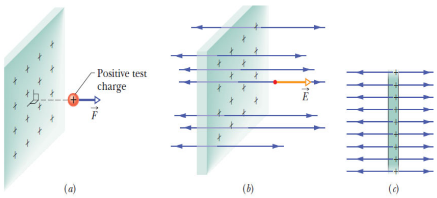
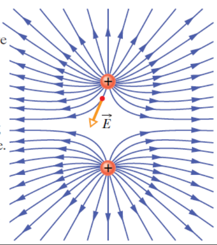


Fig. 22-3 (*a*) The electrostatic force $\vec{F}$ on a positive test charge near a very large, nonconducting sheet with uniformly distributed positive charge on one side. (*b*) The electric field vector $\vec{E}$ at the location of the test charge, and the electric field lines in the space near the sheet. The field lines extend *away from* the positively charged sheet. (*c*) Side view of (*b*).

Fig. 22-4 Field lines for two equal positive point charges. The charges repel each other. (The lines terminate on distant negative charges.) To "see" the actual three-dimensional pattern of field lines, mentally rotate the pattern shown here about an axis passing through both charges in the plane of the page. The three-dimensional pattern and the electric field it represents are said to have *rotational symmetry* about that axis. The electric field vector at one point is shown; note that it is tangent to the field line through that point.



22.4 The Electric Field due to a Point:

To find the electric field due to a point charge q (or charged particle) at any point a distance r from the point charge, we put a positive test charge q_0 at that point.

The direction of E is directly away from the point charge if q is positive, and directly toward the point charge if q is negative. The electric field vector is:

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \quad \text{(point charge).}$$

The net, or resultant, electric field due to more than one point charge can be found by the superposition principle. If we place a positive test charge q_0 near n point charges $q_1, q_2, \ldots, q_n$, then, the net force, $\mathbf{F_o}$, from the n point charges acting on the test charge is

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \cdots + \vec{F}_{0n}.$$

The net electric field at the position of the test charge is

$$\vec{E} = \frac{\vec{F}_0}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \cdots + \frac{\vec{F}_{0n}}{q_0} = \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n.$$

Example, The net electric field due to three charges:

Figure 22-7*a* shows three particles with charges $q_1 = +2Q$, $q_2 = -2Q$, and $q_3 = -4Q$, each a distance d from the origin. What net electric field $\vec{E}$ is produced at the origin?

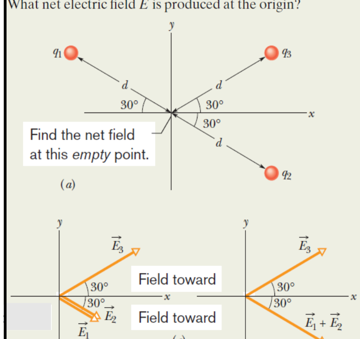


Fig. 22-7 (*a*) Three particles with charges q_1, q_2, and q_3 are at the same distance d from the origin. (*b*) The electric field vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$, at the origin due to the three particles. (*c*) The electric field vector $\vec{E}_3$ and the vector sum $\vec{E}_1 + \vec{E}_2$ at the origin.

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}.$$

$$E_1 + E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} + \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}.$$

From the symmetry of Fig. 22-7c, we realize that the equal y components of our two vectors cancel and the equal x components add.

Thus, the net electric field at the origin is in the positive direction of the x axis and has the magnitude

$$E = 2E_{3x} = 2E_3\cos 30^\circ = (2)\frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}(0.866) = \frac{6.93Q}{4\pi\varepsilon_0 d^2}.$$

22.5 The Electric Field due to an Electric Dipole:

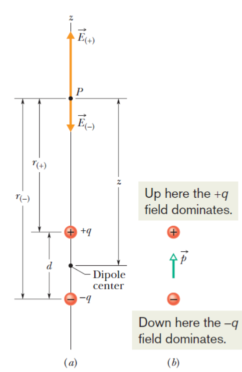
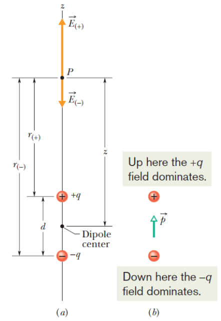


Fig. 22-8 (*a*) An electric dipole. The electric field vectors $\vec{E}_{(+)}$ and $\vec{E}_{(-)}$ at point P on the dipole axis result from the dipole's two charges. Point P is at distances $r_{(+)}$ and $r_{(-)}$ from the individual charges that make up the dipole. (*b*) The dipole moment $\vec{p}$ of the dipole points from the negative charge to the positive charge.

22.5 The Electric Field due to an Electric Dipole:

From symmetry, the electric field E at point P—and also the fields E_+ and E_- due to the separate charges that make up the dipole—must lie along the dipole axis, which we have taken to be a z axis. From the superposition principle for electric fields, the magnitude E of the electric field at P is



$$E = E_{(+)} - E_{(-)} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(-)}^2} = \frac{q}{4\pi\varepsilon_0(z-\frac{1}{2}d)^2} - \frac{q}{4\pi\varepsilon_0(z+\frac{1}{2}d)^2}.$$

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left(\frac{1}{\left(1-\frac{d}{2z}\right)^2} - \frac{1}{\left(1+\frac{d}{2z}\right)^2}\right).$$

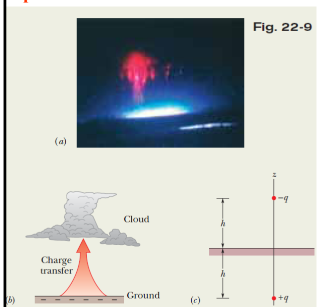
$$E = \frac{q}{4\pi\varepsilon_0 z^2}\,\frac{2d/z}{\left(1-\left(\frac{d}{2z}\right)^2\right)^2} = \frac{q}{2\pi\varepsilon_0 z^3}\,\frac{d}{\left(1-\left(\frac{d}{2z}\right)^2\right)^2}.$$

$d/2z \ll 1 \longrightarrow$ $$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}.$$

The product qd, which involves the two intrinsic properties q and d of the dipole, is the magnitude p of a vector quantity known as the ***electric dipole moment*** of the dipole.

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3} \quad \text{(electric dipole).}$$

Example, Electric Dipole and Atmospheric Sprites:



Sprites (Fig. 22-9*a*) are huge flashes that occur far above a large thunderstorm. They are still not well understood but are believed to be produced when especially powerful lightning occurs between the ground and storm clouds, particularly when the lightning transfers a huge amount of negative charge *-q* from the ground to the base of the clouds (Fig. 22-9*b*).

We can model the electric field due to the charges in the clouds and the ground by assuming a vertical electric dipole that has charge *-q* at cloud height *h* and charge *+q* at below-ground depth *h (Fig.* 22-9*c).* If $q = 200\ C$ and $h = 6.0$ km, what is the magnitude of the dipole's electric field at altitude $z_1 = 30\ km$ somewhat above the clouds and altitude $z_2 = 60$ km somewhat above the stratosphere?

$$E = \frac{1}{2\pi\varepsilon_0}\frac{q(2h)}{z^3},$$

where $2h$ is the separation between $-q$ and $+q$ in Fig. 22-9c. For the electric field at altitude $z_1 = 30$ km, we find

$$E = \frac{1}{2\pi\varepsilon_0}\frac{(200\ \text{C})(2)(6.0\times10^3\ \text{m})}{(30\times10^3\ \text{m})^3} = 1.6\times10^3\ \text{N/C}. \quad \text{(Answer)}$$

Similarly, for altitude $z_2 = 60$ km, we find

$$E = 2.0\times10^2\ \text{N/C}. \quad \text{(Answer)}$$

22.6 The Electric Field due to a Continuous Charge:

When we deal with continuous charge distributions, it is most convenient to express the charge on an object as a *charge density rather than as a total charge.* For a line of charge, for example, we would report the *linear charge density* (or charge per unit length) λ, whose SI unit is the coulomb per meter.

Table 22-2 shows the other charge densities we shall be using.

Table 22-2
Some Measures of Electric Charge

Name	Symbol	SI Unit
Charge	q	C
Linear charge density	λ	C/m
Surface charge density	σ	C/m^2
Volume charge density	ρ	C/m^3

22.6 The Electric Field due to a Line Charge:

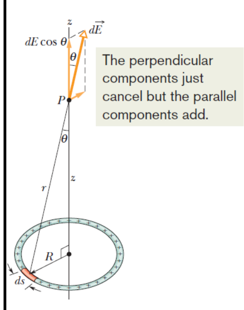


Fig. 22-10 A ring of uniform positive charge. A differential element of charge occupies a length *ds* (greatly exaggerated for clarity). This element sets up an electric field $d\vec{E}$ at point *P*. The component of $d\vec{E}$ along the central axis of the ring is $dE\cos\theta$.

We can mentally divide the ring into differential elements of charge that are so small that they are like point charges, and then we can apply the definition to each of them.

Next, we can add the electric fields set up at *P by all the differential elements.* The vector sum of the fields gives us the field set up at *P by the ring.*

Let *ds* be the (arc) length of any differential element of the ring. Since λ is the charge per unit (arc) length, the element has a charge of magnitude $dq = \lambda\, ds.$

This differential charge sets up a differential electric field ***dE*** at point *P,* a distance *r* from the element.

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{(z^2+R^2)}.$$

All the ***dE*** vectors have components parallel and perpendicular to the central axis; the perpendicular components are identical in magnitude but point in different directions.

The parallel components are $dE\cos\theta = \dfrac{z\lambda}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}\,ds.$

Finally, for the entire ring,

$$E = \int dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}\int_0^{2\pi R} ds = \frac{z\lambda(2\pi R)}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}.$$

Example, Electric Field of a Charged Circular Rod

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}$$

Figure 22-11*a* shows a plastic rod having a uniformly distributed charge $-Q$. The rod has been bent in a 120° circular arc of radius r. We place coordinate axes such that the axis of symmetry of the rod lies along the x axis and the origin is at the center of curvature P of the rod. In terms of Q and r, what is the electric field $\vec{E}$ due to the rod at point P?

This negatively charged rod is obviously not a particle.

These x components add. Our job is to add all such components.

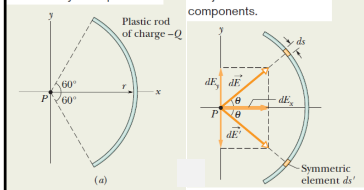


Fig. 22-11 (a) A plastic rod of charge Q is a circular section of radius r and central angle 120°; point P is the center of curvature of the rod. (b) The field components from symmetric elements from the rod.

$$\lambda = \frac{\text{charge}}{\text{length}} = \frac{Q}{2\pi r/3} = \frac{0.477Q}{r}. \qquad dq = \lambda\, ds.$$

Our element has a symmetrically located (mirror image) element *ds* in the bottom half of the rod.

If we resolve the electric field vectors of *ds* and *ds'* into x and y components as shown in we see that their y components cancel (because they have equal magnitudes and are in opposite directions). We also see that their x components have equal magnitudes and are in the same direction.

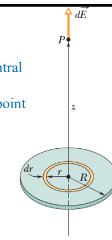
$$E = \int dE_x = \int_{-60^\circ}^{60^\circ} \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r^2}\cos\theta\, r\, d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r}\int_{-60^\circ}^{60^\circ}\cos\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 r}\Big[\sin\theta\Big]_{-60^\circ}^{60^\circ}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r}[\sin 60^\circ - \sin(-60^\circ)]$$

$$= \frac{1.73\lambda}{4\pi\varepsilon_0 r}. = \frac{0.83Q}{4\pi\varepsilon_0 r^2}.$$

22.6 The Electric Field due to a Charged Disk:



We need to find the electric field at point *P*, a distance z from the disk along its central axis.

Divide the disk into concentric flat rings and then to calculate the electric field at point *P* by adding up (that is, by integrating) the contributions of all the rings. The figure shows one such ring, with radius *r* and radial width *dr*. If σ is the charge per unit area, the charge on the ring is

$$dq = \sigma\, dA = \sigma(2\pi r\, dr),$$

$$dE = \frac{z\sigma 2\pi r\, dr}{4\pi\varepsilon_0(z^2+r^2)^{3/2}} = \frac{\sigma z}{4\varepsilon_0}\frac{2r\, dr}{(z^2+r^2)^{3/2}}$$

We can now find *E* by integrating *dE* over the surface of the disk— that is, by integrating with respect to the variable *r* from *r =0 to r =R.*

$$E = \int dE = \frac{\sigma z}{4\varepsilon_0}\int_0^R (z^2+r^2)^{-3/2}(2r)\, dr. = \frac{\sigma z}{4\varepsilon_0}\left[\frac{(z^2+r^2)^{-1/2}}{-\frac{1}{2}}\right]_0^R.$$

$$\Rightarrow \quad E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right) \quad \text{(charged disk)}$$

If we let $R \to \infty$, while keeping z finite, the second term in the parentheses in the above equation approaches zero, and this equation reduces to

$$E = \frac{\sigma}{2\varepsilon_0} \quad \text{(infinite sheet).}$$

22.8: A Point Charge in an Electric Field

The electrostatic force F acting on a charged particle located in an external electric field $\vec{E}$ has the direction of $\vec{E}$ if the charge q of the particle is positive and has the opposite direction if q is negative.

$$\vec{F} = q\vec{E},$$

When a charged particle, of charge *q*, is in an electric field, ***E,*** set up by other stationary or slowly moving charges, an electrostatic force, ***F,*** acts on the charged particle as given by the above equation.

22.8: A Point Charge in an Electric Field:

Measuring the Elementary Charge

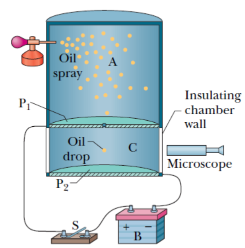


Fig. 22-14 The Millikan oil-drop apparatus for measuring the elementary charge e. When a charged oil drop drifted into chamber C through the hole in plate P_1, its motion could be controlled by closing and opening switch S and thereby setting up or eliminating an electric field in chamber C. The microscope was used to view the drop, to permit timing of its motion.

Ink-Jet Printing

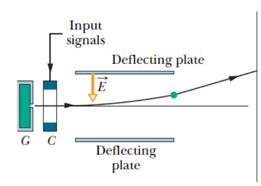


Fig. 22-15 Ink-jet printer. Drops shot from generator G receive a charge in charging unit C. An input signal from a computer controls the charge and thus the effect of field $\vec{E}$ where the drop lands on the paper.

Example, Motion of a Charged Particle in an Electric Field

Figure 22-17 shows the deflecting plates of an ink-jet printer, with superimposed coordinate axes. An ink drop with a mass m of 1.3×10^{-10} kg and a negative charge of magnitude $Q = 1.5 \times 10^{-13}$ C enters the region between the plates, initially moving along the x axis with speed $v_x = 18$ m/s. The length L of each plate is 1.6 cm. The plates are charged and thus produce an electric field at all points between them. Assume that field $\vec{E}$ is downward directed, is uniform, and has a magnitude of 1.4×10^6 N/C. What is the vertical deflection of the drop at the far edge of the plates? (The gravitational force on the drop is small relative to the electrostatic force acting on the drop and can be neglected.)

KEY IDEA

The drop is negatively charged and the electric field is directed *downward*. From Eq. 22-28, a constant electrostatic force of magnitude QE acts *upward* on the charged drop. Thus, as the drop travels parallel to the x axis at constant speed v_x, it accelerates upward with some constant acceleration a_y.

Calculations: Applying Newton's second law ($F = ma$) for components along the y axis, we find that

$$a_y = \frac{F}{m} = \frac{QE}{m}. \qquad (22\text{-}30)$$

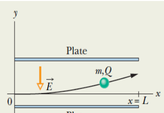


Fig. 22-17 An ink drop of mass m and charge magnitude Q is deflected in the electric field of an ink-jet printer.

Let t represent the time required for the drop to pass through the region between the plates. During t the vertical and horizontal displacements of the drop are

$$y = \tfrac{1}{2}a_y t^2 \quad \text{and} \quad L = v_x t, \qquad (22\text{-}31)$$

respectively. Eliminating t between these two equations and substituting Eq. 22-30 for a_y, we find

$$y = \frac{QEL^2}{2mv_x^2}$$

$$= \frac{(1.5 \times 10^{-13}\ \text{C})(1.4 \times 10^6\ \text{N/C})(1.6 \times 10^{-2}\ \text{m})^2}{(2)(1.3 \times 10^{-10}\ \text{kg})(18\ \text{m/s})^2}$$

$$= 6.4 \times 10^{-4}\ \text{m}$$

$$= 0.64\ \text{mm}. \qquad \text{(Answer)}$$

22.9: A Dipole in an Electric Field

When an electric dipole is placed in a region where there is an external electric field, ***E***, electrostatic forces act on the charged ends of the dipole. If the
electric field is uniform, those forces act in opposite directions and with the same magnitude ***F*** $=q$***E***.

Although the net force on the dipole from the field is zero, and the center of mass of the dipole does not move, the forces on the charged ends do produce
a net torque $\boldsymbol{\tau}$ on the dipole about its center of mass.

The center of mass lies on the line connecting the charged ends, at some distance x from one end and a distance d -x from the other end.

The net torque is:

$$\tau = Fx\sin\theta + F(d-x)\sin\theta = Fd\sin\theta. = pE\sin\theta.$$

$$\vec{\tau} = \vec{p} \times \vec{E} \quad \text{(torque on a dipole).}$$

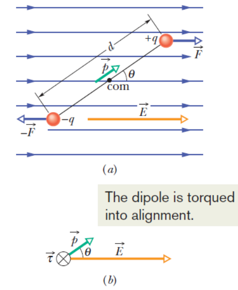


Fig. 22-19 (a) An electric dipole in a uniform external electric field $\vec{E}$. Two centers of equal but opposite charge are separated by distance d. The line between them represents their rigid connection. (b) Field $\vec{E}$ causes a torque $\vec{\tau}$ on the dipole. The direction of $\vec{\tau}$ is into the page, as represented by the symbol ⊗.

22.9: A Dipole in an Electric Field: Potential Energy

Potential energy can be associated with the orientation of an electric dipole in an electric field.

The dipole has its least potential energy when it is in its equilibrium orientation, which is when its moment ***p*** is lined up with the field ***E***.

The expression for the potential energy of an electric dipole in an external electric field is simplest if we choose the potential energy to be zero when the angle θ (Fig.22-19) is 90°.

The potential energy U of the dipole at any other value of θ can be found by calculating the work W done by the field on the dipole when the dipole is rotated to that value of θ from 90°.

$$U = -W = -\int_{90°}^{\theta} \tau\, d\theta = \int_{90°}^{\theta} pE\sin\theta\, d\theta = -pE\cos\theta.$$

$$U = -\vec{p}\cdot\vec{E} \quad \text{(potential energy of a dipole).}$$

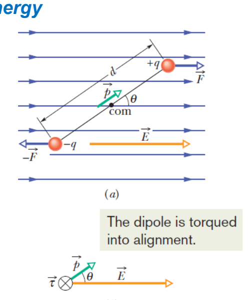


Fig. 22-19 (a) An electric dipole in a uniform external electric field $\vec{E}$. Two centers of equal but opposite charge are separated by distance d. The line between them represents their rigid connection. (b) Field $\vec{E}$ causes a torque $\vec{\tau}$ on the dipole. The direction of $\vec{\tau}$ is into the page, as represented by the symbol ⊗.

Example, Torque, Energy of an Electric Dipole in an Electric Field

A neutral water molecule (H_2O) in its vapor state has an electric dipole moment of magnitude 6.2×10^{-30} C·m.

(a) How far apart are the molecule's centers of positive and negative charge?

KEY IDEA

A molecule's dipole moment depends on the magnitude q of the molecule's positive or negative charge and the charge separation d.

Calculations: There are 10 electrons and 10 protons in a neutral water molecule; so the magnitude of its dipole moment is

$$p = qd = (10e)(d),$$

in which d is the separation we are seeking and e is the elementary charge. Thus,

$$d = \frac{p}{10e} = \frac{6.2 \times 10^{-30}\ \text{C·m}}{(10)(1.60 \times 10^{-19}\ \text{C})}$$

$$= 3.9 \times 10^{-12}\ \text{m} = 3.9\ \text{pm}. \qquad \text{(Answer)}$$

This distance is not only small, but it is also actually smaller than the radius of a hydrogen atom.

(b) If the molecule is placed in an electric field of 1.5×10^4 N/C, what maximum torque can the field exert on it? (Such a field can easily be set up in the laboratory.)

KEY IDEA

The torque on a dipole is maximum when the angle θ between $\vec{p}$ and $\vec{E}$ is 90°.

Calculation: Substituting $\theta = 90°$ in Eq. 22-33 yields

$$\tau = pE\sin\theta$$

$$= (6.2 \times 10^{-30}\ \text{C·m})(1.5 \times 10^4\ \text{N/C})(\sin 90°)$$

$$= 9.3 \times 10^{-26}\ \text{N·m}. \qquad \text{(Answer)}$$

(c) How much work must an *external agent* do to rotate this molecule by 180° in this field, starting from its fully aligned position, for which $\theta = 0$?

KEY IDEA

The work done by an external agent (by means of a torque applied to the molecule) is equal to the change in the molecule's potential energy due to the change in orientation.

Calculation: From Eq. 22-40, we find

$$W_a = U_{180°} - U_0$$

$$= (-pE\cos 180°) - (-pE\cos 0)$$

$$= 2pE = (2)(6.2 \times 10^{-30}\ \text{C·m})(1.5 \times 10^4\ \text{N/C})$$

$$= 1.9 \times 10^{-25}\ \text{J}. \qquad \text{(Answer)}$$