Chapter 27

Circuits

27.2: Pumping Charges:

In order to produce a steady flow of charge through a resistor, one needs a "charge pump," a device that—by doing work on the charge carriers—maintains a potential difference between a pair of terminals.

Such a device is called an **emf** , or *electromotive force*.

A common emf device is the *battery,* used to power a wide variety of machines from wristwatches to submarines. The emf device that most influences our daily lives is the *electric generator,* which, by means of electrical connections (wires) from a generating plant, creates a potential difference in our homes and workplaces.

Some other emf devices known are *solar cells, fuel cells.* An emf device does not have to be an instrument—living systems, ranging from electric eels and human beings to plants, have physiological emf devices.

The world's largest battery energy storage plant (dismantled in 1996) connected over 8000 large lead-acid batteries in 8 strings at 1000 V each with a capability of 10 MW of power for 4 hours. Charged up at night, the batteries were then put to use during peak power demands on the electrical system. *(Courtesy Southern California Edison Company)*

27.3: Work, Energy, and Emf:

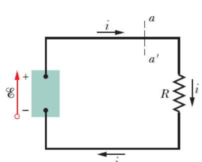


Fig. 27-1 A simple electric circuit, in which a device of emf ℰ does work on the charge carriers and maintains a steady current *i* in a resistor of resistance *R*.

In any time interval *dt,* a charge *dq* passes through any cross section of the circuit shown, such as *aa'.* This same amount of charge must enter the emf device at its low-potential end and leave at its high-potential end.

The emf device must do an amount of work *dW* on the charge *dq* to force it to move in this way.

We define the emf of the emf device in terms of this work:

$$\mathscr{E} = \frac{dW}{dq} \qquad \text{(definition of } \mathscr{E}\text{)}.$$

An **ideal emf device** is one that has no internal resistance to the internal movement of charge from terminal to terminal. The potential difference between the terminals of an ideal emf device is exactly equal to the emf of the device.

A **real emf device,** such as any real battery, has internal resistance to the internal movement of charge. When a real emf device is not connected to a circuit, and thus does not have current through it, the potential difference between its terminals is equal to its emf. However, when that device has current through it, the potential difference between its terminals differs from its emf.

27.4: Calculating the Current in a Single-Loop Circuit:

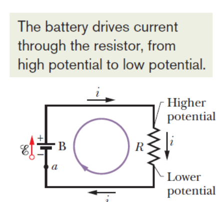


Fig. 27-3 A single-loop circuit in which a resistance *R* is connected across an ideal battery B with emf ℰ. The resulting current *i* is the same throughout the circuit.

The equation $P = i^2R$ tells us that in a time interval *dt* an amount of energy given by $i^2R\,dt$ will appear in the resistor, as shown in the figure, as thermal energy.

During the same interval, a charge $dq = i\,dt$ will have moved through battery B, and the work that the battery will have done on this charge, is

$$dW = \mathscr{E}\,dq = \mathscr{E}i\,dt.$$

From the principle of conservation of energy, the work done by the (ideal) battery must equal the thermal energy that appears in the resistor:

$$\mathscr{E}i\,dt = i^2R\,dt. \Longrightarrow \mathscr{E} = iR. \Longrightarrow i = \frac{\mathscr{E}}{R}.$$

Therefore, the energy per unit charge transferred to the moving charges is equal to the energy per unit charge transferred from them.

27.4: Calculating the Current in a Single-Loop Circuit, Potential Method:

LOOP RULE: The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

In the figure, let us start at point *a*, whose potential is V_a, and mentally go clockwise around the circuit until we are back at *a*, keeping track of potential changes as we move.

Our starting point is at the low-potential terminal of the battery. Since the battery is ideal, the potential difference between its terminals is equal to $\mathscr{E}$.

As we go along the top wire to the top end of the resistor, there is no potential change because the wire has negligible resistance.

When we pass through the resistor, however, the potential decreases by iR.

We return to point *a* by moving along the bottom wire. At point *a*, the potential is again V_a. The initial potential, as modified for potential changes along the way, must be equal to our final potential; that is,

$$V_a + \mathscr{E} - iR = V_a. \quad \longrightarrow \quad \mathscr{E} - iR = 0.$$

The battery drives current through the resistor, from high potential to low potential.

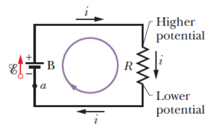


Fig. 27-3 A single-loop circuit in which a resistance R is connected across an ideal battery B with emf $\mathscr{E}$. The resulting current i is the same throughout the circuit.

27.4: Calculating the Current in a Single-Loop Circuit, Potential Method:

For circuits that are more complex than that of the previous figure, two basic rules are usually followed for finding potential differences as we move around a loop:

RESISTANCE RULE: For a move through a resistance in the direction of the current, the change in potential is $-iR$; in the opposite direction it is $+iR$.

EMF RULE: For a move through an ideal emf device in the direction of the emf arrow, the change in potential is $+\mathscr{E}$; in the opposite direction it is $-\mathscr{E}$.

27.5: Other Single-Loop Circuits, Internal Resistance:

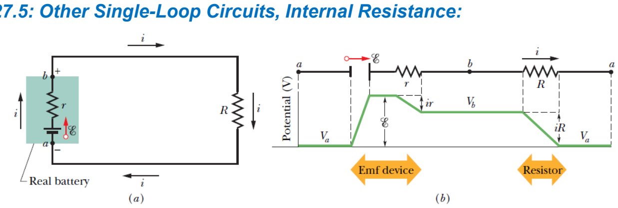


Fig. 27-4 (*a*) A single-loop circuit containing a real battery having internal resistance r and emf $\mathscr{E}$. (*b*) The same circuit, now spread out in a line. The potentials encountered in traversing the circuit clockwise from *a* are also shown. The potential V_a is arbitrarily assigned a value of zero, and other potentials in the circuit are graphed relative to V_a.

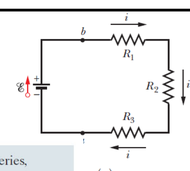
The figure above shows a real battery, with internal resistance *r*, wired to an external resistor of resistance *R*. According to the potential rule,

$$\mathscr{E} - ir - iR = 0.$$

$$\boxed{i = \frac{\mathscr{E}}{R + r}.}$$

27.5: Other Single-Loop Circuits, Resistances in Series:

$$\mathscr{E} - iR_1 - iR_2 - iR_3 = 0, \quad \Longrightarrow \quad i = \frac{\mathscr{E}}{R_1 + R_2 + R_3}.$$

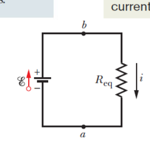


When a potential difference V is applied across resistances connected in series, the resistances have identical currents i. The sum of the potential differences across the resistances is equal to the applied potential difference V.

Resistances connected in series can be replaced with an equivalent resistance R_{eq} that has the same current i and the same *total* potential difference V as the actual resistances.

Series resistors and their equivalent have the same current ("ser-i").

$$\mathscr{E} - iR_{eq} = 0, \quad \Longrightarrow \quad i = \frac{\mathscr{E}}{R_{eq}}.$$



$$R_{eq} = R_1 + R_2 + R_3.$$

$$\Longrightarrow \quad R_{eq} = \sum_{j=1}^{n} R_j \qquad (n \text{ resistances in series}).$$

Fig. 27-5 (*a*) Three resistors are connected in series between points *a* and *b*. (*b*) An equivalent circuit, with the three resistors replaced with their equivalent resistance R_{eq}.

27.6: Potential between two points:

The internal resistance reduces the potential difference between the terminals.

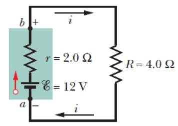


Fig. 27-6 Points a and b, which are at the terminals of a real battery, differ in potential.

Going clockwise from a:

$$V_a + \mathscr{E} - ir = V_b,$$
$$V_b - V_a = \mathscr{E} - ir.$$

$$i = \frac{\mathscr{E}}{R + r}.$$

$$V_b - V_a = \mathscr{E} - \frac{\mathscr{E}}{R + r} r$$
$$= \frac{\mathscr{E}}{R + r} R.$$

$$V_b - V_a = \frac{12\ \text{V}}{4.0\ \Omega + 2.0\ \Omega} 4.0\ \Omega = 8.0\ \text{V}.$$

Going counterclockwise from a: $V_a + iR = V_b$, $V_b - V_a = iR.$ + $i = \frac{\mathscr{E}}{R + r}.$ → $V_b - V_a = 8.0$ V.

To find the potential between any two points in a circuit, start at one point and traverse the circuit to the other point, following any path, and add algebraically the changes in potential you encounter.

27.6: Potential across a real battery:

If the internal resistance r of the battery in the previous case were zero, V would be equal to the emf $\mathscr{E}$ of the battery—namely, *12 V.*
However, since $r = 2.0\Omega$, V is less than $\mathscr{E}$.
The result depends on the value of the current through the battery. If the same battery were in a different circuit and had a different current through it, V would have some other value.

The internal resistance reduces the potential difference between the terminals.

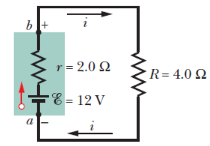
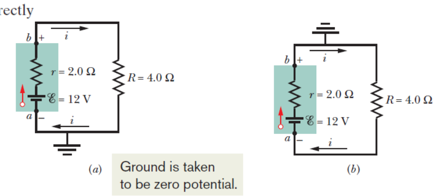


Fig. 27-6 Points a and b, which are at the terminals of a real battery, differ in potential.

27.6: Grounding a Circuit:

Fig. 27-7 (a) Point a is directly connected to ground. (b) Point b is directly connected to ground.



This is the same example as in the previous slide, except that battery terminal a is grounded in Fig. 27-7a. *Grounding a circuit* usually means connecting the circuit to a conducting path to Earth's surface, and such a connection means that the potential is defined to be zero at the grounding point in the circuit.

In Fig. 27-7a, the potential at a is defined to be $V_a = 0$. Therefore, the potential at b is $V_b = 8.0$ V.

27.6: Power, Potential, and Emf:

The net rate P of energy transfer from the emf device to the charge carriers is given by:

$$P = iV$$

where V is the potential across the terminals of the emf device.

But $V = \mathscr{E} - ir$, therefore $P = i(\mathscr{E} - ir) = i\mathscr{E} - i^2 r.$

But P_r is the rate of energy transfer to thermal energy within the emf device:

$$P_r = i^2 r \quad \text{(internal dissipation rate).}$$

Therefore the term $i\mathscr{E}$ must be the rate P_{emf} at which the emf device transfers energy both to the charge carriers and to internal thermal energy.

$$P_{\text{emf}} = i\mathscr{E} \quad \text{(power of emf device).}$$

Example, Single loop circuit with two real batteries:

The emfs and resistances in the circuit of Fig. 27-8*a* have the following values:

$$\mathscr{E}_1 = 4.4\ \text{V}, \quad \mathscr{E}_2 = 2.1\ \text{V},$$
$$r_1 = 2.3\ \Omega, \quad r_2 = 1.8\ \Omega, \quad R = 5.5\ \Omega.$$

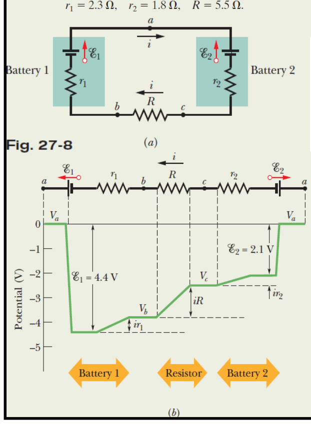


Fig. 27-8 (*a*) (*b*)

Calculations: Although knowing the direction of i is not necessary, we can easily determine it from the emfs of the two batteries. Because $\mathscr{E}_1$ is greater than $\mathscr{E}_2$, battery 1 controls the direction of i, so the direction is clockwise. (These decisions about where to start and which way you go are arbitrary but, once made, you must be consistent with decisions about the plus and minus signs.) Let us then apply the loop rule by going counterclockwise—against the current—and starting at point a. We find

$$-\mathscr{E}_1 + ir_1 + iR + ir_2 + \mathscr{E}_2 = 0.$$

Check that this equation also results if we apply the loop rule clockwise or start at some point other than a. Also, take the time to compare this equation term by term with Fig. 27-8*b*, which shows the potential changes graphically (with the potential at point a arbitrarily taken to be zero).

Solving the above loop equation for the current i, we obtain

$$i = \frac{\mathscr{E}_1 - \mathscr{E}_2}{R + r_1 + r_2} = \frac{4.4\ \text{V} - 2.1\ \text{V}}{5.5\ \Omega + 2.3\ \Omega + 1.8\ \Omega}$$
$$= 0.2396\ \text{A} \approx 240\ \text{mA}. \qquad \text{(Answer)}$$

Example, Single loop circuit with two real batteries, cont.:

(b) What is the potential difference between the terminals of battery 1 in Fig. 27-8*a*?

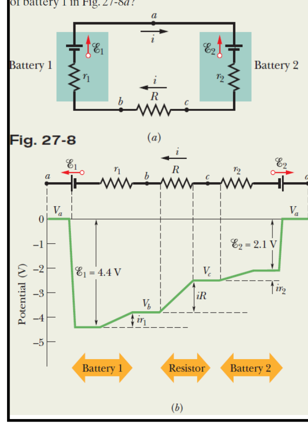


Fig. 27-8 (*a*) (*b*)

KEY IDEA

We need to sum the potential differences between points a and b.

Calculations: Let us start at point b (effectively the negative terminal of battery 1) and travel clockwise through battery 1 to point a (effectively the positive terminal), keeping track of potential changes. We find that

$$V_b - ir_1 + \mathscr{E}_1 = V_a,$$

which gives us

$$V_a - V_b = -ir_1 + \mathscr{E}_1$$
$$= -(0.2396\ \text{A})(2.3\ \Omega) + 4.4\ \text{V}$$
$$= +3.84\ \text{V} \approx 3.8\ \text{V}, \qquad \text{(Answer)}$$

which is less than the emf of the battery. You can verify this result by starting at point b in Fig. 27-8*a* and traversing the circuit counterclockwise to point a. We learn two points here. (1) The potential difference between two points in a circuit is independent of the path we choose to go from one to the other. (2) When the current in the battery is in the "proper" direction, the terminal-to-terminal potential difference is low.

27.7: Multi-loop Circuits:

JUNCTION RULE: The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.

Consider junction d in the circuit. Incoming currents i_1 and i_3, and it leaves via outgoing current i_2. Since there is no variation in the charge at the junction, the total incoming current must equal the total outgoing current: $i_1 + i_3 = i_2$.

This rule is often called *Kirchhoff's junction rule* (or *Kirchhoff's current law).*

The current into the junction must equal the current out (charge is conserved).

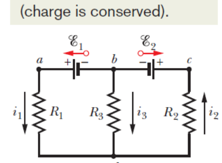


Fig. 27-9 A multiloop circuit consisting of three branches: left-hand branch *bad*, right-hand branch *bcd*, and central branch *bd*. The circuit also consists of three loops: left-hand loop *badb*, right-hand loop *bcdb*, and big loop *badcb*.

For the left-hand loop,

$$\mathscr{E}_1 - i_1R_1 + i_3R_3 = 0.$$

For the right-hand loop,

$$-i_3R_3 - i_2R_2 - \mathscr{E}_2 = 0.$$

And for the entire loop,

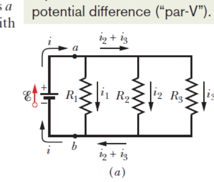
$$\mathscr{E}_1 - i_1R_1 - i_2R_2 - \mathscr{E}_2 = 0.$$

27.7: Multi-loop Circuits, Resistors in Parallel:

- When a potential difference V is applied across resistances connected in parallel, the resistances all have that same potential difference V.
- Resistances connected in parallel can be replaced with an equivalent resistance R_{eq} that has the same potential difference V and the same *total* current i as the actual resistances.

Parallel resistors and their equivalent have the same potential difference ("par-V").

Fig. 27-10 (*a*) Three resistors connected in parallel across points a and b. (*b*) An equivalent circuit, with the three resistors replaced with their equivalent resistance R_{eq}.



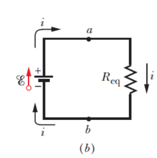
$$i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2}, \quad \text{and} \quad i_3 = \frac{V}{R_3},$$

where V is the potential difference between a and b.

From the junction rule, $i = i_1 + i_2 + i_3 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right).$

$\Rightarrow$ $i = \frac{V}{R_{eq}}.$ $\Rightarrow$ $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$

$\Rightarrow$ $\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$ (*n* resistances in parallel).



27.7: Multi-loop Circuits:

Table 27-1

Series and Parallel Resistors and Capacitors

Series	Parallel	Series	Parallel
Resistors		Capacitors	
$R_{eq} = \sum_{j=1}^{n} R_j$ Eq. 27-7	$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$ Eq. 27-24	$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$ Eq. 25-20	$C_{eq} = \sum_{j=1}^{n} C_j$ Eq. 25-19
Same current through all resistors	Same potential difference across all resistors	Same charge on all capacitors	Same potential difference across all capacitors

Example, Resistors in Parallel and in Series:

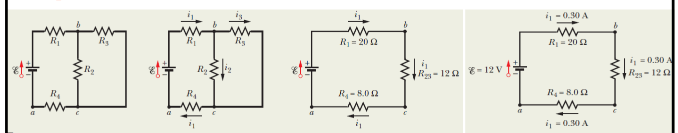


Fig. 27-11 (a) (b) (c) (d)

Figure 27-11a shows a multiloop circuit containing one ideal battery and four resistances with the following values:

$$R_1 = 20\ \Omega, \quad R_2 = 20\ \Omega, \quad \mathscr{E} = 12\ \text{V},$$
$$R_3 = 30\ \Omega, \quad R_4 = 8.0\ \Omega.$$

(a) What is the current through the battery?

Note carefully that R_1 and R_2 are *not* in series and thus cannot be replaced with an equivalent resistance. However, R_2 and R_3 are in parallel, so we can use either Eq. 27-24 or Eq. 27-25 to find their equivalent resistance R_{23}. From the latter,

$$R_{23} = \frac{R_2 R_3}{R_2 + R_3} = \frac{(20\ \Omega)(30\ \Omega)}{50\ \Omega} = 12\ \Omega.$$

We can now redraw the circuit as in Fig. 27-11c; note that the current through R_{23} must be i_1 because charge that moves through R_1 and R_4 must also move through R_{23}. For this simple one-loop circuit, the loop rule (applied clockwise from point a as in Fig. 27-11d) yields

$$+\mathscr{E} - i_1 R_1 - i_1 R_{23} - i_1 R_4 = 0.$$

Substituting the given data, we find

$$12\ \text{V} - i_1(20\ \Omega) - i_1(12\ \Omega) - i_1(8.0\ \Omega) = 0,$$

which gives us

$$i_1 = \frac{12\ \text{V}}{40\ \Omega} = 0.30\ \text{A}. \qquad \text{(Answer)}$$

Example, Resistors in Parallel and in Series, cont.:

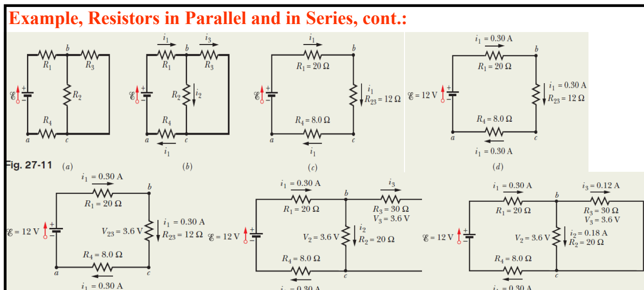


Fig. 27-11 (a) (b) (c) (d) (e) (f) (g)

Figure 27-11a shows a multiloop circuit containing one ideal battery and four resistances with the following values:

$$R_1 = 20\ \Omega, \quad R_2 = 20\ \Omega, \quad \mathscr{E} = 12\ \text{V},$$
$$R_3 = 30\ \Omega, \quad R_4 = 8.0\ \Omega.$$

(b) What is the current i_2 through R_2?

Working backward: We know that the current through R_{23} is $i_1 = 0.30$ A. Thus, we can use Eq. 26-8 ($R = V/i$) and Fig. 27-11e to find the potential difference V_{23} across R_{23}. Setting $R_{23} = 12\ \Omega$ from (a), we write Eq. 26-8 as

$$V_{23} = i_1 R_{23} = (0.30\ \text{A})(12\ \Omega) = 3.6\ \text{V}.$$

The potential difference across R_2 is thus also 3.6 V (Fig. 27-11f), so the current i_2 in R_2 must be, by Eq. 26-8 and Fig. 27-11g,

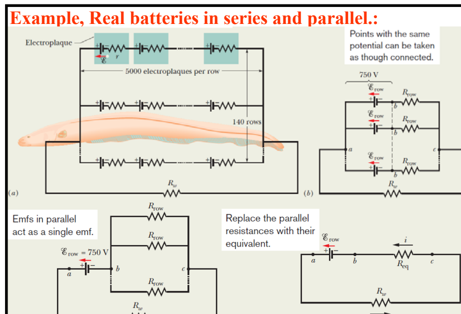
$$i_2 = \frac{V_2}{R_2} = \frac{3.6\ \text{V}}{20\ \Omega} = 0.18\ \text{A}. \qquad \text{(Answer)}$$

(c) What is the current i_3 through R_3?

Calculation: Rearranging this junction-rule result yields the result displayed in Fig. 27-11g:

$$i_3 = i_1 - i_2 = 0.30\ \text{A} - 0.18\ \text{A}$$
$$= 0.12\ \text{A}. \qquad \text{(Answer)}$$

Example, Real batteries in series and parallel.:



(a) If the water surrounding the eel has resistance R_w= *800 Ω,* how much current can the eel produce in the water?

Electric fish are able to generate current with biological cells called *electroplaques,* which are physiological emf devices. The electroplaques in the type of electric fish known as a South American eel are arranged in 140 rows, each row stretching horizontally along the body and each containing 5000 electroplaques. The arrangement is suggested in Fig. 27-12a; each electroplaque has an emf $\mathscr{E}$ of 0.15 V and an internal resistance r of 0.25 Ω. The water surrounding the eel completes a circuit between the two ends of the electroplaque array, one end at the animal's head and the other near its tail.

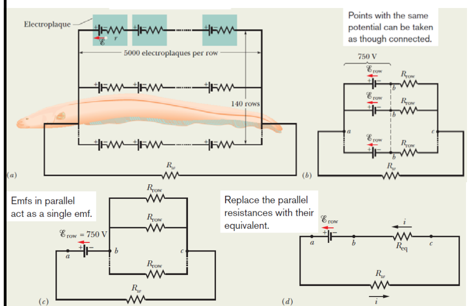
$$\mathscr{E}_{row} = 5000\mathscr{E} = (5000)(0.15\ \text{V}) = 750\ \text{V}.$$

$$R_{row} = 5000r = (5000)(0.25\ \Omega) = 1250\ \Omega.$$

$$\frac{1}{R_{eq}} = \sum_{j=1}^{140} \frac{1}{R_j} = 140\frac{1}{R_{row}}, \qquad R_{eq} = \frac{R_{row}}{140} = \frac{1250\ \Omega}{140} = 8.93\ \Omega.$$

$$\mathscr{E}_{row} - iR_w - iR_{eq} = 0. \qquad i = \frac{\mathscr{E}_{row}}{R_w + R_{eq}} = 0.927\ \text{A} \approx 0.93\ \text{A}.$$

Example, Real batteries in series and parallel.:



Electric fish are able to generate current with biological cells called *electroplaques*, which are physiological emf devices. The electroplaques in the type of electric fish known as a South American eel are arranged in 140 rows, each row stretching horizontally along the body and each containing 5000 electroplaques. The arrangement is suggested in Fig. 27-12*a*; each electroplaque has an emf $\mathscr{E}$ of 0.15 V and an internal resistance r of 0.25 Ω. The water surrounding the eel completes a circuit between the two ends of the electroplaque array, one end at the animal's head and the other near its tail.

(b) How much current i_{row} travels through each row of Fig. 27-12*a*?

Calculation: Thus, we write

$$i_{row} = \frac{i}{140} = \frac{0.927 \text{ A}}{140} = 6.6 \times 10^{-3} \text{ A}. \quad \text{(Answer)}$$

Thus, the current through each row is small, about two orders of magnitude smaller than the current through the water. This tends to spread the current through the eel's body, so that the eel need not stun or kill itself when it stuns or kills a fish.

Example, Multi-loop circuit and simultaneous loop equations:

Figure 27-13 shows a circuit whose elements have the following values:

$$\mathscr{E}_1 = 3.0 \text{ V}, \quad \mathscr{E}_2 = 6.0 \text{ V},$$
$$R_1 = 2.0\ \Omega, \quad R_2 = 4.0\ \Omega.$$

The three batteries are ideal batteries. Find the magnitude and direction of the current in each of the three branches.

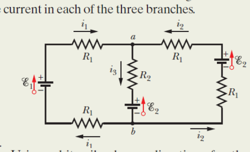


Fig. 27-13 A multiloop circuit with three ideal batteries and five resistances.

Junction rule: Using arbitrarily chosen directions for the currents as shown in Fig. 27-13, we apply the junction rule at point a by writing

$$i_3 = i_1 + i_2. \tag{27-26}$$

Left-hand loop: We first arbitrarily choose the left-hand loop, arbitrarily start at point b, and arbitrarily traverse the loop in the clockwise direction, obtaining

$$-i_1R_1 + \mathscr{E}_1 - i_1R_1 - (i_1 + i_2)R_2 - \mathscr{E}_2 = 0,$$

where we have used $(i_1 + i_2)$ instead of i_3 in the middle branch. Substituting the given data and simplifying yield

$$i_1(8.0\ \Omega) + i_2(4.0\ \Omega) = -3.0 \text{ V}. \tag{27-27}$$

Right-hand loop: For our second application of the loop rule, we arbitrarily choose to traverse the right-hand loop counterclockwise from point b, finding

$$-i_2R_1 + \mathscr{E}_2 - i_2R_1 - (i_1 + i_2)R_2 - \mathscr{E}_2 = 0.$$

Substituting the given data and simplifying yield

$$i_1(4.0\ \Omega) + i_2(8.0\ \Omega) = 0. \tag{27-28}$$

Combining equations: We now have a system of two equations (Eqs. 27-27 and 27-28) in two unknowns (i_1 and i_2) to solve either "by hand" (which is easy enough here) or with a "math package." (One solution technique is Cramer's rule, given in Appendix E.) We find

$$i_1 = -0.50 \text{ A}. \tag{27-29}$$

(The minus sign signals that our arbitrary choice of direction for i_1 in Fig. 27-13 is wrong, but we must wait to correct it.) Substituting $i_1 = -0.50$ A into Eq. 27-28 and solving for i_2 then give us

$$i_2 = 0.25 \text{ A}. \quad \text{(Answer)}$$

With Eq. 27-26 we then find that

$$i_3 = i_1 + i_2 = -0.50 \text{ A} + 0.25 \text{ A}$$
$$= -0.25 \text{ A}.$$

The positive answer we obtained for i_2 signals that our choice of direction for that current is correct. However, the negative answers for i_1 and i_3 indicate that our choices for those currents are wrong. Thus, as a *last step* here, we correct the answers by reversing the arrows for i_1 and i_3 in Fig. 27-13 and then writing

$$i_1 = 0.50 \text{ A} \quad \text{and} \quad i_3 = 0.25 \text{ A}. \quad \text{(Answer)}$$

27.8: Ammeter and Voltmeter:

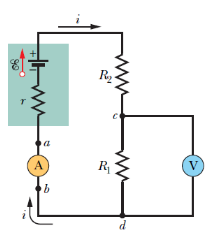


Fig. 27-14 A single-loop circuit, showing how to connect an ammeter (A) and a voltmeter (V).

An instrument used to measure currents is called an *ammeter*. It is essential that the resistance R_A of the ammeter be very much smaller than other resistances in the circuit.

A meter used to measure potential differences is called a *voltmeter*. It is essential that the resistance R_V of a voltmeter be very much larger than the resistance of any circuit element across which the voltmeter is connected.

27.9: RC Circuits, Charging a Capacitor:

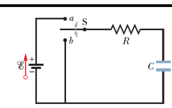
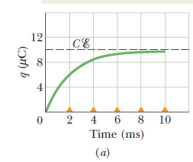


Fig. 27-15 When switch S is closed on a, the capacitor is *charged* through the resistor. When the switch is afterward closed on b, the capacitor *discharges* through the resistor.

The capacitor's charge grows as the resistor's current dies out.



It turns out that: $\mathscr{E} - iR - \dfrac{q}{C} = 0.$

We know that: $i = \dfrac{dq}{dt}.$ ➡ $R\dfrac{dq}{dt} + \dfrac{q}{C} = \mathscr{E}$ (charging equation).

$$q = C\mathscr{E}(1 - e^{-t/RC}) \quad \text{(charging a capacitor).}$$

$$V_C = \frac{q}{C} = \mathscr{E}(1 - e^{-t/RC}) \quad \text{(charging a capacitor).}$$

A capacitor that is being charged initially acts like ordinary connecting wire relative to the charging current. A long time later, it acts like a broken wire.

27.9: RC Circuits, Time Constant:

The product RC is called the capacitive time constant of the circuit and is represented with the symbol τ:

$$\tau = RC \qquad \text{(time constant).}$$

At time $t = \tau = (RC)$, the charge on the initially uncharged capacitor increases from zero to:

$$q = C\mathscr{E}(1 - e^{-1}) = 0.63C\mathscr{E}.$$

During the first time constant τ the charge has increased from zero to 63% of its final value $C\mathscr{E}$.

27.9: RC Circuits, Discharging a Capacitor:

Assume that the capacitor of the figure is fully charged to a potential V_0 equal to the emf of the battery $\mathscr{E}$.

At a new time $t = 0$, switch S is thrown from a to b so that the capacitor can discharge through resistance R.

$$R\frac{dq}{dt} + \frac{q}{C} = 0 \qquad \text{(discharging equation).}$$

$$q = q_0 e^{-t/RC} \qquad \text{(discharging a capacitor),}$$

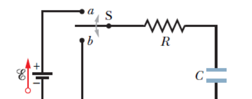


Fig. 27-15 When switch S is closed on a, the capacitor is *charged* through the resistor. When the switch is afterward closed on b, the capacitor *discharges* through the resistor.

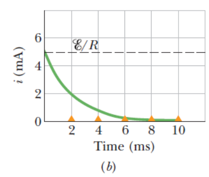
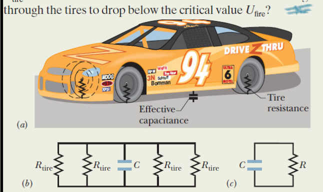


Fig. 27-16 (b) This shows the decline of the charging current in the circuit. The curves are plotted for $R = 2000\ \Omega$, $C = 1\ \mu F$, and $\mathscr{E} = 10\ V$; the small triangles represent successive intervals of one time constant τ.

Example, Discharging an RC circuit :

As a car rolls along pavement, electrons move from the pavement first onto the tires and then onto the car body. The car stores this excess charge and the associated electric potential energy as if the car body were one plate of a capacitor and the pavement were the other plate (Fig. 27-17a). When the car stops, it discharges its excess charge and energy through the tires, just as a capacitor can discharge through a resistor. If a conducting object comes within a few centimeters of the car before the car is discharged, the remaining energy can be suddenly transferred to a spark between the car and the object. Suppose the conducting object is a fuel dispenser. The spark will not ignite the fuel and cause a fire if the spark energy is less than the critical value $U_{fire} = 50$ mJ.

When the car of Fig. 27-17a stops at time $t = 0$, the car–ground potential difference is $V_0 = 30$ kV. The car–ground capacitance is $C = 500$ pF, and the resistance of *each* tire is $R_{tire} = 100$ GΩ. How much time does the car take to discharge through the tires to drop below the critical value U_{fire}?



Calculations: We can treat the tires as resistors that are connected to one another at their tops via the car body and at their bottoms via the pavement. Figure 27-17b shows how the four resistors are connected in parallel across the car's capacitance, and Fig. 27-17c shows their equivalent resistance R.

$$\frac{1}{R} = \frac{1}{R_{tire}} + \frac{1}{R_{tire}} + \frac{1}{R_{tire}} + \frac{1}{R_{tire}},$$

or
$$R = \frac{R_{tire}}{4} = \frac{100 \times 10^9\ \Omega}{4} = 25 \times 10^9\ \Omega. \qquad (27\text{-}44)$$

When the car stops, it discharges its excess charge and energy through R.

$$U = \frac{q^2}{2C} = \frac{(q_0 e^{-t/RC})^2}{2C}$$

$$= \frac{q_0^2}{2C} e^{-2t/RC}. \qquad (27\text{-}45)$$

$$U = \frac{(CV_0)^2}{2C} e^{-2t/RC} = \frac{CV_0^2}{2} e^{-2t/RC},$$

or
$$e^{-2t/RC} = \frac{2U}{CV_0^2}. \qquad (27\text{-}46)$$

Taking the natural logarithms of both sides, we obtain

$$-\frac{2t}{RC} = \ln\left(\frac{2U}{CV_0^2}\right),$$

$$t = -\frac{(25 \times 10^9\ \Omega)(500 \times 10^{-12}\ \text{F})}{2} \times \ln\left(\frac{2(50 \times 10^{-3}\ \text{J})}{(500 \times 10^{-12}\ \text{F})(30 \times 10^3\ \text{V})^2}\right)$$

$$= 9.4\ \text{s}. \qquad \text{(Answer)}$$