Chapter 28

Magnetic Fields

28.2: What Produces Magnetic Field?:

Fig. 28-1 Using an electromagnet to collect and transport scrap metal at a steel mill. *(Digital Vision/Getty Images)*

One way that magnetic fields are produced is to use moving electrically charged particles, such as a current in a wire, to make an **electromagnet.** The current produces a magnetic field that is utilizable.

The other way to produce a magnetic field is by means of elementary particles such as electrons, because these particles have an *intrinsic magnetic field* around them.

The magnetic fields of the electrons in certain materials add together to give a net magnetic field around the material. Such addition is the reason why a **permanent magnet**, has a permanent magnetic field.

In other materials, the magnetic fields of the electrons cancel out, giving no net magnetic field surrounding the material.

28.3: The Definition of B:

We can define a **magnetic field, *B*,** by firing a charged particle through the point at which is to be defined, using various directions and speeds for the particle and determining the force that acts on the particle at that point. ***B*** is then defined to be a vector quantity that is directed along the zero-force axis.

The magnetic force on the charged particle, F_B, is defined to be:

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

Here q is the charge of the particle, ***v*** is its velocity, and ***B*** the magnetic field in the region. The magnitude of this force is then:

$$F_B = |q| vB \sin\phi,$$

Here ϕ is the angle between vectors ***v*** and ***B***.

28.3: Finding the Magnetic Force on a Particle:

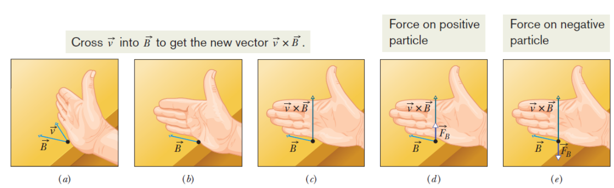


Fig. 28-2 (*a*) – (*c*) The right-hand rule (in which $\vec{v}$ is swept into $\vec{B}$ through the smaller angle ϕ between them) gives the direction of $\vec{v} \times \vec{B}$ as the direction of the thumb. (*d*) If q is positive, then the direction of $\vec{F}_B = q\vec{v} \times \vec{B}$ is in the direction of $\vec{v} \times \vec{B}$. (*e*) If q is negative, then the direction of $\vec{F}_B$ is opposite that of $\vec{v} \times \vec{B}$.

The force $\vec{F}_B$ acting on a charged particle moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ is *always* perpendicular to $\vec{v}$ and $\vec{B}$.

28.3: The Definition of B:

Table 28-1

Some Approximate Magnetic Fields

At surface of neutron star	10^8 T
Near big electromagnet	1.5 T
Near small bar magnet	10^{-2} T
At Earth's surface	10^{-4} T
In interstellar space	10^{-10} T
Smallest value in magnetically shielded room	10^{-14} T

The SI unit for ***B*** that follows is newton per coulomb-meter per second. For convenience, this is called the **tesla (T):**

$$1 \text{ tesla} = 1 \text{ T} = 1 \frac{\text{newton}}{(\text{coulomb})(\text{meter/second})}$$

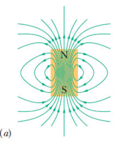
$$= 1 \frac{\text{newton}}{(\text{coulomb/second})(\text{meter})} = 1 \frac{\text{N}}{\text{A} \cdot \text{m}}$$

An earlier (non-SI) unit for ***B*** is the *gauss (G),* and

$$1 \text{ tesla} = 10^4 \text{ gauss.}$$

28.3: Magnetic Field Lines:

- The direction of the tangent to a magnetic field line at any point gives the direction of ***B*** at that point.
- The spacing of the lines represents the magnitude of ***B*** —the magnetic field is stronger where the lines are closer together, and conversely.



(*a*)


(*b*)

Fig. 28-4 (*a*) The magnetic field lines for a bar magnet. (*b*) A "cow magnet"—a bar magnet that is intended to be slipped down into the rumen of a cow to prevent accidentally ingested bits of scrap iron from reaching the cow's intestines. The iron filings at its ends reveal the magnetic field lines. *(Courtesy Dr. Richard Cannon, Southeast Missouri State University, Cape Girardeau)*

Opposite magnetic poles attract each other, and like magnetic poles repel each other.

Example, Magnetic Force on a Moving Charged Particle :

A uniform magnetic field $\vec{B}$, with magnitude 1.2 mT, is directed vertically upward throughout the volume of a laboratory chamber. A proton with kinetic energy 5.3 MeV enters the chamber, moving horizontally from south to north. What magnetic deflecting force acts on the proton as it enters the chamber? The proton mass is 1.67×10^{-27} kg. (Neglect Earth's magnetic field.)

KEY IDEAS

Because the proton is charged and moving through a magnetic field, a magnetic force $\vec{F}_B$ can act on it. Because the initial direction of the proton's velocity is not along a magnetic field line, $\vec{F}_B$ is not simply zero.

Magnitude: To find the magnitude of $\vec{F}_B$, we can use Eq. 28-3 ($F_B = |q|vB \sin\phi$) provided we first find the proton's speed v. We can find v from the given kinetic energy because $K = \frac{1}{2}mv^2$. Solving for v, we obtain

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{(2)(5.3 \text{ MeV})(1.60 \times 10^{-13} \text{ J/MeV})}{1.67 \times 10^{-27} \text{ kg}}} = 3.2 \times 10^7 \text{ m/s.}$$

Equation 28-3 then yields

$$\begin{aligned} F_B &= |q|vB \sin\phi \\ &= (1.60 \times 10^{-19} \text{ C})(3.2 \times 10^7 \text{ m/s}) \\ &\quad \times (1.2 \times 10^{-3} \text{ T})(\sin 90^\circ) \\ &= 6.1 \times 10^{-15} \text{ N.} \qquad \text{(Answer)} \end{aligned}$$

This may seem like a small force, but it acts on a particle of small mass, producing a large acceleration; namely,

$$a = \frac{F_B}{m} = \frac{6.1 \times 10^{-15} \text{ N}}{1.67 \times 10^{-27} \text{ kg}} = 3.7 \times 10^{12} \text{ m/s}^2.$$

Direction: To find the direction of $\vec{F}_B$, we use the fact that $\vec{F}_B$ has the direction of the cross product $q\vec{v} \times \vec{B}$. Because the charge q is positive, $\vec{F}_B$ must have the same direction as $\vec{v} \times \vec{B}$, which can be determined with the right-hand rule for cross products (as in Fig. 28-2*d*). We know that $\vec{v}$ is directed horizontally from south to north and $\vec{B}$ is directed vertically up. The right-hand rule shows us that the deflecting force $\vec{F}_B$ must be directed horizontally from west to east, as Fig. 28-6 shows. (The array of dots in the figure represents a magnetic field directed out of the plane of the figure. An array of **X**s would have represented a magnetic field directed into that plane.)

If the charge of the particle were negative, the magnetic deflecting force would be directed in the opposite direction—that is, horizontally from east to west. This is predicted automatically by Eq. 28-2 if we substitute a negative value for q.

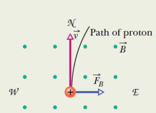


Fig. 28-6 An overhead view of a proton moving from south to north with velocity $\vec{v}$ in a chamber. A magnetic field is directed vertically upward in the chamber, as represented by the array of dots (which resemble the tips of arrows). The proton is deflected toward the east.

28.4: Crossed Fields, Discovery of an Electron:

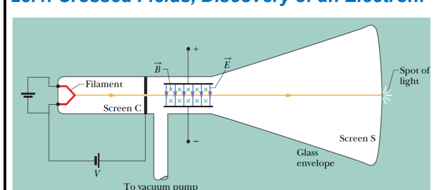


Fig. 28-7 A modern version of J. J. Thomson's apparatus for measuring the ratio of mass to charge for the electron. An electric field $\vec{E}$ is established by connecting a battery across the deflecting-plate terminals. The magnetic field $\vec{B}$ is set up by means of a current in a system of coils (not shown). The magnetic field shown is into the plane of the figure, as represented by the array of **X**s (which resemble the feathered ends of arrows).

When the two fields in Fig. 28-7 are adjusted so that the two deflecting forces acting on the charged particle cancel, we have

$$|q|E = |q|vB\sin(90^\circ) = |q|vB \Rightarrow v = \frac{E}{B}.$$

Thus, the crossed fields allow us to measure the speed of the charged particles passing through them.

The deflection of a charged particle, moving through an electric field, ***E,*** between two plates, at the far end of the plates (in the previous problem) is

$$y = \frac{|q|EL^2}{2mv^2} \Rightarrow \frac{m}{|q|} = \frac{B^2L^2}{2yE}$$

Here, v is the particle's speed, m its mass, q its charge, and L is the length of the plates.

28.5: Crossed Fields, The Hall Effect:

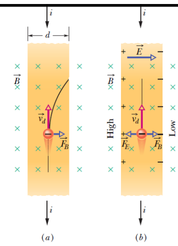


Fig. 28-8 A strip of copper carrying a current *i is immersed in a magnetic field* . (*a*)*The situation immediately after the* magnetic field is turned on. The curved path that will then be taken by an electron is shown. (*b*) *The situation at equilibrium,* which quickly follows. Note that negative charges pile up on the right side of the strip, leaving uncompensated positive charges on the left. Thus, the left side is at a higher potential than the right side. (*c*) *For the same* current direction, if the charge carriers were positively charged, *they would pile up* on the right side, and the right side would be at the higher potential.

A Hall potential difference V is associated with the electric field across strip width d, and the magnitude of that potential difference is $V = Ed$. When the electric and magnetic forces are in balance (Fig. 28-8*b),*

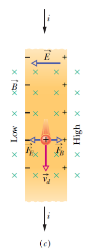
$$eE = ev_dB$$

where v_d is the drift speed. But,

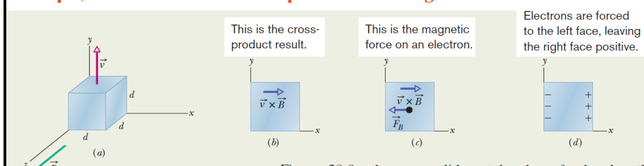
$$v_d = \frac{J}{ne} = \frac{i}{neA}$$

Where J is the current density, A the cross-sectional area, e the electronic charge, and n the number of charges per unit volume.

Therefore, $n = \dfrac{Bi}{Vle}$, Here, $l=(A/d)$, the thickness of the strip.



Example, Potential Difference Setup Across a Moving Conductor:



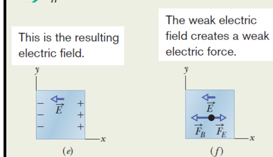


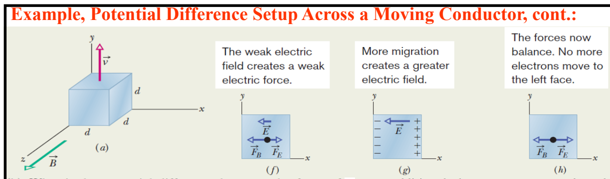
Figure 28-9*a* shows a solid metal cube, of edge length $d = 1.5$ cm, moving in the positive y direction at a constant velocity $\vec{v}$ of magnitude 4.0 m/s. The cube moves through a uniform magnetic field $\vec{B}$ of magnitude 0.050 T in the positive z direction.

(a) Which cube face is at a lower electric potential and which is at a higher electric potential because of the motion through the field?

Reasoning: When the cube first begins to move through the magnetic field, its electrons do also. Because each electron has charge q and is moving through a magnetic field with velocity $\vec{v}$, the magnetic force $\vec{F}_B$ acting on the electron is given by Eq. 28-2. Because q is negative, the direction of $\vec{F}_B$ is opposite the cross product $\vec{v} \times \vec{B}$, which is in the positive direction of the x axis (Fig. 28-9*b*). Thus, $\vec{F}_B$ acts in the negative direction of the x axis, toward the left face of the cube (Fig. 28-9*c*).

Most of the electrons are fixed in place in the atoms of the cube. However, because the cube is a metal, it contains conduction electrons that are free to move. Some of those conduction electrons are deflected by $\vec{F}_B$ to the left cube face, making that face negatively charged and leaving the right face positively charged (Fig. 28-9*d*). This charge separation produces an electric field $\vec{E}$ directed from the positively charged right face to the negatively charged left face (Fig. 28-9*e*). Thus, the left face is at a lower electric potential, and the right face is at a higher electric potential.

Example, Potential Difference Setup Across a Moving Conductor, cont.:



(b) What is the potential difference between the faces of higher and lower electric potential?

1. The electric field $\vec{E}$ created by the charge separation produces an electric force $\vec{F}_E = q\vec{E}$ on each electron (Fig. 28-9*f*). Because q is negative, this force is directed opposite the field $\vec{E}$—that is, rightward. Thus on each electron, $\vec{F}_E$ acts toward the right and $\vec{F}_B$ acts toward the left.
2. When the cube had just begun to move through the magnetic field and the charge separation had just begun, the magnitude of $\vec{E}$ began to increase from zero. Thus, the magnitude of $\vec{F}_E$ also began to increase from zero and was initially smaller than the magnitude $\vec{F}_B$. During this early stage, the net force on any electron was dominated by $\vec{F}_B$, which continuously moved additional electrons to the left cube face, increasing the charge separation (Fig. 28-9*g*).
3. However, as the charge separation increased, eventually magnitude F_E became equal to magnitude F_B (Fig. 28-9*h*). The net force on any electron was then zero, and no additional electrons were moved to the left cube face. Thus, the magnitude of $\vec{F}_E$ could not increase further, and the electrons were then in equilibrium.

Calculations: We seek the potential difference V between the left and right cube faces after equilibrium was reached (which occurred quickly). We can obtain V with Eq. 28-9 ($V = Ed$) provided we first find the magnitude E of the electric field at equilibrium. We can do so with the equation for the balance of forces ($F_E = F_B$).

For F_E, we substitute $|q|E$, and then for F_B, we substitute $|q|vB \sin\phi$ from Eq. 28-3. From Fig. 28-9*a*, we see that the angle ϕ between velocity vector $\vec{v}$ and magnetic field vector $\vec{B}$ is 90°; thus $\sin\phi = 1$ and $F_E = F_B$ yields

$$|q|E = |q|vB \sin 90^\circ = |q|vB.$$

This gives us $E = vB$; so $V = Ed$ becomes

$$V = vBd. \tag{28-13}$$

Substituting known values gives us

$$V = (4.0\text{ m/s})(0.050\text{ T})(0.015\text{ m})$$
$$= 0.0030\text{ V} = 3.0\text{ mV}. \quad \text{(Answer)}$$

28.6: A Circulating Charged Particle:

Consider a particle of charge magnitude $|q|$ and mass m moving perpendicular to a uniform magnetic field $\boldsymbol{B}$, at speed v.

The magnetic force continuously deflects the particle, and since $\boldsymbol{B}$ and $\boldsymbol{v}$ are always perpendicular to each other, this deflection causes the particle to follow a circular path.

The magnetic force acting on the particle has a magnitude of $|q|vB$.

For uniform circular motion $F = m\dfrac{v^2}{r}$,

$$\Longrightarrow \quad |q|vB = \frac{mv^2}{r}.$$

$$r = \frac{mv}{|q|B} \quad \text{(radius).}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{|q|B} = \frac{2\pi m}{|q|B} \quad \text{(period).}$$

$$f = \frac{1}{T} = \frac{|q|B}{2\pi m} \quad \text{(frequency).}$$

$$\omega = 2\pi f = \frac{|q|B}{m} \quad \text{(angular frequency).}$$

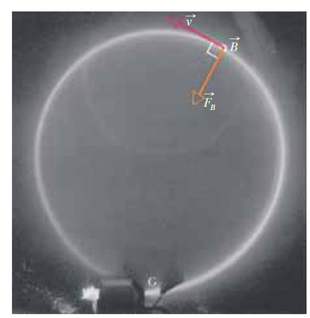


Fig. 28-10 Electrons circulating in a chamber containing gas at low pressure (their path is the glowing circle). A uniform magnetic field, B, pointing directly out of the plane of the page, fills the chamber. Note the radially directed magnetic force F_B ; for circular motion to occur, F_B must point toward the center of the circle, *(Courtesy John Le P.Webb, Sussex University, England)*

28.6: Helical Paths:

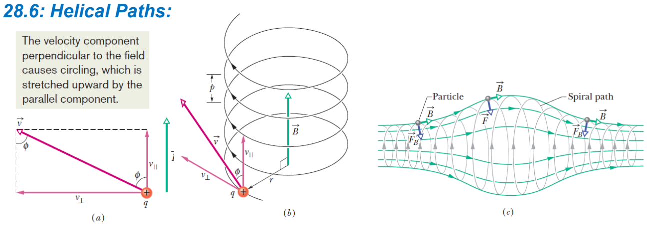


Fig. 28-11 (a) A charged particle moves in a uniform magnetic field , the particle's velocity *v* making an angle f with the field direction. (b) The particle follows a helical path of radius r and pitch p. (c) A charged particle spiraling in a nonuniform magnetic field. (The particle can become trapped, spiraling back and forth between the strong field regions at either end.) Note that the magnetic force vectors at the left and right sides have a component pointing toward the center of the figure.

The velocity vector, ***v***, of such a particle resolved into two components, one parallel to and one perpendicular to it: $v_{\parallel} = v\cos\phi$ and $v_{\perp} = v\sin\phi$.

The parallel component determines the pitch *p* of the helix (the distance between adjacent turns (Fig. 28-11*b*)). The perpendicular component determines the radius of the helix.
The more closely spaced field lines at the left and right sides indicate that the magnetic field is stronger there. When the field at an end is strong enough, the particle "reflects" from that end. If the particle reflects from both ends, it is said to be trapped in a *magnetic bottle.*

Example, Helical Motion of a Charged Particle in a Magnetic Field:

An electron with a kinetic energy of 22.5 eV moves into a region of uniform magnetic field $\vec{B}$ of magnitude 4.55×10^{-4} T. The angle between the directions of $\vec{B}$ and the electron's velocity $\vec{v}$ is 65.5°. What is the pitch of the helical path taken by the electron?

KEY IDEAS

(1) The pitch *p* is the distance the electron travels parallel to the magnetic field $\vec{B}$ during one period *T* of circulation. (2) The period *T* is given by Eq. 28-17 regardless of the angle between the directions of $\vec{v}$ and $\vec{B}$ (provided the angle is not zero, for which there is no circulation of the electron).

Calculations: Using Eqs. 28-20 and 28-17, we find

$$p = v_{\parallel}T = (v\cos\phi)\frac{2\pi m}{|q|B}. \qquad (28\text{-}21)$$

Calculating the electron's speed *v* from its kinetic energy, find that $v = 2.81 \times 10^6$ m/s. Substituting this and known data in Eq. 28-21 gives us

$$p = (2.81 \times 10^6\ \text{m/s})(\cos 65.5°) \times \frac{2\pi(9.11 \times 10^{-31}\ \text{kg})}{(1.60 \times 10^{-19}\ \text{C})(4.55 \times 10^{-4}\ \text{T})}$$
$$= 9.16\ \text{cm}. \qquad \text{(Answer)}$$

Example, Uniform Circular Motion of a Charged Particle in a Magnetic Field:

Figure 28-12 shows the essentials of a *mass spectrometer,* which can be used to measure the mass of an ion; an ion of mass *m* (to be measured) and charge *q* is produced in source *S*. The initially stationary ion is accelerated by the electric field due to a potential difference *V*. The ion leaves *S* and enters a separator chamber in which a uniform magnetic field $\vec{B}$ is perpendicular to the path of the ion. A wide detector lines the bottom wall of the chamber, and the $\vec{B}$ causes the ion to move in a semicircle and thus strike the detector. Suppose that $B = 80.000$ mT, $V = 1000.0$ V, and ions of charge $q = +1.6022 \times 10^{-19}$ C strike the detector at a point that lies at $x = 1.6254$ m. What is the mass *m* of the individual ions, in atomic mass units (Eq. 1-7: 1 u = 1.6605×10^{-27} kg)?

Finding speed: When the ion emerges from the source, its kinetic energy is approximately zero. At the end of the acceleration, its kinetic energy is $\frac{1}{2}mv^2$. Also, during the acceleration, the positive ion moves through a change in potential of $-V$. Thus, because the ion has positive charge *q*, its potential energy changes by $-qV$. If we now write the conservation of mechanical energy as

$$\Delta K + \Delta U = 0,$$

we get

$$\tfrac{1}{2}mv^2 - qV = 0$$

or

$$v = \sqrt{\frac{2qV}{m}}. \qquad (28\text{-}22)$$

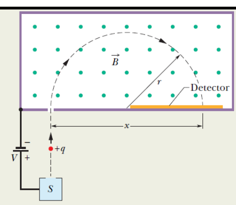


Fig. 28-12 Essentials of a mass spectrometer. A positive ion, after being accelerated from its source *S* by a potential difference *V*, enters a chamber of uniform magnetic field $\vec{B}$. There it travels through a semicircle of radius *r* and strikes a detector at a distance *x* from where it entered the chamber.

Finding mass: Substituting this value for *v* into Eq. 28-16 gives us

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}}.$$

Thus,

$$x = 2r = \frac{2}{B}\sqrt{\frac{2mV}{q}}.$$

Solving this for *m* and substituting the given data yield

$$m = \frac{B^2qx^2}{8V}$$
$$= \frac{(0.080000\ \text{T})^2(1.6022 \times 10^{-19}\ \text{C})(1.6254\ \text{m})^2}{8(1000.0\ \text{V})}$$
$$= 3.3863 \times 10^{-25}\ \text{kg} = 203.93\ \text{u}. \qquad \text{(Answer)}$$

28.7: Cyclotrons :

Suppose that a proton, injected by source *S* at the center of the cyclotron in Fig. 28-13, initially moves toward a negatively charged dee. It will accelerate toward this dee and enter it. Once inside, it is shielded from electric fields by the copper walls of the dee; that is, the electric field does not enter the dee. The magnetic field, however, is not screened by the (nonmagnetic) copper dee, so the proton moves in a circular path whose radius, which depends on its speed, is given by ($r = mv/|q|B$).

Let us assume that at the instant the proton emerges into the center gap from the first dee, the potential difference between the dees is reversed. Thus, the proton again faces a negatively charged dee and is again accelerated. This process continues, the circulating proton always being in step with the oscillations of the dee potential, until the proton has spiraled out to the edge of the dee system. There a deflector plate sends it out through a portal.

The frequency *f* at which the proton circulates in the magnetic field (and that does not depend on its speed*)* must be equal to the fixed frequency f_{osc} of the electrical oscillator:

$$f = f_{osc} \qquad \text{(resonance condition)}.$$

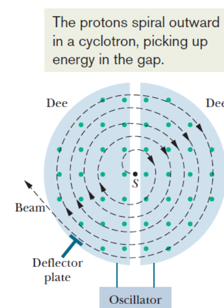


Fig. 28-13 The elements of a cyclotron, showing the particle source *S* and the dees. A uniform magnetic field is directed up from the plane of the page. Circulating protons spiral outward within the hollow dees, gaining energy every time they cross the gap between the dees.

28.7: The Proton Synchrotron :

At proton energies above 50 MeV, the conventional cyclotron begins to fail. Also, for a 500 GeV proton in a magnetic field of 1.5 T, the path radius is 1.1 km. The corresponding magnet for a conventional cyclotron of the proper size would be impossibly expensive.

In the *proton synchrotron* the magnetic field *B,* and the oscillator frequency f_{osc}, instead of having fixed values as in the conventional cyclotron, are made to vary with time during the accelerating cycle.

When this is done properly,
(1) the frequency of the circulating protons remains in step with the oscillator at all times, and
(2) the protons follow a circular—not a spiral—path. Thus, the magnet need extend only along that circular path, not over some 4×10^6 m^2. The circular path, however, still must be large if high energies are to be achieved.

The proton synchrotron at the Fermi National Accelerator Laboratory (Fermilab) in Illinois has a circumference of 6.3 km and can produce protons with energies of about 1 TeV (10^{12} eV).

Example, Accelerating a Charged Particle in a Synchrotron:

Suppose a cyclotron is operated at an oscillator frequency of 12 MHz and has a dee radius $R = 53$ cm.

(a) What is the magnitude of the magnetic field needed for deuterons to be accelerated in the cyclotron? The deuteron mass is $m = 3.34 \times 10^{-27}$ kg (twice the proton mass).

KEY IDEA

For a given oscillator frequency f_{osc}, the magnetic field magnitude B required to accelerate any particle in a cyclotron depends on the ratio $m/|q|$ of mass to charge for the particle, according to Eq. 28-24 ($|q|B = 2\pi m f_{osc}$).

Calculation: For deuterons and the oscillator frequency $f_{osc} =$ 12 MHz, we find

$$B = \frac{2\pi m f_{osc}}{|q|} = \frac{(2\pi)(3.34 \times 10^{-27}\,\text{kg})(12 \times 10^6\,\text{s}^{-1})}{1.60 \times 10^{-19}\,\text{C}}$$
$$= 1.57\ \text{T} \approx 1.6\ \text{T}. \qquad \text{(Answer)}$$

Note that, to accelerate protons, B would have to be reduced by a factor of 2, provided the oscillator frequency remained fixed at 12 MHz.

(b) What is the resulting kinetic energy of the deuterons?

KEY IDEAS

(1) The kinetic energy ($\frac{1}{2}mv^2$) of a deuteron exiting the cyclotron is equal to the kinetic energy it had just before exiting, when it was traveling in a circular path with a radius approximately equal to the radius R of the cyclotron dees. (2) We can find the speed v of the deuteron in that circular path with Eq. 28-16 ($r = mv/|q|B$).

Calculations: Solving that equation for v, substituting R for r, and then substituting known data, we find

$$v = \frac{R|q|B}{m} = \frac{(0.53\,\text{m})(1.60 \times 10^{-19}\,\text{C})(1.57\,\text{T})}{3.34 \times 10^{-27}\,\text{kg}}$$
$$= 3.99 \times 10^7\ \text{m/s}.$$

This speed corresponds to a kinetic energy of

$$K = \tfrac{1}{2}mv^2$$
$$= \tfrac{1}{2}(3.34 \times 10^{-27}\,\text{kg})(3.99 \times 10^7\,\text{m/s})^2$$
$$= 2.7 \times 10^{-12}\ \text{J}, \qquad \text{(Answer)}$$

or about 17 MeV.

28.8: Magnetic Force on a Current-Carrying Wire:

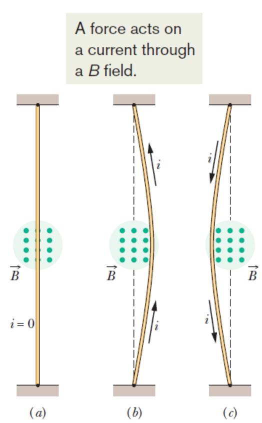


Fig. 28-14 A flexible wire passes between the pole faces of a magnet (only the farther pole face is shown). (*a*) Without current in the wire, the wire is straight. (*b*) With upward current, the wire is deflected rightward. (*c*) With downward current, the deflection is leftward. The connections for getting the current into the wire at one end and out of it at the other end are not shown.

28.8: Magnetic Force on a Current-Carrying Wire:

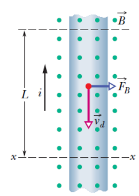


Fig. 28-15 A close-up view of a section of the wire of Fig. 28-14*b*. The current direction is upward, which means that electrons drift downward. A magnetic field that emerges from the plane of the page causes the electrons and the wire to be deflected to the right.

Consider a length L of the wire in the figure. All the conduction electrons in this section of wire will drift past plane *xx* in a time $t = L/vd$.

Thus, in that time a charge will pass through that plane that is given by

$$q = it = i\frac{L}{v_d}$$

$$F_B = qv_d B \sin\phi = \frac{iL}{v_d} v_d B \sin 90°$$

$$F_B = iLB.$$

$$\vec{F}_B = i\vec{L} \times \vec{B} \qquad \text{(force on a current).}$$

Here $\boldsymbol{L}$ is a length vector that has magnitude L and is directed along the wire segment in the direction of the (conventional) current.

If a wire is not straight or the field is not uniform, we can imagine the wire broken up into small straight segments . The force on the wire as a whole is then the vector sum of all the forces on the segments that make it up. In the differential limit, we can write $d\vec{F}_B = i\,d\vec{L} \times \vec{B}$, and we can find the resultant force on any given arrangement of currents by integrating Eq. 28-28 over that arrangement.

Example, Magnetic Force on a Wire Carrying Current:

A straight, horizontal length of copper wire has a current $i = 28$ A through it. What are the magnitude and direction of the minimum magnetic field $\vec{B}$ needed to suspend the wire—that is, to balance the gravitational force on it? The linear density (mass per unit length) of the wire is 46.6 g/m.

KEY IDEAS

(1) Because the wire carries a current, a magnetic force $\vec{F}_B$ can act on the wire if we place it in a magnetic field $\vec{B}$. To balance the downward gravitational force $\vec{F}_g$ on the wire, we want $\vec{F}_B$ to be directed upward (Fig. 28-17). (2) The direction of $\vec{F}_B$ is related to the directions of $\vec{B}$ and the wire's length vector $\vec{L}$ by Eq. 28-26 ($\vec{F}_B = i\vec{L} \times \vec{B}$).

Calculations: Because $\vec{L}$ is directed horizontally (and the current is taken to be positive), Eq. 28-26 and the right-hand rule for cross products tell us that $\vec{B}$ must be horizontal and rightward (in Fig. 28-17) to give the required upward $\vec{F}_B$.

The magnitude of $\vec{F}_B$ is $F_B = iLB \sin \phi$ (Eq. 28-27). Because we want $\vec{F}_B$ to balance $\vec{F}_g$, we want

$$iLB \sin \phi = mg, \tag{28-29}$$

where mg is the magnitude of $\vec{F}_g$ and m is the mass of the wire.

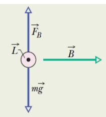


Fig. 28-17 A wire (shown in cross section) carrying current out of the page.

We also want the minimal field magnitude B for $\vec{F}_B$ to balance $\vec{F}_g$. Thus, we need to maximize $\sin \phi$ in Eq. 28-29. To do so, we set $\phi = 90°$, thereby arranging for $\vec{B}$ to be perpendicular to the wire. We then have $\sin \phi = 1$, so Eq. 28-29 yields

$$B = \frac{mg}{iL \sin \phi} = \frac{(m/L)g}{i}. \tag{28-30}$$

We write the result this way because we know m/L, the linear density of the wire. Substituting known data then gives us

$$B = \frac{(46.6 \times 10^{-3} \text{ kg/m})(9.8 \text{ m/s}^2)}{28 \text{ A}}$$

$$= 1.6 \times 10^{-2} \text{ T}. \quad \text{(Answer)}$$

This is about 160 times the strength of Earth's magnetic field.

28.9: Torque on a Current Loop:

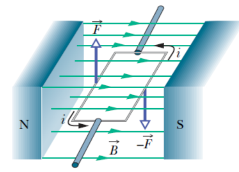
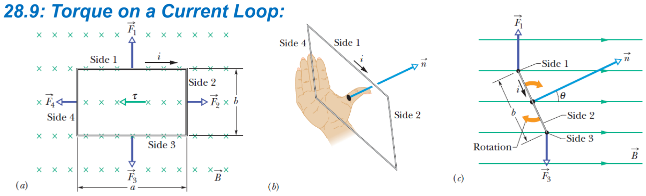


Fig. 28-18 The elements of an electric motor. A rectangular loop of wire, carrying a current and free to rotate about a fixed axis, is placed in a magnetic field. Magnetic forces on the wire produce a torque that rotates it. A commutator (not shown) reverses the direction of the current every half-revolution so that the torque always acts in the same direction.

The two magnetic forces ***F*** and –***F*** produce a torque on the loop, tending to rotate it about its central axis.

28.9: Torque on a Current Loop:



To define the orientation of the loop in the magnetic field, we use a normal vector ***n*** that is perpendicular to the plane of the loop. Figure 28-19*b* shows a right-hand rule for finding the direction of ***n***. In Fig. 28-19*c*, the normal vector of the loop is shown at an arbitrary angle θ to the direction of the magnetic field.

For side 2 the magnitude of the force acting on this side is $F_2 = ibB \sin(90° - \theta) = ibB \cos\theta = F_4$. F_2 and F_4 cancel out exactly.

Forces F_1 and F_3 have the common magnitude iaB. As Fig. 28-19*c* shows, these two forces do not share the same line of action; so they produce a net torque.

$$\tau' = \left(iaB\,\frac{b}{2}\sin\theta\right) + \left(iaB\,\frac{b}{2}\sin\theta\right) = iabB\sin\theta.$$

For N loops, when $A=ab$, the area of the loop, the total torque is:

$$\tau = N\tau' = NiabB\sin\theta = (NiA)B\sin\theta,$$

28.10: The Magnetic Dipole Moment, μ:

Definition: $\mu = NiA$ (magnetic moment),

Here, N is the number of turns in the coil, i is the current through the coil, and A is the area enclosed by each turn of the coil.

Direction: The direction of μ is that of the normal vector to the plane of the coil.

The magnetic moment vector attempts to align with the magnetic field.

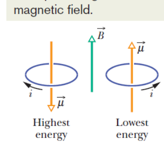


Fig. 28-20 The orientations of highest and lowest energy of a magnetic dipole (here a coil carrying current) in an external magnetic field $\vec{B}$. The direction of the current i gives the direction of the magnetic dipole moment $\vec{\mu}$ via the right-hand rule shown for $\vec{n}$ in Fig. 28-19*b*.

The definition of torque can be rewritten as:

$$\tau = \mu B \sin\theta,$$

$$\vec{\tau} = \vec{\mu} \times \vec{B},$$

Just as in the electric case, the magnetic dipole in an external magnetic field has an energy that depends on the dipole's orientation in the field:

$$U(\theta) = -\vec{\mu}\cdot\vec{B}.$$

A magnetic dipole has its lowest energy ($-\mu B \cos 0 = -\mu B$) when its dipole moment μ is lined up with the magnetic field. It has its highest energy ($-\mu B \cos 180° = +\mu B$) when μ is directed opposite the field.

28.10: The Magnetic Dipole Moment, μ:

$$\mu = NiA \quad \text{(magnetic moment)},$$

$$U(\theta) = -\vec{\mu} \cdot \vec{B}.$$

Table 28-2

Some Magnetic Dipole Moments

Small bar magnet	5 J/T
Earth	8.0×10^{22} J/T
Proton	1.4×10^{-26} J/T
Electron	9.3×10^{-24} J/T

From the above equations, one can see thatthe unit of $\boldsymbol{\mu}$ can be the joule per tesla (J/T), or the ampere–square meter.