Review for Celebration #1

$\boxed{\text{Chapter 21 : Electric Charge}}$

charge on proton $q_p = e = 1.60 \times 10^{-19}\,C$

charge on electron $q_e = -e = -1.60 \times 10^{-19}\,C$

charge is quantized $\Rightarrow$ $\boxed{q = ne}$ $\quad n = \pm 1, \pm 2, \ldots$

conservation of charge

Coulomb's law $\Rightarrow$ $\boxed{F = \dfrac{K|q_1||q_2|}{r^2}}$

$$K = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9\ Nm^2/C^2$$

$$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$$

$\Rightarrow$ direction of force along line joining two particles

$\downarrow$

like charges repel

opposite charges attract

Two Shell Theorems

1) a shell of uniform charge can be treated as if all the charge were concentrated at its center

2) there is no net electrostatic force on a charged particle inside a shell of uniform charge

$\downarrow$ both theorems are proved using Gauss' law principle of superposition $\Rightarrow$ $\vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \ldots$

must add forces like vectors

$\boxed{\text{Chapter 22: Electric Fields}}$

$\Rightarrow$ you measure $\vec{E}$ at a given point by placing a small positive test charge q_0 at that point

$$\boxed{\vec{E} = \vec{F}/q_0}$$

for a point charge $\Rightarrow$ $\boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{|q|}{r^2}}$

$\vec{E}$ points away from a + charge
$\vec{E}$ points towards a − charge

know electric field lines

Binomial expansion : if $x << 1$ $\quad (1+x)^n \approx 1+nx$

$(1-x)^n \approx 1-nx$

electric field from dipole $\Rightarrow$ $\boxed{E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}}$

dipole moment $\boxed{p = qd}$

$\vec{p}$ points from − to + charge

continuous charge distributions $\lambda = q/L$

$\sigma = q/A$ $\quad \rho = q/V$

$\Rightarrow$ for a continuous charge distribution $\boxed{dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}}$

$$E = \int dE = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}$$

know how to use!

* must take into account fact that $\vec{E}$ is a vector

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{qz}{(z^2+R^2)^{3/2}}}$$

$\vec{E}$ a distance z above a charged ring along central axis

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)}$$

$\vec{E}$ a distance z above the center of a charged disk

$\Rightarrow$ force on a point charge in an external electric field :

$$\boxed{\vec{F} = q\vec{E}}$$

if $q > 0$, $\vec{F}$ is in same direction as $\vec{E}$
if $q < 0$, $\vec{F}$ is in opposite direction as $\vec{E}$

SKIP

$\boxed{\tau = \vec{p} \times \vec{E}}$ torque on dipole in uniform $\vec{E}$-field

$\boxed{U = -\vec{p} \cdot \vec{E}}$ potential energy of dipole in uniform $\vec{E}$-field

Chapter 23: Gauss' Law

electric flux $\boxed{\Phi = \vec{E} \cdot \vec{A}}$

↓ applies for uniform $\vec{E}$ field and flat surface of area A

in general ⇒ $\boxed{\Phi = \int \vec{E} \cdot d\vec{A}}$

Gauss' Law ⇒ $\boxed{\oint \vec{E} \cdot d\vec{A} = q_{encl}/\varepsilon_0}$

* Gauss' law is always true but it is not always useful

* know the properties of an isolated conductor (isolated from ground):

1) $\vec{E} = 0$ everywhere inside conductor

2) all excess charge resides on surface

3) $\vec{E} \perp$ to surface of conductor & has magnitude $\boxed{E = \sigma/\varepsilon_0}$

$\boxed{q = \int \rho \, dV}$

⇒ if ρ is only a function of r:

$q = \int \rho(r) \, dV$ $\qquad dV = 2\pi r L \, dr$ cylinder
$\qquad\qquad\qquad\qquad dV = 4\pi r^2 \, dr$ sphere

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

→ E a distance r from an infinite line of charge

$$\boxed{E = \sigma/2\varepsilon_0}$$

→ E from an infinite nonconducting sheet of charge

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$$

→ E outside sphere of charge

$$\boxed{E = \frac{q\,r}{4\pi\varepsilon_0 R^3}}$$

→ E inside uniformly charged sphere

Chapter 24: Electric Potential

$V = U/q \longrightarrow \boxed{\Delta V = \Delta U/q = -W/q}$ W → work done by electric field

$1\,V/m = 1\,J/C$ $1\,eV = 1.60 \times 10^{-19}\,J$

$$\boxed{V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}}$$

$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}}$ → potential from a point charge assuming $V = 0$ at $r = \infty$

$V > 0$ for a + charge
$V < 0$ for a − charge

* V is a scalar

⇒ for a collection of point charges $\boxed{V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n} q_i/r_i}$

for a conducting sphere of radius R:

$V = \frac{1}{4\pi\varepsilon_0}\, q/R \qquad r \le R$

$V = \frac{1}{4\pi\varepsilon_0}\, q/r \qquad r > R$

* know properties of equipotential surfaces

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}}$$ → potential from a continuous charge distribution assuming $V = 0$ at ∞

$$E = -\frac{\partial V}{\partial s} \longrightarrow \boxed{E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}}$$

* $\vec{E}$ points in direction of decreasing potential

⇒ for spherical symmetry $E_r = -\frac{\partial V}{\partial r}$

⇒ if $\vec{E}$ is uniform $E = -\frac{\Delta V}{\Delta s}$

$$\boxed{U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}}$$ electric potential energy between two point charges (assuming $U = 0$ at $r = \infty$)

* any excess charge on an isolated conductor will arrange itself on the surface such that all points of the conductor are at the same potential

↓

if one conductor touches another conductor, both conductors will come to the same potential

Chapter 25: Capacitance

$\boxed{C = q/V}$ $\boxed{q = CV}$

C only depends on the geometry of the capacitor, not q or V

for a parallel plate capacitor: $\boxed{V = Ed}$

$$\boxed{C = \varepsilon_0 A/d}$$

for a cylindrical capacitor: $\boxed{C = 2\pi\varepsilon_0 \dfrac{L}{\ln(b/a)}}$

for a spherical capacitor: $\boxed{C = 4\pi\varepsilon_0 \dfrac{ab}{b-a}}$

for an isolated sphere: $\boxed{C = 4\pi\varepsilon_0 R}$

SKIP

Combinations of Capacitors

capacitors in parallel: $\boxed{C_{eq} = C_1 + C_2 + \cdots}$

capacitors in series: $\boxed{\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots}$

* capacitors in parallel have the same V as their C_{eq}

* capacitors in series have the same q as their C_{eq}

energy stored by a capacitor: $\boxed{U = \frac{1}{2} q^2/C = \frac{1}{2} CV^2 = \frac{1}{2} qV}$

energy density $\boxed{u = \frac{1}{2} \varepsilon_0 E^2}$

⇒ the capacitance increases when a dielectric material is inserted between the plates

$$\boxed{C = K C_0}$$

$K \rightarrow$ dielectric constant

* when a region of space is completely filled with a dielectric, all electrostatic equations having ε_0 are replaced with $K \varepsilon_0$

⇒ the effect of a dielectric is to weaken the electric field that is already present

$$\boxed{E = E_0 / K}$$