

Review for Exam 1

Chapter 21 : Electric Charge

charge on proton $q_p = e = 1.60 \times 10^{-19} \text{ C}$

charge on electron $q_e = -e = -1.60 \times 10^{-19} \text{ C}$

charge is quantized \Rightarrow $q = ne$ $n = \pm 1, \pm 2, \dots$

conservation of charge

Coulomb's law \Rightarrow $F = \frac{K |q_1| |q_2|}{r^2}$

$$K = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

\Rightarrow direction of force along line joining two particles



like charges repel
opposite charges attract

Two Shell Theorems

- 1) a shell of uniform charge can be treated as if all the charge were concentrated at its center
- 2) there is no net electrostatic force on a charged particle inside a shell of uniform charge

↓ both theorems are proved using Gauss' law

principle of superposition $\Rightarrow \vec{F}_{1, \text{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \dots$



must add forces like vectors

Chapter 22: Electric Fields

\Rightarrow you measure \vec{E} at a given point by placing a small positive test charge q_0 at that point



$$\vec{E} = \vec{F} / q_0$$

for a point charge \Rightarrow

$$E = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2}$$



\vec{E} points away from a + charge
 \vec{E} points towards a - charge

know electric field lines

Binomial expansion: if $x \ll 1$ $(1+x)^n \approx 1+nx$
 $(1-x)^n \approx 1-nx$

electric field from dipole \Rightarrow $E = \frac{1}{2\pi\epsilon_0} \frac{p}{z^3}$

dipole moment $p = qd$

\downarrow \vec{p} points from - to + charge

continuous charge distributions

$\lambda = q/L$

$\sigma = q/A$

$\rho = q/V$

\Rightarrow for a continuous charge distribution

$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2}$

$E = \int dE = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2}$

\downarrow know how to use!

* must take into account fact that \vec{E} is a vector

$E = \frac{1}{4\pi\epsilon_0} \frac{qz}{(z^2+R^2)^{3/2}}$

\vec{E} a distance z above a charged ring along central axis

$E = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{z}{\sqrt{z^2+R^2}} \right)$

\vec{E} a distance z above the center of a charged disk

⇒ force on a point charge in an external electric field:

$$\vec{F} = q \vec{E}$$

↓
if $q > 0$, \vec{F} is in same direction as \vec{E}
if $q < 0$, \vec{F} is in opposite direction as \vec{E}

SKIP

$$\tau = \vec{p} \times \vec{E}$$

torque on dipole in uniform \vec{E} -field

$$U = -\vec{p} \cdot \vec{E}$$

potential energy of dipole in uniform \vec{E} -field

Chapter 23: Gauss' Law

electric flux $\Phi = \vec{E} \cdot \vec{A}$

↓ applies for uniform \vec{E} field and flat surface of area A

in general $\Rightarrow \Phi = \int \vec{E} \cdot d\vec{A}$

Gauss' Law $\Rightarrow \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}} / \epsilon_0$

* Gauss' law is always true but it is not always useful

* know the properties of an isolated conductor (isolated from ground):

1) $\vec{E} = 0$ everywhere inside conductor

2) all excess charge resides on surface

3) $\vec{E} \perp$ to surface of conductor & has magnitude $E = \sigma / \epsilon_0$

$$q = \int \rho dV$$

\Rightarrow if ρ is only a function of r :

$$q = \int \rho(r) dV \quad \begin{array}{l} dV = 2\pi r L dr \quad \text{cylinder} \\ dV = 4\pi r^2 dr \quad \text{sphere} \end{array}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

→ E a distance r from an infinite line of charge

$$E = \sigma / 2\epsilon_0$$

→ E from an infinite nonconducting sheet of charge

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

→ E outside sphere of charge

$$E = \frac{q r}{4\pi\epsilon_0 R^3}$$

→ E inside uniformly charged sphere

Chapter 24: Electric Potential

$$V = U/q \rightarrow \Delta V = \Delta U/q = -W/q$$

$W \rightarrow$ work done by electric field

$$1 \text{ V/m} = 1 \text{ J/C} \quad 1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

$$V_f - V_i = - \int_i^f \vec{E} \cdot d\vec{s}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

\rightarrow potential from a point charge assuming $V=0$ at $r=\infty$

$V > 0$ for a + charge

$V < 0$ for a - charge

* V is a scalar

\Rightarrow for a collection of point charges

$$V = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n q_i / r_i$$

for a conducting sphere of radius R :

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{R} \quad r \leq R$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \quad r > R$$

* know properties of equipotential surfaces

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

→ potential from a continuous charge distribution assuming $V=0$ at ∞

$$E = -\frac{\partial V}{\partial s} \rightarrow$$

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

* \vec{E} points in direction of decreasing potential

⇒ for spherical symmetry $E_r = -\frac{\partial V}{\partial r}$

⇒ if \vec{E} is uniform $E = -\frac{\Delta V}{\Delta s}$

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

electric potential energy between two point charges (assuming $U=0$ at $r=\infty$)

* any excess charge on an isolated conductor will arrange itself on the surface such that all points of the conductor are at the same potential



if one conductor touches another conductor, both conductors will come to the same potential

Chapter 25: Capacitance

$$C = q/V$$

$$q = CV$$

↓
C only depends on the geometry of the capacitor, not q or V

for a parallel plate capacitor:

$$V = Ed$$

$$C = \epsilon_0 A/d$$

for a cylindrical capacitor:

$$C = 2\pi\epsilon_0 L / \ln(b/a)$$

for a spherical capacitor:

$$C = 4\pi\epsilon_0 \frac{ab}{b-a}$$

for an isolated sphere:

$$C = 4\pi\epsilon_0 R$$

SKIP

Combinations of Capacitors

capacitors in parallel:

$$C_{eq} = C_1 + C_2 + \dots$$

capacitors in series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

* capacitors in parallel have the same V as their C_0

* capacitor in series have the same q as their C_0

energy stored by a capacitor:

$$U = \frac{1}{2} \frac{q^2}{C} = \frac{1}{2} C V^2 = \frac{1}{2} q V$$

energy density

$$u = \frac{1}{2} \epsilon_0 E^2$$

\Rightarrow the capacitance increases when a dielectric material is inserted between the plates

\downarrow

$$C = K C_0$$

$K \rightarrow$ dielectric constant

* when a region of space is completely filled with a dielectric, all electrostatic equations having E_0 are replaced with $K E_0$.

\Rightarrow the effect of a dielectric is to weaken the electric field that is already present

$$E = E_0 / K$$

Review for Test #2

Chapter 26: Current & Resistance

current $i = \frac{dq}{dt}$ units of $i \rightarrow$ amp (A) $1 \text{ A} = 1 \text{ C/s}$

conventional current \Rightarrow flow of hypothetical + charge from + to - terminal of battery

\downarrow in reality, electrons flow in opposite direction

$i = \int \vec{J} \cdot d\vec{A}$ \rightarrow always applies
current density $\vec{J} = i/A$ $[J] = \text{A/m}^2$

drift speed $V_d = \frac{i}{nAe}$

\downarrow applies if i is uniform and parallel to $d\vec{A}$

$V_d = J/ne \rightarrow \vec{J} = (ne)\vec{V}_d$

definition of resistance $R = V/i$ units of $R \rightarrow$ ohm (Ω)

$1 \Omega = 1 \text{ V/A}$

resistivity $\rho = E/J$ units of $\rho \rightarrow \Omega \text{ m}$

$$\vec{E} = \rho \vec{J}$$

conductivity $\sigma = 1/\rho$ \rightarrow units of $(\Omega m)^{-1}$

$$\vec{J} = 1/\rho \vec{E} = \sigma \vec{E}$$

resistance of a conducting wire $\Rightarrow R = \rho L/A$

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$$
$$R - R_0 = R_0 \alpha (T - T_0)$$

$\alpha \rightarrow$ temp. coeff. of resistivity
 \downarrow
units of α are $(K)^{-1}$ or $(C^\circ)^{-1}$

"Ohm's Law" $\rightarrow V = iR \quad i = V/R \quad R = V/i$

\downarrow
a conductor obeys Ohm's law if R is a constant independent of magnitude & polarity of voltage

$P = iV$ \rightarrow holds true for any electrical device

$P = i^2 R$
 $P = V^2/R$ \rightarrow only holds true for resistors

Chapter 27: Circuits

emf $\mathcal{E} \rightarrow$ defined as $\mathcal{E} = dW/dq$

voltage across a battery with internal resistance r is given by:

$$V = \mathcal{E} - i r$$

for n resistors in series:

$$R_{eq} = \sum_{i=1}^n R_i$$

for n resistors in parallel:

$$\frac{1}{R_{eq}} = \sum_{i=1}^n \frac{1}{R_i}$$

* resistors in series have same i as R_{eq}

* resistors in parallel have same V as R_{eq}

Kirchoff's junction rule $\Rightarrow i_{in} = i_{out}$

Kirchoff's loop rule $\Rightarrow \sum V = 0$ around any closed loop



algebraic sum of the changes in potential around any closed loop is zero

sign conventions for Kirchhoff's loop rule:

emf devices \rightarrow going through emf device from $-$ to $+$ terminal,
the change in potential is $+E$

\downarrow from $+$ to $-$ terminal, change in potential is $-E$

resistor \rightarrow going through resistor in same direction as current,
the change in potential is $-iR$

\downarrow in opposite direction as current, the change in
potential is $+iR$

* if you do not know the direction of the current,
assume a direction \rightarrow if i comes out negative, your
assumed direction was wrong

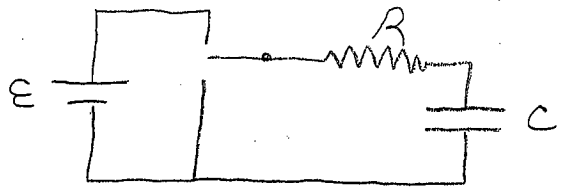
\Rightarrow for an emf device $P = iE$

* know how to calculate the potential difference between
2 points or the potential at one point given the potential
at another point

ammeter \rightarrow very small R (connected in series)

voltmeter \rightarrow very large R (connected in parallel)

RC Circuits



charging a capacitor:

$$q = q_0 (1 - e^{-t/\tau}) = C\mathcal{E} (1 - e^{-t/\tau})$$

$$V = q/C = \mathcal{E} (1 - e^{-t/\tau})$$

$$i = dq/dt = \mathcal{E}/R e^{-t/\tau}$$

$$\tau = RC$$

↓
capacitive time
constant

discharging a capacitor:

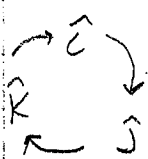
$$q = q_0 e^{-t/\tau}$$

$$i = dq/dt = - (q_0/RC) e^{-t/\tau}$$

Chapter 28: Magnetic Fields

$$\boxed{\vec{F}_B = q \vec{v} \times \vec{B}} \rightarrow \text{magnitude of } F_B = q v B \sin \theta$$

↓
direction given by RHR \Rightarrow curl fingers of right hand from \vec{v} into \vec{B} , thumb points in direction of \vec{F} on a positive charge (opposite direction if charge is negative)



$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{k} \times \hat{j} = -\hat{i}$$

$$\hat{i} \times \hat{k} = -\hat{j}$$

Joe's RHR \Rightarrow

thumb $\rightarrow \vec{v}$

fingers $\rightarrow \vec{B}$

palm $\rightarrow \vec{F}$

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i} \begin{vmatrix} v_y & v_z \\ B_y & B_z \end{vmatrix} - \hat{j} \begin{vmatrix} v_x & v_z \\ B_x & B_z \end{vmatrix} + \hat{k} \begin{vmatrix} v_x & v_y \\ B_x & B_y \end{vmatrix}$$

* \vec{F}_B is always \perp to both \vec{v} and \vec{B}

↓
 magnetic fields can not change the speed of a moving charged particle, only the direction of the particle

unit of $\vec{B} \rightarrow$ tesla (T)

$$\boxed{1 \text{ T} = 1 \text{ N/A}\cdot\text{m}}$$

\Rightarrow if both \vec{E} + \vec{B} fields are present :

$$\boxed{\vec{F} = q (\vec{E} + \vec{v} \times \vec{B})}$$

⇒ for crossed $\vec{E} + \vec{B}$ fields in opposition, the net force on a charged particle is zero if:

$$v = E/B$$

Circulating Charged Particle

$$r = \frac{mv}{qB}$$

$$T = \frac{2\pi m}{qB}$$

$$f = \frac{qB}{2\pi m}$$

↓
above equations apply if $\vec{v} \perp \vec{B}$

⇒ if \vec{v} is not \perp to \vec{B} , the particle moves in a helical path

radius of helix

$$r = \frac{m v_{\perp}}{qB}$$

$$v_{\perp} = v \sin \theta$$

pitch

$$p = v_{\parallel} T$$

$$v_{\parallel} = v \cos \theta$$

magnetic force on a current-carrying wire

$$\vec{F} = i \vec{L} \times \vec{B}$$

↓
magnitude of $F = i L B \sin \theta$

direction of \vec{F} from RHR ⇒ curl fingers of RH from \vec{L} (which is in direction of conventional current) into \vec{B} , thumb points in direction of \vec{F}

⇒ the net force on a closed current-carrying loop of wire in a uniform \vec{B} -field is zero

↓ however, current loop may experience a torque

$$\tau = N i A B \sin \theta$$

magnetic dipole moment

$$\mu = N i A$$

↓

direction of $\vec{\mu}$ ⇒ curl fingers of RH in direction of conventional current, thumb points in direction of $\vec{\mu}$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

⇒ potential energy of a magnetic dipole in an external magnetic field is given by:

$$U = -\vec{\mu} \cdot \vec{B}$$

Chapter 29: Magnetic Fields Due to Currents

Biot-Savart law \Rightarrow

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \vec{r}}{r^3}$$



magnitude

$$dB = \frac{\mu_0}{4\pi} \frac{i ds \sin\theta}{r^2}$$

↓ direction of $d\vec{B}$ is given by $d\vec{s} \times \vec{r}$

$$\vec{B} = \int d\vec{B}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ Tm/A} \approx 1.26 \times 10^{-6} \text{ Tm/A}$$

long (infinite) straight wire \Rightarrow

$$B = \frac{\mu_0 i}{2\pi r}$$

semi-infinite straight wire \Rightarrow

$$B = \frac{\mu_0 i}{4\pi r}$$

center of circular arc of radius R
and angle ϕ (in radians) \Rightarrow

$$B = \frac{\mu_0 i \phi}{4\pi R}$$

force between two parallel currents \Rightarrow

$$F = \frac{\mu_0 L i_1 i_2}{2\pi d}$$

* parallel currents attract, antiparallel currents repel

Ampere's law \Rightarrow

$$\oint \vec{B} \cdot d\vec{S} = \mu_0 i_{\text{encl}}$$

RHR for Ampere's law \Rightarrow curl fingers of RH in direction of Amperian loop; currents in general direction of thumb are +, currents in opposite direction are -

inside long straight wire with uniform current density \Rightarrow

$$B = \left(\frac{\mu_0 i}{2\pi R^2} \right) r$$

ideal solenoid \Rightarrow

$$B = \mu_0 i n$$

$n = \# \text{ turns per unit length}$



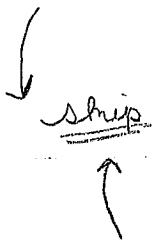
B inside solenoid

B = 0 outside ideal solenoid

inside toroid

$$B = \frac{\mu_0 i N}{2\pi r}$$

B = 0 outside toroid



$$\vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3}$$

\rightarrow \vec{B} a distance z above center of a current carrying coil

$$\mu = N i A$$

Chapter 30: Induction and Inductance

magnetic flux $\Phi_B = \int \vec{B} \cdot d\vec{A}$ units of $\Phi_B \rightarrow$ weber Wb

$$1 \text{ Wb} = 1 \text{ Tm}^2$$

\Rightarrow if \vec{B} is uniform and in same direction as \vec{A} $\Phi = BA$

* anytime Φ_B through a conducting loop changes, an emf \mathcal{E} is induced in the loop



Faraday's law $\Rightarrow \mathcal{E} = -N d\Phi_B/dt$

Lenz's law \Rightarrow an induced emf gives rise to a current whose magnetic field opposes the original change in flux that caused it

EMF Produced in a Moving Conductor

$$\mathcal{E} = BLv \quad (\text{motional emf})$$

$$i = \mathcal{E}/R = BLv/R$$

$$F = \frac{B^2 L^2 v}{R} \rightarrow \text{force required to pull loop at constant velocity}$$

$$P = B^2 L^2 v^2 / R \rightarrow \text{rate at which force does work + rate at which resistor dissipates energy}$$

* understand eddy currents conceptually

* a changing magnetic flux produces an electric field



Faraday's law of induction \Rightarrow

$$\oint \vec{E} \cdot d\vec{s} = -d\Phi_B/dt$$

inductance of an inductor \Rightarrow

$$L = N\Phi_B/I$$



units of L is henry (H)

$$1H = 1Tm^2/A$$

for an ideal solenoid \rightarrow

$$L = \mu_0 n^2 A l$$

$$\begin{aligned}\mu_0 &= 4\pi \times 10^{-7} Tm/A \\ &= 4\pi \times 10^{-7} H/m\end{aligned}$$

self-induced emf \Rightarrow

$$\mathcal{E}_L = -L di/dt$$



\mathcal{E}_L opposes change of current through it

RL circuits

rise of current :

$$i = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau_L})$$

inductive time constant

$$\tau_L = L/R$$

* can get V_R from $V_R = iR$ and \mathcal{E}_L from $\mathcal{E}_L = -L di/dt$

$$\mathcal{E}_L = -L \frac{di}{dt} = -\mathcal{E} e^{-t/\tau_L}$$

decay of current :

$$i = i_0 e^{-t/\tau_L}$$

$$\mathcal{E}_L = -L \frac{di}{dt} = i_0 R e^{-t/\tau_L}$$

energy stored in \vec{B} -field of inductor \Rightarrow

$$U_B = \frac{1}{2} L i^2$$

magnetic energy density \Rightarrow

$$u = \frac{1}{2} B^2 / \mu_0$$

↓
energy per unit volume at any point in a \vec{B} -field

Review of Chapter 31: EM Oscillations + AC

⇒ an EM oscillator (LC circuit) is analogous to a mechanical oscillator (mass on a spring)

$$q = Q \cos(\omega t + \phi) \quad \omega = 2\pi f = \frac{1}{\sqrt{LC}}$$

$$i = -I \sin(\omega t + \phi) \quad I = \omega Q$$

$$E = U_E + U_B = \frac{1}{2} Q^2/C + \frac{1}{2} L i^2$$

$$E = \frac{1}{2} Q^2/C = \frac{1}{2} L I^2$$

⇒ if there is resistance in the circuit, the EM oscillations die out because energy is dissipated by the resistor

$$\downarrow \quad q = Q e^{-Rt/2L} \cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\omega^2 - (R/2L)^2}$$

Three Simple Circuits

↓ R, L, or C connected to alternating emf source

$$E = E_m \sin \omega_d t = E_m \sin 2\pi f_d t$$

$$i = I \sin(\omega_d t - \phi)$$

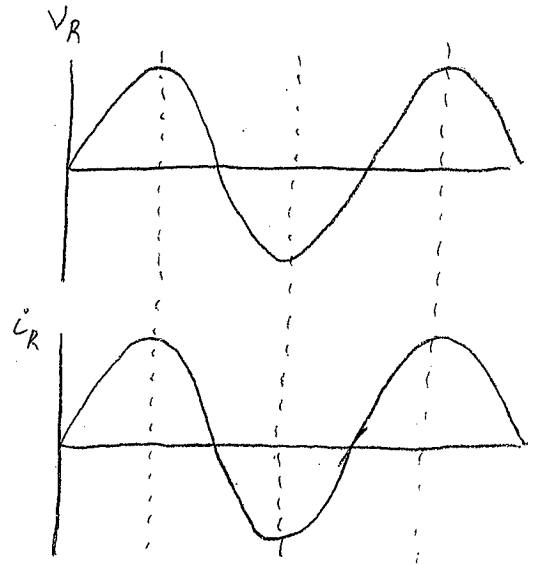
resistive load

$$i_R = I_R \sin(\omega_d t - \phi)$$

$$I_R = V_R / R$$

$$\phi = 0$$

↓
current in phase with
voltage



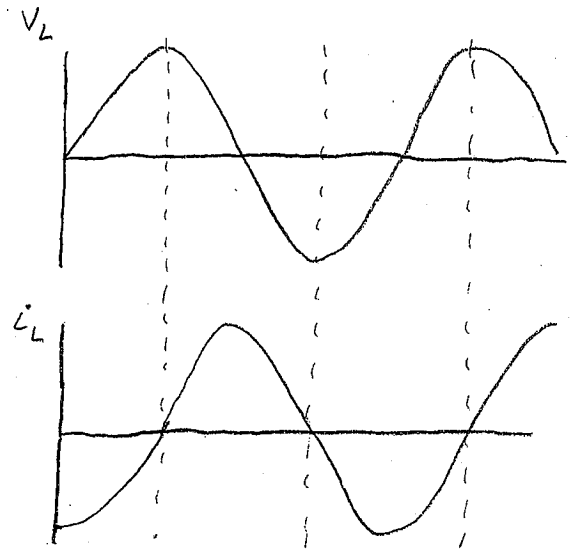
inductive load

$$i_L = I_L \sin(\omega_d t - \phi)$$

$$I_L = V_L / X_L \quad X_L = \omega_d L$$

$$\phi = 90^\circ$$

↓
current peaks after voltage



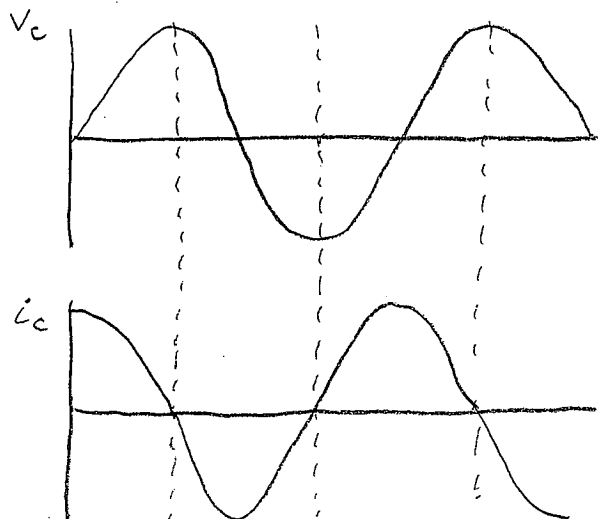
capacitive load

$$i_C = I_C \sin(\omega_d t - \phi)$$

$$I_C = V_C / X_C \quad X_C = \frac{1}{\omega_d C}$$

$$\phi = -90^\circ$$

↓
current peaks before voltage



<u>circuit element</u>	<u>resistance or reactance</u>	<u>amplitude relation</u>	<u>phase constant</u>	<u>phase of current</u>
resistor	R	$V_R = I_R R$	0	in phase with V_R
inductor	$X_L = \omega_d L$	$V_L = I_L X_L$	$+90^\circ$	lags V_L by 90°
capacitor	$X_C = 1/\omega_d C$	$V_C = I_C X_C$	-90°	leads V_C by 90°

ELI "positively" is the ICE man

⇒ know phasors conceptually

Series RLC circuit → $R, L, \text{ \& } C$ all connected in series to an alternating emf source

$$E = E_m \sin \omega_d t$$

$$i = I \sin(\omega_d t - \phi)$$

$$I = E_m / Z \quad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan \phi = \frac{X_L - X_C}{R}$$

- 1) $X_L > X_C$ (mainly inductive) $\phi > 0$ current lags emf
- 2) $X_C > X_L$ (mainly capacitive) $\phi < 0$ current leads emf
- 3) $X_L = X_C$ (resonance) $\phi = 0$ current in phase with emf

⇒ resonance occurs when $\omega_d = \omega = \frac{1}{\sqrt{LC}}$

↓
at resonance, Z is a minimum ($Z = R$) and current is a maximum

1) $\omega_d = \omega$ $X_c = X_L$ resonance ($\phi = 0$)

2) $\omega_d < \omega$ $X_c > X_L$ mainly capacitive ($\phi < 0$)

3) $\omega_d > \omega$ $X_L > X_c$ mainly inductive ($\phi > 0$)

Power in AC Circuits

$$I_{\text{rms}} = I/\sqrt{2} \quad V_{\text{rms}} = V/\sqrt{2} \quad E_{\text{rms}} = \mathcal{E}/\sqrt{2}$$

$$P_{\text{ave}} = I_{\text{rms}}^2 R$$

$$P_{\text{ave}} = I_{\text{rms}} E_{\text{rms}} \cos \phi$$

↓
power factor

Transformers

$$V_s = V_p \frac{N_s}{N_p}$$

$N_s > N_p$ step-up transformer

$N_s < N_p$ step-down transformer

$$I_s = I_p \frac{N_p}{N_s}$$