Physics 4B
Fall 2022

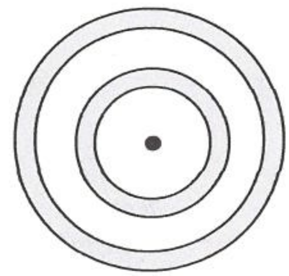
Name: Answer Key

Lab (circle one): 8:00 am 11:15 am 2:30 pm

Celebration #1: Chapters 21 – 25

Short Answer Questions (5 or 6 points each)

Question 1 (6 points)
A $+20\mu C$ point charge is surrounded by 2 conducting spherical shells as shown in the figure to the right. The inner shell has a charge of +15μC and has an inner radius R_a and outer radius R_b. The outer shell has a charge of $-10\mu C$ and has an inner radius R_c and outer radius R_d



What is the charge on the **inner** surface of the **inner** shell (at radius R_a)?

$\boxed{-20\mu C}$ $\quad (q_{encl} = +20\mu C + q_{inner} = 0)$

What is the charge on the **outer** surface of the **inner** shell (at radius R_b)?

$\boxed{35\mu C}$ $\quad (+15\mu C = q_{inner} + q_{outer})$

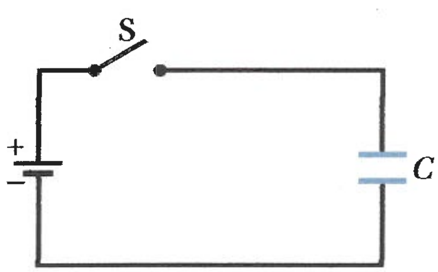
What is the charge on the **inner** surface of the **outer** shell (at radius R_c)?

$\boxed{-35\mu C}$ $\quad (q_{encl} = +20\mu C + 15\mu C + q_{inner} = 0)$

What is the charge on the **outer** surface of the **outer** shell (at radius R_d)?

$\boxed{25\mu C}$ $\quad (-10\mu C = q_{inner} + q_{outer})$

Question 2 (6 points)
The capacitor in the figure below has a capacitance of 25.0 μF and is initially uncharged. The battery provides a potential difference is 15.0 V. After the switch is closed, how many electrons will flow from the battery to the bottom plate of the capacitor?



$$q = CV = (25.0\times10^{-6}\,F)(15.0\,V)$$

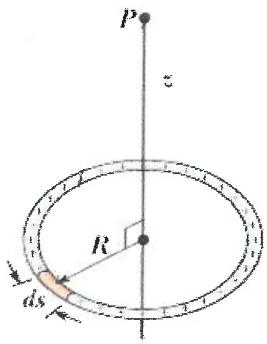
$$q = 3.75\times10^{-4}\,C$$

$$n = q/e = \frac{3.75\times10^{-4}\,C}{1.602\times10^{-19}\,C}$$

$$\boxed{n = 2.34\times10^{15}\text{ electrons}}$$

Question 3 (5 points)
The electric potential at point P, a distance z from central axis of a thin ring of radius R and linear charge density λ, can be written as $V = \lambda R/(2\varepsilon_0\sqrt{z^2+R^2})$. Show that both sides of this equation have the same units.

$[V] = V \quad [\lambda] = C/m \quad [R] = m \quad [\varepsilon_0] = C^2/Nm^2$



$[\sqrt{z^2+R^2}] = m$

$$V = \frac{\lambda R}{2\varepsilon_0\sqrt{z^2+R^2}}$$

$$V = \frac{(C/m)(m)}{(C^2/Nm^2)(m)} \longrightarrow V = Nm/C = J/C$$

$$V = J/C \rightarrow V = V$$

Question 4 (5 points)
You are given the potential function V(x,y) = $6xy^3 + 3x^2y$, where V is in volts and x and y are in meters. Determine the magnitude of the electric field **E** at the point x = 1, y = 2.

$$E_x = -\partial V/\partial x = -6y^3 - 6xy$$

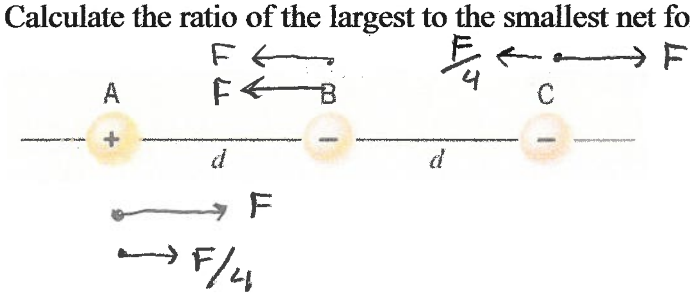
$$E_x\,(x=1, y=2) = -6(2)^3 - 6(1)(2) = -60\ V/m$$

$$E_y = -\partial V/\partial y = -18xy^2 - 3x^2$$

$$E_y\,(x=1, y=2) = -18(1)(2)^2 - 3(1)^3 = -75\ V/m$$

$$E = \sqrt{E_x^2 + E_y^2} \longrightarrow \boxed{E = 96\ V/m}$$

Question 5 (6 points)
Three point charges have equal magnitudes. They are fixed in place on the same straight line, and are equally separated by a distance d. Consider the net electrostatic force acting on each charge. Calculate the ratio of the largest to the smallest net force.



let $F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}$

A → $5F/4$

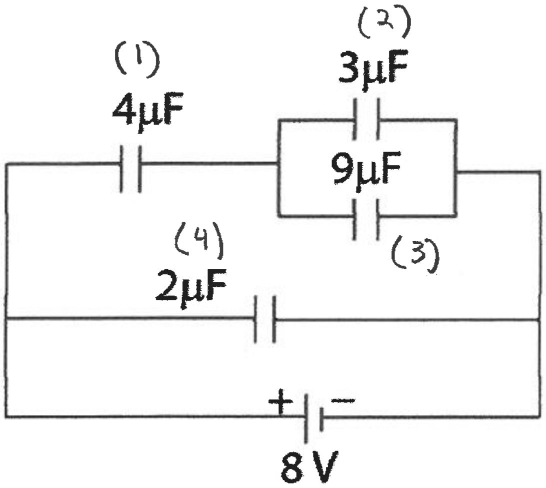
B → $2F$

C → $3F/4$

$$\frac{\text{largest}}{\text{smallest}} = \frac{2F}{(3F/4)} = \boxed{8/3}$$

Problems (12 points each)

Problem 1
In the circuit shown below, what is the charge on and the potential difference across each capacitor?

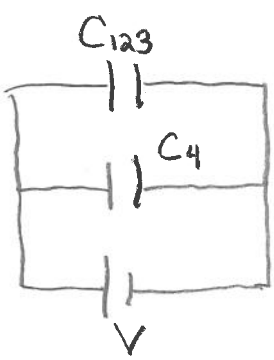


$C_{23} = C_2 + C_3 = 3\mu F + 9\mu F$

$C_{23} = 12\mu F$

$\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_{23}} = \frac{1}{4\mu F} + \frac{1}{12\mu F}$

$C_{123} = 3\mu F$



$\boxed{V_4 = 8V}$ $q_4 = C_4 V_4 = (2\mu F)(8V) = \boxed{16\mu C}$

$V_{123} = 8V$ $q_{123} = C_{123} V_{123} = (3\mu F)(8V) = 24\mu C$

$q_1 = q_{23} = q_{123} = 24\mu C$ $\boxed{q_1 = 24\mu C}$ $V_1 = \frac{q_1}{C_1} = \frac{24\mu C}{4\mu F} = \boxed{6V}$

$q_{23} = 24\mu C$ $V_{23} = \frac{q_{23}}{C_{23}} = \frac{24\mu C}{12\mu F} = 2V$

$V_2 = V_3 = V_{23} = 2V$ $\boxed{V_2 = 2V}$ $q_2 = C_2 V_2 = (3\mu F)(2V)$

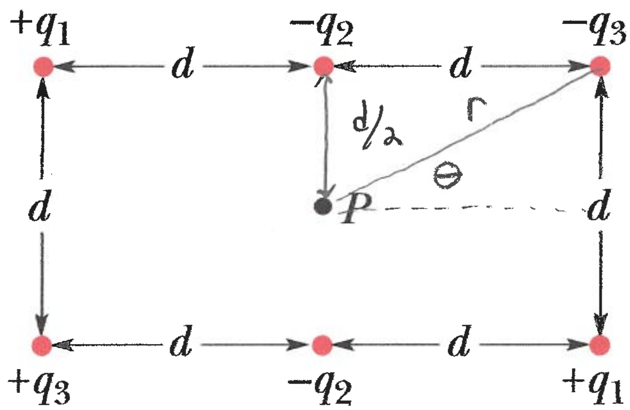
$\boxed{q_2 = 6\mu C}$

$\boxed{V_3 = 2V}$ $q_3 = C_3 V_3 = (9\mu F)(2V) = \boxed{18\mu C}$

$$\boxed{\begin{array}{llll} 4\mu F & \longrightarrow & V = 6V & q = 24\mu C \\ 3\mu F & \longrightarrow & V = 2V & q = 6\mu C \\ 9\mu F & \longrightarrow & V = 2V & q = 18\mu C \\ 2\mu F & \longrightarrow & V = 8V & q = 16\mu C \end{array}}$$

Problem 2

In the figure below, point P is at the center of the rectangle, $q_1 = 4.00$ pC, $q_2 = 2.50$ pC, $q_3 = 5.00$ pC, and $d = 3.50$ cm. **(a)** What is the net electric potential at point P due to the six charged particles? **(b)** What is the magnitude and direction of the net electric field at point P due to the six charged particles?



halliday_9e_fig_24_65

$$r = \sqrt{d^2 + (d/2)^2} = \sqrt{\tfrac{5}{4}}\, d = 3.91\text{ cm}$$

$$\theta = \tan^{-1}\left(\frac{d/2}{d}\right) = \tan^{-1}\left(\tfrac{1}{2}\right)$$

$$\theta = 26.6^\circ$$

$$V = \frac{1}{4\pi\varepsilon_0}\sum_i q/r$$

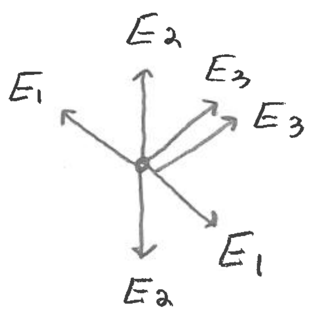
starting in upper left corner and going clockwise:

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r} - \frac{q_2}{(d/2)} - \frac{q_3}{r} + \frac{q_1}{r} - \frac{q_2}{(d/2)} + \frac{q_3}{r}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{2q_1}{r} - \frac{4q_2}{d}\right]$$

$$V = (8.99\times10^{9}\text{ N m}^2/\text{C}^2)\left[\frac{2(4.00\times10^{-12}\text{ C})}{(0.0391\text{ m})} - \frac{4(2.50\times10^{-12}\text{ C})}{(0.0350\text{ m})}\right] = \boxed{-0.729\text{ V}}$$

(b)



$\vec{E}$ from $+q_1$ charges cancel

$\vec{E}$ from $-q_2$ charges cancel

$\vec{E}$ from $-q_3$ and $+q_3$ add together

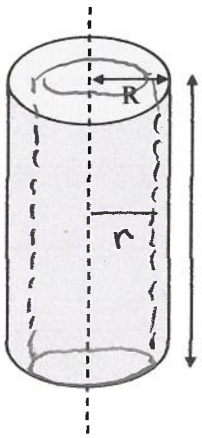
$$E_{net} = \frac{1}{4\pi\varepsilon_0}\frac{|q_3|}{r^2}(2) = \frac{(8.99\times10^{9}\text{ Nm}^2/\text{C}^2)(5.00\times10^{-12}\text{ C})(2)}{(0.0391\text{ m})^2}$$

$$\boxed{\vec{E} = 58.8\text{ N/C at } \theta = 26.6^\circ}$$

Problem 3

A long, solid *nonconducting* cylinder of length L and radius R has a nonuniform charge distribution of volume charge density $\rho = Ar/R$, where r is the radial distance from the cylindrical axis and A is a constant. Using Gauss' law, derive an expression for the electric field a radial distance r from the axis of the cylinder for points **(a)** inside the cylinder ($r < R$), and **(b)** outside the cylinder ($r > R$).

Hint: $q = \int \rho dV = \int \rho(2\pi r L)dr$



(a) Gaussian surface → cylinder of radius $r < R$ centered on nonconducting cylinder

$$\oint \vec{E} \cdot d\vec{A} = q_{encl}/\varepsilon_0$$

$$\oint \vec{E} \cdot d\vec{A} = \underbrace{\int \vec{E} \cdot dA}_{\substack{\text{top cap} \\ = 0}} + \underbrace{\int \vec{E} \cdot d\vec{A}}_{\substack{\text{bottom cap} \\ = 0}} + \underbrace{\int \vec{E} \cdot d\vec{A}}_{\text{cylinder}}$$

$$\int E\, dA \cos 0^\circ = E \int dA = E(2\pi r L)$$

$$E(2\pi r L) = q_{encl}/\varepsilon_0$$

$$q_{encl} = \int \rho(2\pi r L)\, dr = \int (Ar/R)\, 2\pi r L\, dr$$

$$q_{encl} = \frac{2\pi L A}{R} \int_0^r r^2 dr = \frac{2\pi r^3 L A}{3R}$$

$$E(2\pi r L) = \frac{2\pi r^3 L A}{3R} \frac{1}{\varepsilon_0} \rightarrow \boxed{E = \frac{A r^2}{3 R \varepsilon_0}}$$

(b) Gaussian surface → cylinder of radius $r > R$

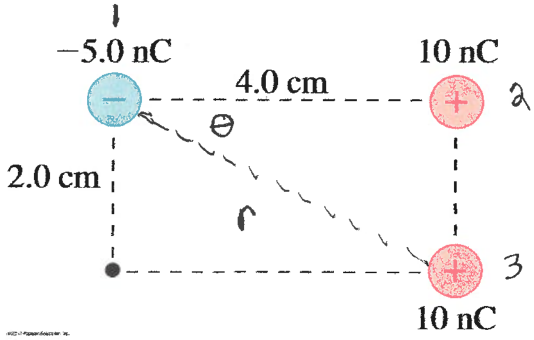
$$\oint \vec{E} \cdot d\vec{A} = q_{encl}/\varepsilon_0 \qquad q_{encl} = \frac{2\pi L A}{R} \int_0^R r^2 dr = \frac{2\pi R^2 L A}{3}$$

$$\oint \vec{E} \cdot d\vec{A} = E(2\pi r L) = \frac{2\pi R^2 L A}{3\varepsilon_0} \rightarrow \boxed{E = \frac{A R^2}{3 \varepsilon_0 r}}$$

* note: results give same answer for $r = R$

Problem 4

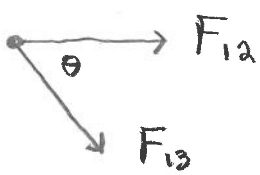
What is the magnitude and direction of the net electrostatic force on the -5.0 nC charge?



$$r = \sqrt{(4.0\,\text{cm})^2 + (2.0\,\text{cm})^2}$$

$$\underline{r = 4.47\,\text{cm}}$$

$$\theta = \tan^{-1}\left(\frac{2.0\,\text{cm}}{4.0\,\text{cm}}\right) \rightarrow \underline{\theta = 26.6^\circ}$$



$$F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r_{12}^2}$$

$$= \frac{(8.99\times10^9\,\text{N m}^2/\text{c}^2)(5.0\times10^{-9}\,\text{c})(10\times10^{-9}\,\text{c})}{(0.040\,\text{m})^2}$$

$$F_{12} = \underline{2.81\times10^{-4}}\,\text{N}$$

$$F_{13} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_3|}{r_{13}^2} = \frac{(8.99\times10^9\,\text{N m}^2/\text{c}^2)(5.0\times10^{9}\,\text{c})(10\times10^{-9}\,\text{c})}{(0.0447\,\text{m})^2}$$

$$F_{13} = \underline{2.25\times10^{-4}}\,\text{N}$$

$$\sum_i F_x = F_{12} + F_{13}\cos\theta = (2.81\times10^{-4}\,\text{N}) + (2.25\times10^{-4}\,\text{N})\cos 26.6^\circ$$

$$\sum_i F_x = \underline{4.82\times10^{-4}}\,\text{N}$$

$$\sum_i F_y = 0 - F_{13}\sin\theta = -(2.25\times10^{-4}\,\text{N})\sin 26.9^\circ = \underline{-1.02\times10^{-4}}\,\text{N}$$

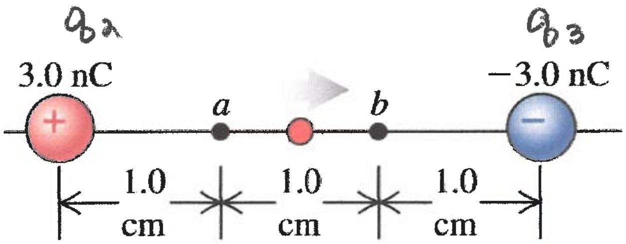
$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = \boxed{4.93\times10^{-4}\,\text{N}}$$

$$\theta = \tan^{-1}(F_y/F_x) \rightarrow \boxed{\theta = -11.9^\circ \text{ or } 348^\circ}$$

q_1

Problem 5

A dust particle with mass 5.0 μg and charge 2.0 nC starts from rest at point a and moves in a straight line to point b. What is its speed at point b?



$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

$$U_a = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}}\right)$$

$$U_a = (8.99\times10^9\,\mathrm{Nm^2/c^2})\left[\frac{(2.0\times10^{-9}\mathrm{c})(3.0\times10^{-9}\mathrm{c})}{(0.010\,\mathrm{m})} + \frac{(2.0\times10^{-9}\mathrm{c})(-3.0\times10^{-9}\mathrm{c})}{(0.020\,\mathrm{m})}\right]$$

$$U_a = 2.697\times10^{-6}\,\mathrm{J}$$

$$U_b = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}}\right)$$

$$= (8.99\times10^9\,\mathrm{Nm^2/c^2})\left[\frac{(2.0\times10^{-9}\mathrm{c})(3.0\times10^{-9}\mathrm{c})}{(0.020\,\mathrm{m})} + \frac{(2.0\times10^{-9}\mathrm{c})(-3.0\times10^{-9}\mathrm{c})}{(0.010\,\mathrm{m})}\right]$$

$$= -2.697\times10^{-6}\,\mathrm{J}$$

from conservation of energy: $\underbrace{K_a}_{=0} + U_a = K_b + U_b$

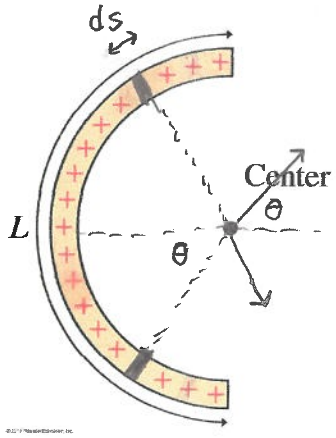
$$K_b = U_a - U_b \longrightarrow \tfrac{1}{2} m v_b^2 = (U_a - U_b)$$

$$V_b = \sqrt{\frac{2(U_a - U_b)}{m}} = \sqrt{\frac{2\left[2.697\times10^{-6}\,\mathrm{J} - (-2.697\times10^{-6}\,\mathrm{J})\right]}{5.0\times10^{-9}\,\mathrm{Kg}}}$$

$$\boxed{V_b = 46.4\ \mathrm{m/s}}$$

Problem 6

Charge Q = 30.0 nC is uniformly distributed along a thin, flexible rod of length L = 15.0 cm. The rod is bent into a semicircle as shown in the figure below. What is the magnitude and direction of the electric field at the center of the semicircle?



⇒ vertical components cancel, horizontal components ($\cos\theta$) add

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

$$E = \int dE\cos\theta = \int \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\cos\theta$$

$$\lambda = q/L \rightarrow q = \lambda L \rightarrow dq = \lambda\, dL = \lambda\, ds$$

$$E = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda\cos\theta\, ds}{r^2} \qquad s = r\theta \rightarrow ds = r\, d\theta$$

$$E = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda\cos\theta\, r\, d\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r}\sin\theta\Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r}(1-(-1)) = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r} \longrightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}\ (\hat{i})$$

$$\lambda = q/L = \frac{30.0\times10^{-9}\,C}{0.15\,m} = \underline{2.0\times10^{-7}\,C/m}$$

$$L = \pi r \longrightarrow r = L/\pi = \frac{0.15\,m}{\pi} = \underline{0.0477\,m}$$

$$E = \frac{(8.99\times10^{9}\,Nm^2/C^2)\,2\,(2.0\times10^{-7}\,C/m)}{(0.0477\,m)}\ (\hat{i})$$

$$\boxed{E = 7.54\times10^{4}\,N/C\ (\hat{i})}$$