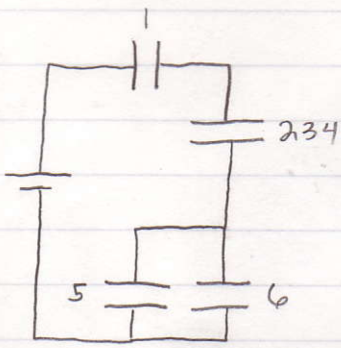
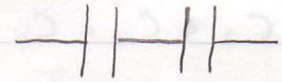
Review Problems for Exam 1

1) C_2, C_3, and C_4 are in parallel

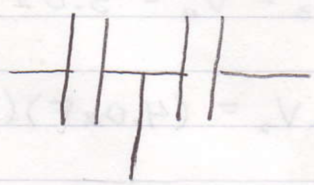
$C_{234} = C_2 + C_3 + C_4 = 12\,\mu F$



note: C_{234} and C_6 are not in series

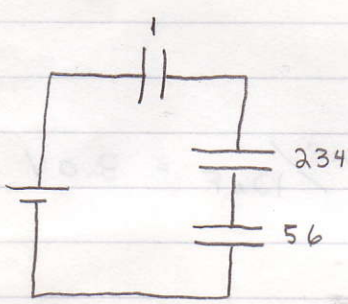


series



not series!

C_5 & C_6 are in parallel $\longrightarrow$ $C_{56} = C_5 + C_6 = 12\,\mu F$

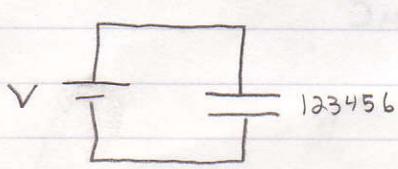


C_1 + C_{234} + C_{56} in series

$$1/C_{123456} = 1/C_1 + 1/C_{234} + 1/C_{56}$$

$$= 1/6.0\,\mu F + 1/12\,\mu F + 1/12\,\mu F$$

$$C_{123456} = 3.0\,\mu F$$



$$q_{123456} = C_{123456}\,V = (3.0\,\mu F)(12\,V)$$

$$q_{123456} = 36\,\mu C$$

$C_{123456} \rightarrow C_1$ & C_{234} & C_{56} in series

$q_1 = q_{234} = q_{56} = q_{123456} = 36\,\mu C$

$V_1 = q_1/C_1 = 36\,\mu C / 6\,\mu F = 6.0\,V$

$V_{234} = q_{234}/C_{234} = 36\,\mu C / 12\,\mu F = 3.0\,V$

$C_{234} \rightarrow C_2$ & C_3 & C_4 in parallel

$V_2 = V_3 = V_4 = 3.0\,V$

$q_2 = C_2 V_2 = (4.0\,\mu F)(3.0\,V) = 12\,\mu C$

$q_3 = C_3 V_3 = 12\,\mu C$

$q_4 = C_4 V_4 = 12\,\mu C$

$q_{56} = 36\,\mu C \qquad V_{56} = q_{56}/C_{56} = 36\,\mu C / 12\,\mu F = 3.0\,V$

$C_{56} \rightarrow C_5$ and C_6 in parallel

$V_5 = V_6 = V_{56} = 3.0\,V$

$q_5 = C_5 V_5 = (6.0\,\mu F)(3.0\,V) = 18\,\mu C$

$q_6 = C_6 V_6 = 18\,\mu C$

$q_1 = 36\mu C$ $\quad V_1 = 6.0V$

$q_2 = 12\mu C$ $\quad V_2 = 3.0V$

$q_3 = 12\mu C$ $\quad V_3 = 3.0V$

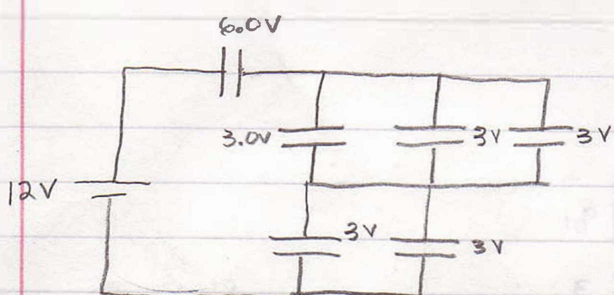
$q_4 = 12\mu C$ $\quad V_4 = 3.0V$

$q_5 = 18\mu C$ $\quad V_5 = 3.0V$

$q_6 = 18\mu C$ $\quad V_6 = 3.0V$

Two checks → 1) does $C = q/V$ for every capacitor?

2) along any closed path including battery, voltage across all capacitors must equal battery voltage



Problem 2

$$\sigma = q/A \rightarrow q = \sigma A = (15\times10^{-9}\,C/m^2)\,4\pi(0.125\,m)^2$$

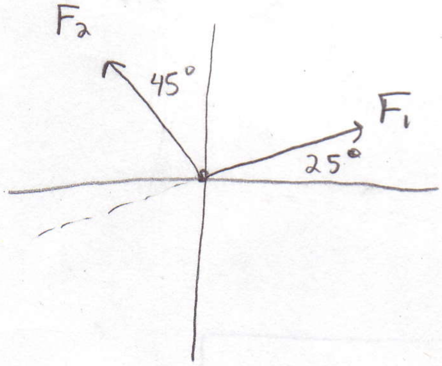
$$q = 2.95\times10^{-9}\,C$$

note: $r_1 = 0.80\,m + 0.125\,m = 0.925\,m$

$r_2 = 1.0\,m + 0.125\,m = 1.125\,m$

$$F_1 = \frac{Kq_1q_p}{r_1^2} = \frac{(8.99\times10^9\,Nm^2/c^2)(2.95\times10^{-9}\,c)(1.602\times10^{-19}\,c)}{(0.925\,m)^2} = 4.97\times10^{-18}\,N$$

$$F_2 = \frac{Kq_2q_p}{r_2^2} = 3.36\times10^{-18}$$



$$\sum F_x = F_1\cos25° - F_2\cos45° = 2.13\times10^{-18}\,N$$

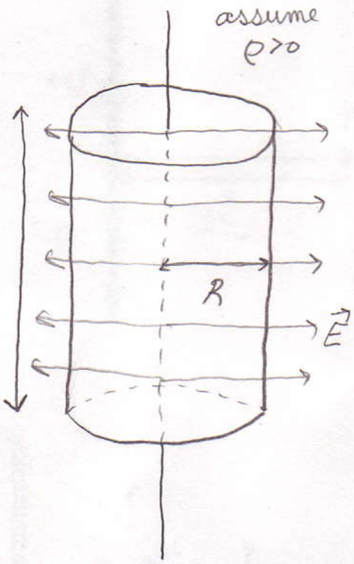
$$\sum F_y = F_1\sin25° + F_2\sin45° = 4.48\times10^{-18}$$

$$|\vec{F}| = \sqrt{F_x^2+F_y^2} = \boxed{4.96\times10^{-18}\,N}$$

$$\theta = \tan^{-1}(F_y/F_x) = \boxed{64.6°}$$

Problem 3
Charge is distributed uniformly throughout the volume of a very long solid nonconducting cylinder of radius R and uniform volume charge density ρ. Using Gauss' law, derive an expression for the electric field a distance r from the axis of the cylinder for points **(a)** inside the cylinder, and **(b)** outside the cylinder. Express your answers in terms ρ, r, and R.

(**Note**: you must start with Gauss's law and show all work to get credit.)



(a) choose a cylinder for Gaussian surface with radius of cylinder $r < R$

$$\oint \vec{E}\cdot d\vec{A} = q_{encl}/\varepsilon_0$$

$$\| $$

$$\underbrace{\int \vec{E}\cdot d\vec{A}_{\text{top cap}}}_{\substack{0 \text{ because} \\ d\vec{A} \perp \vec{E}}} + \underbrace{\int \vec{E}\cdot d\vec{A}_{\substack{\text{cylindrical} \\ \text{surface}}}}_{E(2\pi r L)} + \underbrace{\int \vec{E}\cdot d\vec{A}_{\text{bottom cap}}}_{\substack{0 \text{ because} \\ d\vec{A} \perp \vec{E}}}$$

$$\oint \vec{E}\cdot d\vec{A} = q_{encl}/\varepsilon_0 \longrightarrow E(2\pi r L) = q_{encl}/\varepsilon_0$$

$$q_{encl} = \int \rho \, dV = \int \rho(2\pi r L)\,dr = 2\pi\rho L \int r\,dr = (\pi r^2 L)\rho$$

$$q_{encl} = q\left(\frac{V_{encl}}{V}\right) = \frac{q}{V} V_{encl} = \rho(\pi r^2 L)$$

note: this is really just $q_{encl} = \rho V_{encl}$

$$E(2\pi r L) = \frac{\pi r^2 L \rho}{\varepsilon_0}$$

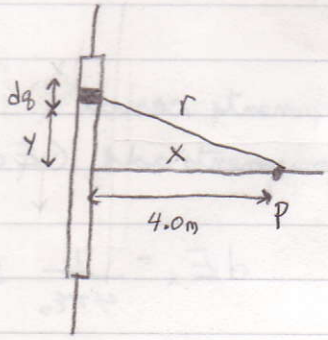
$$\boxed{E = \frac{\rho r}{2\varepsilon_0}}$$

(b) choose a cylinder for Gaussian surface with $r > R$

$\Rightarrow$ every step is the same except $q_{encl} = (\pi R^2 L)\rho$

$$E(2\pi r L) = \frac{\pi R^2 L \rho}{\varepsilon_0} \longrightarrow \boxed{E = \frac{\rho R^2}{2\varepsilon_0 r}}$$

4)



$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$$

$$dq = \lambda\, dy$$

$$r = \sqrt{x^2+y^2} \qquad x = 4.0\text{m}$$

$$V = \frac{\lambda}{4\pi\varepsilon_0}\int_{-L/2}^{L/2} (x^2+y^2)^{-1/2}\, dy \qquad L = 6.0\text{m}$$

I would give this integral

$$= \frac{2\lambda}{4\pi\varepsilon_0}\int_0^{L/2} (x^2+y^2)^{-1/2}\, dy \qquad \int (x^2+y^2)^{-1/2} dy = \ln\left(y+\sqrt{y^2+x^2}\right)$$

$$V = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(y+\sqrt{y^2+x^2}\right)\Big|_0^{L/2}$$

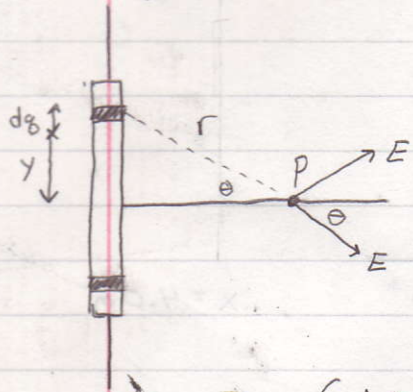
$$= \frac{\lambda}{2\pi\varepsilon_0}\left[\ln\left(L/2+\sqrt{L^2/4+x^2}\right) - \ln x\right] \qquad L = 6.0\text{m},\ x = 4.0\text{m}$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\left[\ln\left(3.0\text{m}+\sqrt{9.0\text{m}^2+16\text{m}^2}\right) - \ln 4.0\text{m}\right]$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\left[\ln 8.0\text{m} - \ln 4.0\text{m}\right] = \frac{\lambda}{2\pi\varepsilon_0}\ln 2$$

$$\boxed{V = 31{,}200\,\text{V}}$$

Find $\vec{E}$



⇒ vertical components cancel

⇒ horizontal components add ($E\cos\theta$)

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} \qquad dE_x = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\cos\theta$$

$$E_x = \int dE_x = \frac{1}{4\pi\varepsilon_0}\int \frac{dq\cos\theta}{r^2}$$

$$dq = \lambda\, dy \qquad \cos\theta = \frac{x}{\sqrt{x^2+y^2}} \qquad r^2 = x^2+y^2$$

$$E_x = \frac{1}{4\pi\varepsilon_0}\int_{-L/2}^{L/2} \frac{\lambda\, dy\, x}{(x^2+y^2)^{3/2}} \longrightarrow E_x = \frac{2\lambda x}{4\pi\varepsilon_0}\int_0^{L/2}\frac{dy}{(x^2+y^2)^{3/2}}$$

$$\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{y}{x^2(x^2+y^2)^{1/2}}$$ → I would give this integral

$$E_x = \frac{\lambda x}{2\pi\varepsilon_0}\left[\frac{y}{x^2(x^2+y^2)^{1/2}}\right]\Bigg/_0^{L/2} = \frac{\lambda x}{2\pi\varepsilon_0}\left[\frac{(L/2)}{x^2\sqrt{x^2+L^2/4}}\right]$$

$$E_x = \frac{\lambda L}{4\pi\varepsilon_0 x}\frac{1}{\sqrt{x^2+L^2/4}}$$

$\lambda = 2.5\times10^{-6}\,C/m$

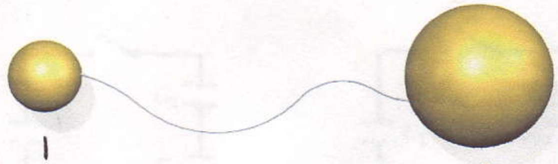
$x = 4.0\,m$

$L = 6.0\,m$

$$\boxed{E = (6.7\times10^3\,N/C)\,\hat{i}}$$

Problem 6

Charge is placed on two conducting spheres that are very far apart and connected by a long thin wire as shown in the figure below. The radius of the smaller sphere is 5.0 cm and the radius of the larger sphere is 12 cm. The electric field at the surface of the larger sphere is 200 kV/m. Find the surface charge density σ on each sphere.



on the surface of a sphere $\longrightarrow$ $E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}$ $\longrightarrow$ $q = 4\pi\varepsilon_0 R^2 E$

for sphere two: $q_2 = 4\pi\varepsilon_0 R^2 E = 4\pi(8.85\times10^{-12}\,C^2/Nm^2)(0.12m)^2(200\times10^3\,V/m)$

$q_2 = 0.32\,\mu C$

$\Rightarrow$ both sphere must be at the same potential so $V_1 = V_2$ or

$\frac{1}{4\pi\varepsilon_0}\frac{q_1}{R_1} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{R_2} \longrightarrow \frac{q_1}{R_1} = \frac{q_2}{R_2} \longrightarrow q_1 = \frac{R_1}{R_2}q_2$

$q_1 = \frac{(0.05m)}{(0.12m)}(0.32\mu C) = 0.133\,\mu C$

$\sigma = q/A = q/4\pi R^2$

$\sigma_1 = q_1/4\pi R_1^2 = \frac{(0.133\mu C)}{4\pi(0.05m)^2} \longrightarrow \boxed{\sigma_1 = 4.2\,\mu C/m^2}$

$\sigma_2 = q_2/4\pi R_2^2 = \frac{(0.32\mu C)}{4\pi(0.12m)^2} \longrightarrow \boxed{\sigma_2 = 1.8\,\mu C/m^2}$