Answer Key

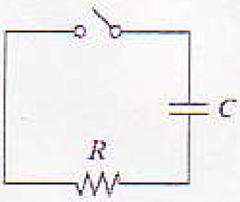
# Review for Test #2: Circuits and Magnetism

## Multiple Choice Questions

**1)** A simple circuit consists of a resistor $R$, a capacitor $C$ charged to a potential $V_0$, and a switch that is initially open but then thrown closed. Immediately after the switch is thrown closed, the current in the circuit is

A

[✓ a) $V_0/R$.]
b) zero.
c) need more information



$i = \mathcal{E}/R\, e^{-t/2}$

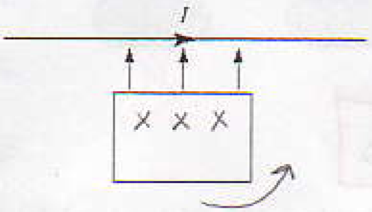
at $t=0$ $i = \mathcal{E}/R = V_0/R$

⇒ note; initially a capacitor acts like a wire

**2)** A long, straight wire carries a steady current $I$. A rectangular conducting loop lies in the same plane as the wire, with two sides parallel to the wire and two sides perpendicular. Suppose the loop is pushed toward the wire as shown. Given the direction of $I$, the induced current in the loop is

B

a) clockwise.
[✓ b) counterclockwise.]
c) need more information



This is from Chapter 30 and will not be on Celebration #2

**3)** A negative particle moves upward along the trajectory shown. A magnetic field points toward the right. In which direction is the magnetic force on the particle?

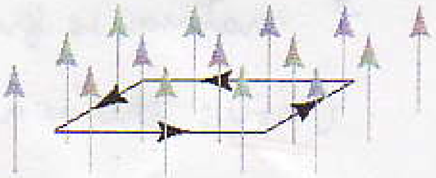
D

a) left
b) right
c) into the page
[✓ d) out of the page]

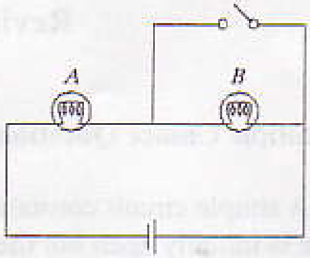
**4)** A rectangular loop is placed in a uniform magnetic field with the plane of the loop perpendicular to the direction of the field. If a current is made to flow through the loop in the sense shown by the arrows, the field exerts on the loop:

D

a) a net force.
b) a net torque.
c) a net force and a net torque.
[✓ d) neither a net force nor a net torque.]



5) The circuit below consists of two identical light bulbs burning with equal brightness and a single 12 V battery. When the switch is closed, the brightness of bulb *A*



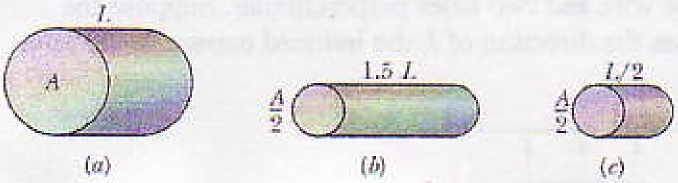
$\boxed{1}$

[✓ 1. increases]
2. decreases
3. remains the same
4. need more information

**Short Answer Questions**

1) The figure here shows three cylindrical copper conductors along with their face areas and lengths. Rank them according to the current through them, greatest first, when the same potential difference V is placed across their lengths.

$R = \rho L/A \rightarrow$ smaller R, greater i



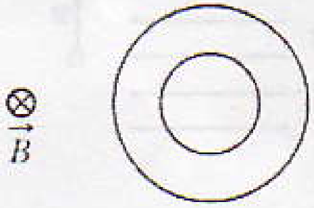
$1 \rightarrow R = \rho L/A$

$2 \rightarrow R = \frac{\rho(1.5L)}{(A/2)} = 3\rho L/A$

$3 \rightarrow R = \rho(L/2)/(A/2) = \rho L/A$

$\boxed{\text{a \& c tie, b}}$

2) The figure here shows the circular paths of two particles that travel at the same speed in a uniform magnetic field, which is directed into the page. One particle is a proton; the other is an electron (which is less massive). (a) Which particle follows the smaller circle, and (b) does that particle travel clockwise or counterclockwise?



$r = \frac{mv}{qB} \rightarrow$ smaller m, smaller r

(a) $\boxed{\text{electron}}$

(b) $\boxed{\text{clockwise (from RHR)}}$

3) Suppose two charged conducting spheres of different radii $r_1$ and $r_2 > r_1$ are connected by a conducting wire. Which sphere has the greater charge density?

material is from Test #1

(note: this is a good question to review for the final)

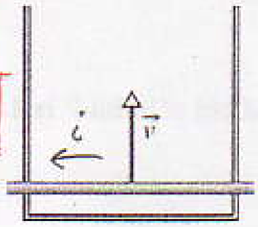
**4)** The figure below shows two circuits in which a conducting bar is slid at the same speed v through the same uniform magnetic field and along a U-shaped wire. The parallel lengths of the wire are separated by 2L in circuit 1 and by L in circuit 2. The current induced in circuit 1 is counterclockwise. (a) Is the direction of the magnetic field into or out of the page? (b) Is the direction of the current induced in circuit 2 clockwise or counterclockwise? (c) Is the emf induced in circuit 1 larger than, smaller than, or the same as that in circuit 2?

(a) $\boxed{\text{into the page}}$

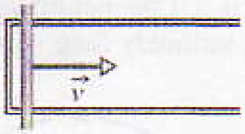
(b) $\boxed{\text{counterclockwise}}$

(c) $\boxed{\text{larger}}$

since $d\Phi/dt$ is bigger since $dA/dt$



(1)



(2)

⇒ the flux is increasing so the induced current creates a $\vec{B}$-field in the opposite direction of the external field

**5)** The energy stored in the magnetic field of an inductor is given by $U_B = \frac{1}{2} Li^2$. Verify that both sides of this equation have the same units.

This is from Chapter 30 and will not be on Celebration #2

$[U] = J$

$[L] = H = Tm^2/A$

$[i] = A$

$U_B = \frac{1}{2} L i^2$

$J = (Tm^2/A)(A)^2 = Tm^2A$

$1T = 1N/Am$

$J = (N/Am)m^2A = Nm$

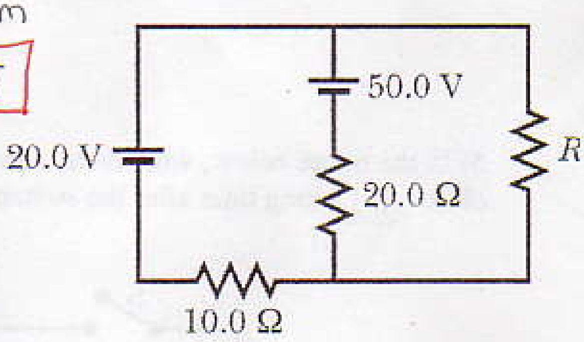
$\boxed{J = J}$

**Problems**

**1)** In the circuit shown to the right, R = 25.0 Ω.

**(a)** Use Kirchoff's laws to find the current through each resistor. (10 points)

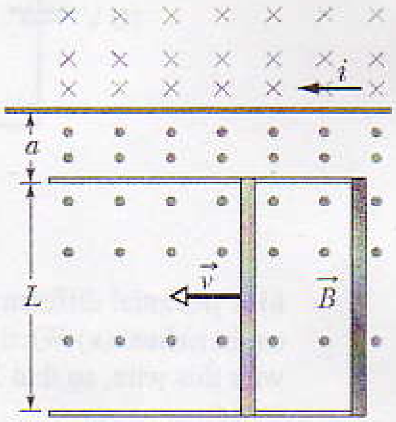
**(b)** Calculate the power from each battery and each resistor. (6 points)



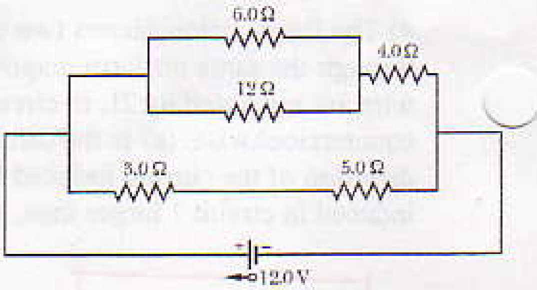
**2)** The figure to the right shows a rod of length L caused to move at a constant speed v along horizontal rails. The magnetic field in which the rod moves is not uniform but is provided by a current i in a long wire parallel to the rails. Assume that v = 3.00 m/s, a = 5.0 cm, L = 20.0 cm, and i = 5.0 A.

**a)** What is the direction of the induced current? (4 points)

**b)** Assuming that the resistance of the rod is 2.00 Ω and the resistance of the rails and the strip that connects them is negligible, at what rate is thermal energy being generated in the rod? (12 points)



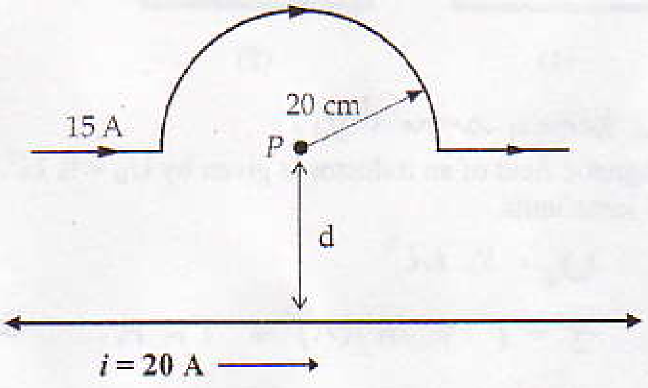
**3)** A circuit containing five resistors connected to a 12 V battery is shown to the right.



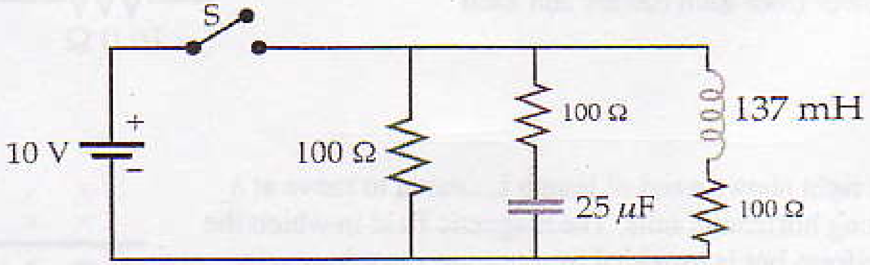
**(a)** What is the equivalent resistance of the circuit? (8 points)

**(b)** What is the current through and the potential drop across the $3.0\,\Omega$ and $6.0\,\Omega$ resistors? (8 points)

**4)** In the figure below, what is d if the net magnetic field at point P is 8.60 μT into the page? (**Note**: the bottom wire is an infinitely long wire.)



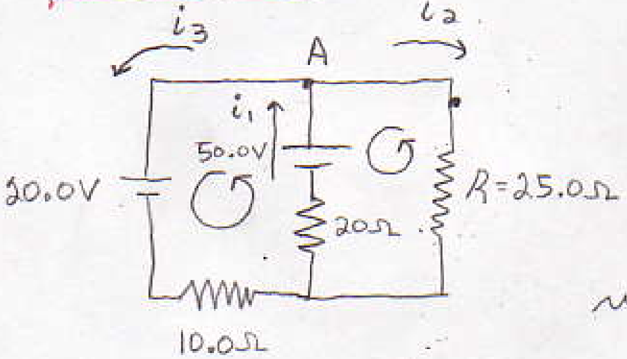
**5)** In the figure below, what is the current through the battery **(a)** immediately after the switch is closed, **(b)** a long time after the switch is closed, and **(c)** 2.0 ms after the switch is closed?



**6)** A potential difference of 2.0 V produces a current of 3.4 A in a 250-m length of wire that is 0.30 cm in radius. **(a)** What is the resistivity of the wire? **(b)** What resistance R could you put in parallel with this wire, so that 25% of the current in the circuit flows through R?

# Review for Test #2

## Problem 1



from junction A $\rightarrow$ $i_1 = i_2 + i_3$ (1)

left loop:

$$-20.0V - i_3(10.0\Omega) - i_1(20.0\Omega) + 50.0V = 0 \quad (2)$$

right loop:

$$-50.0V + i_1(20.0\Omega) + i_2(25.0\Omega) = 0 \quad (3)$$

① $i_1 = i_2 + i_3$

② $20i_1 + 10i_3 = 30 \longrightarrow i_3 = 3 - 2i_1$ ④

③ $20i_1 + 25i_2 = 50 \longrightarrow i_2 = 2 - 0.80i_1$ ⑤

put ④ + ⑤ into ① $\quad i_1 = (2 - 0.80i_1) + (3 - 2i_1) \longrightarrow i_1 = 5 - 2.8i_1$

$$\boxed{i_1 = 1.316A}$$

from ④ $\quad i_3 = 3 - 2(1.32A) \longrightarrow \boxed{i_3 = 0.368A}$

from ⑤ $\quad i_2 = 2 - 0.80(1.32A) \longrightarrow \boxed{i_2 = 0.947A}$

(b) $P = i\mathcal{E}$ battery

$P = i^2R$ resistor

20.0V battery $\rightarrow$ $P = i_3\mathcal{E} = 7.36W$

50.0V battery $\rightarrow$ $P = i_1\mathcal{E} = 65.8W$

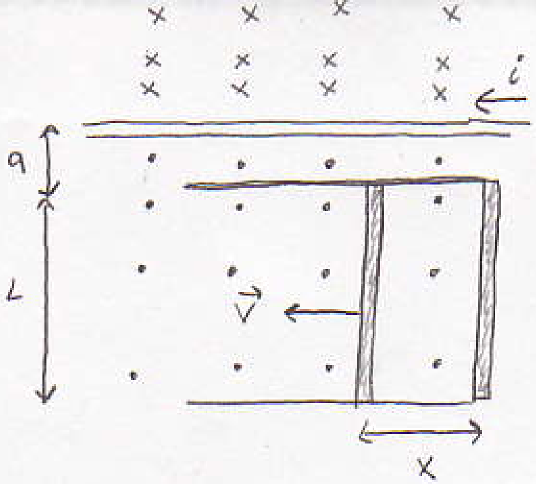
10.0Ω resistor $\rightarrow$ $P = i_3^2R = 1.35W$

20.0Ω resistor $\rightarrow$ $P = i_1^2R = 34.6W$

25.0Ω resistor $\rightarrow$ $P = i_2^2R = 22.5W$

note: $P_{50V\ battery} = P_{20V\ battery} + P_{resistors}$ (20V battery is being charged)

# Problem 2



This is from Chapter 30 and will not be on Celebration #2

a) Since $\vec{A}$ is increasing, $\Phi_B$ is increasing so the induced $\vec{B}$ points into the page. Therefore the induced current is

$\boxed{\text{clockwise.}}$

b) $\Phi_B = \int \vec{B}\cdot d\vec{A} = \int B\,dA$

for a long wire $B = \frac{\mu_0 i}{2\pi r} = \frac{\mu_0 i}{2\pi y}$

$dA = x\,dy$

$$\Phi = \int_a^{L+a} \left(\frac{\mu_0 i}{2\pi y}\right) x\,dy = \frac{\mu_0 i x}{2\pi}\int_a^{L+a} \frac{dy}{a} = \frac{\mu_0 i x}{2\pi}\ln\left(\frac{L+a}{a}\right)$$

$$|\mathcal{E}| = N\,d\Phi/dt = (1)\,d/dt\left[\frac{\mu_0 i x}{2\pi}\ln\left(\frac{L+a}{a}\right)\right]$$

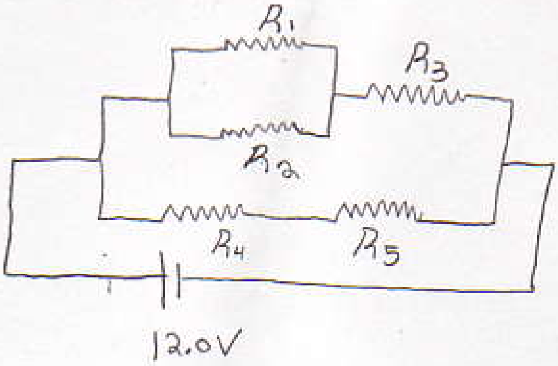
$$= \frac{\mu_0 i}{2\pi}\ln\left(\frac{L+a}{a}\right)\frac{dx}{dt} = \frac{\mu_0 i v}{2\pi}\ln\left(\frac{L+a}{a}\right)$$

$$P = i^2 R = \mathcal{E}^2/R \longrightarrow P = \left[\frac{\mu_0 i v}{2\pi}\ln\left(\frac{L+a}{a}\right)\right]^2/R$$

$$P = \left[\frac{(4\pi\times10^{-7}\,\text{Tm/A})(5.0\,\text{A})(3.00\,\text{m/s})}{2\pi}\ln\left(\frac{25\,\text{cm}}{5\,\text{cm}}\right)\right]^2/(2.00\,\Omega)$$

$$\boxed{P = 1.17\times10^{-11}\,\text{W}}$$

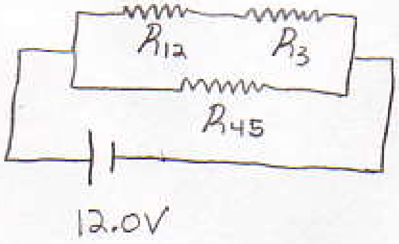
# $\boxed{\text{Problem 3}}$



$R_1 = 6.0\Omega$ $R_3 = 4.0\Omega$ $R_5 = 5.0\Omega$

$R_2 = 12\Omega$ $R_4 = 3.0\Omega$

$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} \longrightarrow R_{12} = 4.0\Omega$$



$$R_{45} = R_4 + R_5 = 8.0\Omega$$

$$R_{123} = R_{12} + R_3 = 8.0\Omega$$

$$1/R_{12345} = \frac{1}{R_{123}} + \frac{1}{R_{45}} \longrightarrow \boxed{R_{12345} = 4.0\Omega}$$

(b) $i_{45} = V_{45}/R_{45} = \frac{12.0V}{8.0\Omega} = 1.5A$

$$i_4 = i_{3.0\Omega} = \boxed{1.5A}$$

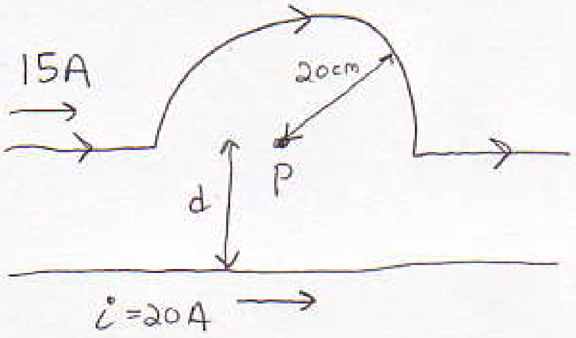
$$V_4 = i_4 R_4 = (1.5A)(3.0\Omega) = \boxed{4.5V}$$

$$i_{123} = \frac{V_{123}}{R_{123}} = \frac{12V}{8.0\Omega} = 1.5A \longrightarrow i_{12} = 1.5A$$

$$V_{12} = i_{12} R_{12} = (1.5A)(4.0\Omega) = 6.0V \longrightarrow \boxed{V_2 = V_{12\Omega} = 6.0V}$$

$$i_2 = V_2/R_2 = \frac{6.0V}{12\Omega} = \boxed{0.50A}$$

# Problem 4



arc: $B = \frac{\mu_0 i \phi}{4\pi R}$

long wire: $B = \frac{\mu_0 i}{2\pi R}$

$\Rightarrow$ $\vec{B}$ from the arc is into the page and $\vec{B}$ from the long wire is out of the page; define into page as +

$$B_{net} = B_{arc} - B_{wire} = 8.60\,\mu T$$

$$B_{wire} = B_{arc} - 8.60\,\mu T = \frac{\mu_0 i \pi}{4\pi R} - 8.60\,\mu T$$

$$B_{wire} = \frac{(4\pi \times 10^{-7}\,Tm/A)(15A)\pi}{4\pi\,(0.20m)} - 8.60 \times 10^{-6}\,T$$

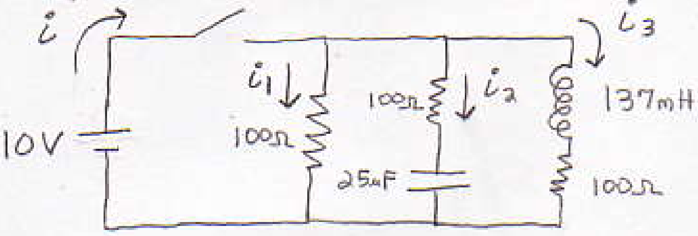
$$B_{wire} = \underline{\underline{1.50 \times 10^{-5}\,T}}$$

$$B_{wire} = \frac{\mu_0 i}{2\pi d} \longrightarrow d = \frac{\mu_0 i}{2\pi B}$$

$$d = \frac{(4\pi \times 10^{-7}\,Tm/A)(20A)}{2\pi\,(1.50 \times 10^{-5}\,T)}$$

$$\boxed{d = 0.267m}$$

# Problem 5



$$i = i_1 + i_2 + i_3$$

This is from Chapter 30 and will not be on Celebration #2

$i_1 = 10\,V / 100\,\Omega = 0.10\,A$ always

$\Rightarrow$ for an RC circuit $i_2 = \mathcal{E}/R\, e^{-t/\tau_c}$ $\quad \tau_c = RC = 0.0025\,s$

$\Rightarrow$ for an RL circuit $i_3 = \mathcal{E}/R\,(1 - e^{-t/\tau_L})$ $\quad \tau_L = L/R = 0.00137\,s$

(a) $$i = i_1 + i_2 + i_3 = 0.10\,A + \frac{(10\,V)}{100\,\Omega} e^{0} + 0 = 0.10\,A + 0.10\,A + 0$$

$$\boxed{i = 0.20\,A}$$

(b) $$i = 0.10\,A + \frac{(10\,V)}{100\,\Omega} e^{-\infty} + \frac{(10\,V)}{100\,\Omega}(1 - e^{-\infty}) = 0.10\,A + 0 + 0.10\,A$$

$$\boxed{i = 0.20\,A}$$

(c) $i_1 = 0.10\,A$

$$i_2 = \frac{(10\,V)}{100\,\Omega} e^{-\frac{0.0020\,s}{0.0025\,s}} = 0.045\,A$$

$$i_3 = \frac{(10\,V)}{100\,\Omega} \left[1 - e^{-\frac{0.0020\,s}{0.0025\,s}}\right] = 0.055\,A$$

$$\boxed{i = i_1 + i_2 + i_3 = 0.20\,A}$$

# Problem 6

$$\left.\begin{array}{l} V = 2.0\,\text{V} \\ i = 3.4\,\text{A} \end{array}\right\} \quad R = V/i = \frac{2.0\,\text{V}}{3.4\,\text{A}} = \underline{0.588\,\Omega}$$

$L = 250\,\text{m}$

$A = \pi r^2 = \pi\,(0.30\times10^{-2}\,\text{m})^2 = 2.83\times10^{-5}\,\text{m}^2$

(a) $R = \rho L/A \longrightarrow \rho = RA/L = \dfrac{(0.588\,\Omega)(2.83\times10^{-5}\,\text{m}^2)}{(250\,\text{m})}$

$$\boxed{\rho = 6.65\times10^{-8}\,\Omega\cdot\text{m}}$$

(b)

$$\left.\begin{array}{l} \text{25\% of current through } R \\ \text{75\% of current through wire} \end{array}\right\} \quad \underline{\underline{i_{wire} = 3\,i_R}}$$

⇒ since resistance R and wire are in parallel, each has the same potential difference V across it

$$i_{wire} = 3\,i_R \longrightarrow V/R_{wire} = 3\,(V/R)$$

$$R = 3\,R_{wire} = 3\,(0.588\,\Omega)$$

$$\boxed{R = 1.8\,\Omega}$$