Physics 4B

Solutions to Chapter 21 HW

Chapter 21: Questions: 4, 8, 10
Problems: 11, 13, 15, 22, 27, 39, 56, 63, 64

Question 21-4
(a) between; (b) positively charged; (c) unstable

Question 21-8
a and *d* tie, then *b* and *c* tie

Question 21-10
$6kq^2/d^2$, leftward

Problem 21-11
The force experienced by q_3 is

$$\vec{F}_3 = \vec{F}_{31} + \vec{F}_{32} + \vec{F}_{34} = \frac{1}{4\pi\varepsilon_0}\left(-\frac{|q_3||q_1|}{a^2}\hat{\mathrm{j}} + \frac{|q_3||q_2|}{(\sqrt{2}a)^2}(\cos 45°\hat{\mathrm{i}} + \sin 45°\hat{\mathrm{j}}) + \frac{|q_3||q_4|}{a^2}\hat{\mathrm{i}} \right)$$

(a) Therefore, the *x*-component of the resultant force on q_3 is

$$F_{3x} = \frac{|q_3|}{4\pi\varepsilon_0 a^2}\left(\frac{|q_2|}{2\sqrt{2}} + |q_4|\right) = \left(8.99\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{2\left(1.0\times10^{-7}\,\mathrm{C}\right)^2}{(0.050\ \mathrm{m})^2}\left(\frac{1}{2\sqrt{2}} + 2\right) = 0.17\ \mathrm{N}.$$

(b) Similarly, the *y*-component of the net force on q_3 is

$$F_{3y} = \frac{|q_3|}{4\pi\varepsilon_0 a^2}\left(-|q_1| + \frac{|q_2|}{2\sqrt{2}}\right) = \left(8.99\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{2\left(1.0\times10^{-7}\,\mathrm{C}\right)^2}{(0.050\ \mathrm{m})^2}\left(-1 + \frac{1}{2\sqrt{2}}\right) = -0.046\ \mathrm{N}.$$

Problem 21-13
(a) There is no equilibrium position for q_3 *between* the two fixed charges, because it is being pulled by one and pushed by the other (since q_1 and q_2 have different signs); in this region this means the two force arrows on q_3 are in the same direction and cannot cancel.
It should also be clear that off-axis (with the axis defined as that which passes through the two fixed charges) there are no equilibrium positions. On the semi-infinite region of the axis that is nearest q_2 and furthest from q_1 an equilibrium position for q_3 cannot be found because $|q_1| < |q_2|$ and the magnitude of force exerted by q_2 is everywhere (in that region) stronger than that exerted by q_1 on q_3. Thus, we must look in the semi-infinite region of the axis which is nearest q_1 and furthest from q_2, where the net force on q_3 has magnitude

$$\left| k\frac{|q_1q_3|}{L_0^2} - k\frac{|q_2q_3|}{(L+L_0)^2} \right|$$

with L = 10 cm and L_0 is assumed to be *positive*. We set this equal to zero, as required by the problem, and cancel k and q_3. Thus, we obtain

$$\frac{|q_1|}{L_0^2} - \frac{|q_2|}{(L+L_0)^2} = 0 \;\Rightarrow\; \left(\frac{L+L_0}{L_0}\right)^2 = \left|\frac{q_2}{q_1}\right| = \left|\frac{-3.0\;\mu\text{C}}{+1.0\;\mu\text{C}}\right| = 3.0$$

which yields (after taking the square root)

$$\frac{L+L_0}{L_0} = \sqrt{3} \;\Rightarrow\; L_0 = \frac{L}{\sqrt{3}-1} = \frac{10\text{ cm}}{\sqrt{3}-1} \approx 14\,\text{cm}$$

for the distance between q_3 and q_1. That is, q_3 should be placed at $x = -14$ cm along the x-axis.

(b) As stated above, y = 0.

Problem 21-15

(a) The distance between q_1 and q_2 is

$$r_{12} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(-0.020\text{ m} - 0.035\text{ m})^2 + (0.015\text{ m} - 0.005\text{ m})^2} = 0.056\text{ m}.$$

The magnitude of the force exerted by q_1 on q_2 is

$$F_{21} = k\frac{|q_1q_2|}{r_{12}^2} = \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\,(3.0\times10^{-6}\,\text{C})\,(4.0\times10^{-6}\,\text{C})}{(0.056\text{ m})^2} = 35\text{ N}.$$

(b) The vector $\vec{F}_{21}$ is directed toward q_1 and makes an angle θ with the $+x$ axis, where

$$\theta = \tan^{-1}\left(\frac{y_2-y_1}{x_2-x_1}\right) = \tan^{-1}\left(\frac{1.5\text{ cm} - 0.5\text{ cm}}{-2.0\text{ cm} - 3.5\text{ cm}}\right) = -10.3° \approx -10°.$$

(c) Let the third charge be located at (x_3, y_3), a distance r from q_2. We note that q_1, q_2, and q_3 must be collinear; otherwise, an equilibrium position for any one of them would be impossible to find. Furthermore, we cannot place q_3 on the same side of q_2 where we also find q_1, since in that region both forces (exerted on q_2 by q_3 and q_1) would be in the same direction (since q_2 is attracted to both of them). Thus, in terms of the angle found in part (a), we have $x_3 = x_2 - r\cos\theta$ and $y_3 = y_2 - r\sin\theta$ (which means $y_3 > y_2$ since θ is negative). The magnitude of force exerted on q_2 by q_3 is $F_{23} = k|q_2q_3|/r^2$, which must equal that of the force exerted on it by q_1 (found in part (a)). Therefore,

$$k\frac{|q_2q_3|}{r^2} = k\frac{|q_1q_2|}{r_{12}^2} \quad \Rightarrow \quad r = r_{12}\sqrt{\frac{q_3}{q_1}} = 0.0645\,\text{m} = 6.45\text{ cm}\ .$$

Consequently, $x_3 = x_2 - r\cos\theta = -2.0\text{ cm} - (6.45\text{ cm})\cos(-10°) = -8.4\text{ cm}$,

(d) and $y_3 = y_2 - r\sin\theta = 1.5\text{ cm} - (6.45\text{ cm})\sin(-10°) = 2.7\text{ cm}$.

Problem 21-22

(a) We note that $\cos(30°) = \frac{1}{2}\sqrt{3}$, so that the dashed line distance in the figure is $r = 2d/\sqrt{3}$.

The net force on q_1 due to the two charges q_3 and q_4 (with $|q_3| = |q_4| = 1.60 \times 10^{-19}$ C) on the y axis has magnitude

$$2\frac{|q_1q_3|}{4\pi\varepsilon_0 r^2}\cos(30°) = \frac{3\sqrt{3}\,|q_1q_3|}{16\pi\varepsilon_0 d^2}\ .$$

This must be set equal to the magnitude of the force exerted on q_1 by $q_2 = 8.00 \times 10^{-19}\text{ C} = 5.00\,|q_3|$ in order that its net force be zero:

$$\frac{3\sqrt{3}\,|q_1q_3|}{16\pi\varepsilon_0 d^2} = \frac{|q_1q_2|}{4\pi\varepsilon_0 (D+d)^2} \quad \Rightarrow \quad D = d\left(2\sqrt{\frac{5}{3\sqrt{3}}} - 1\right) = 0.9245\,d.$$

Given $d = 2.00$ cm, this then leads to $D = 1.92$ cm.

(b) As the angle decreases, its cosine increases, resulting in a larger contribution from the charges on the y axis. To offset this, the force exerted by q_2 must be made stronger, so that it must be brought closer to q_1 (keep in mind that Coulomb's law is *inversely* proportional to distance-squared). Thus, D must be decreased.

Problem 21-27

(a) The magnitude of the force between the (positive) ions is given by

$$F = \frac{(q)(q)}{4\pi\varepsilon_0 r^2} = k\frac{q^2}{r^2}$$

where q is the charge on either of them and r is the distance between them. We solve for the charge:

$$q = r\sqrt{\frac{F}{k}} = \left(5.0\times10^{-10}\text{ m}\right)\sqrt{\frac{3.7\times10^{-9}\text{ N}}{8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2}} = 3.2\times10^{-19}\text{ C}.$$

(b) Let n be the number of electrons missing from each ion. Then, $ne = q$, or

$$n = \frac{q}{e} = \frac{3.2\times10^{-9}\text{ C}}{1.6\times10^{-19}\text{ C}} = 2.$$

Problem 21-39

Using Coulomb's law, the magnitude of the force of particle 1 on particle 2 is $F_{21} = k\dfrac{q_1 q_2}{r^2}$,

where $r = \sqrt{d_1^2 + d_2^2}$ and $k = 1/4\pi\varepsilon_0 = 8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2$. Since both q_1 and q_2 are positively charged, particle 2 is repelled by particle 1, so the direction of $\vec{F}_{21}$ is away from particle 1 and toward 2. In unit-vector notation, $\vec{F}_{21} = F_{21}\hat{\text{r}}$, where

$$\hat{\text{r}} = \frac{\vec{r}}{r} = \frac{(d_2\hat{\text{i}} - d_1\hat{\text{j}})}{\sqrt{d_1^2 + d_2^2}}.$$

The x component of $\vec{F}_{21}$ is $F_{21,x} = F_{21}d_2/\sqrt{d_1^2 + d_2^2}$. Combining the expressions above, we obtain

$$\begin{aligned}
F_{21,x} &= k\frac{q_1 q_2 d_2}{r^3} = k\frac{q_1 q_2 d_2}{(d_1^2 + d_2^2)^{3/2}} \\
&= \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(4\cdot1.60\times10^{-19}\text{ C})(6\cdot1.60\times10^{-19}\text{ C})(6.00\times10^{-3}\text{ m})}{\left[(2.00\times10^{-3}\text{ m})^2 + (6.00\times10^{-3}\text{ m})^2\right]^{3/2}} \\
&= 1.31\times10^{-22}\text{ N}
\end{aligned}$$

Note: In a similar manner, we find the y component of $\vec{F}_{21}$ to be

$$\begin{aligned}
F_{21,y} &= -k\frac{q_1 q_2 d_1}{r^3} = -k\frac{q_1 q_2 d_1}{(d_1^2 + d_2^2)^{3/2}} \\
&= -\frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(4\cdot1.60\times10^{-19}\text{ C})(6\cdot1.60\times10^{-19}\text{ C})(2.00\times10^{-3}\text{ m})}{\left[(2.00\times10^{-3}\text{ m})^2 + (6.00\times10^{-3}\text{ m})^2\right]^{3/2}} \\
&= -0.437\times10^{-22}\text{ N}
\end{aligned}$$

Thus, $\vec{F}_{21} = (1.31\times10^{-22}\text{ N})\hat{\text{i}} - (0.437\times10^{-22}\text{ N})\hat{\text{j}}$.

Problem 21-56

(a) Equation 21-11 (in absolute value) gives

$$n = \frac{|q|}{e} = \frac{2.00\times10^{-6}\text{ C}}{1.60\times10^{-19}\text{ C}} = 1.25\times10^{13}\text{ electrons}.$$

(b) Since you have the excess electrons (and electrons are lighter and more mobile than protons) then the electrons "leap" from you to the faucet instead of protons moving from the faucet to you (in the process of neutralizing your body).

(c) Unlike charges attract, and the faucet (which is grounded and is able to gain or lose any number of electrons due to its contact with Earth's large reservoir of mobile charges) becomes positively charged, especially in the region closest to your (negatively charged) hand, just before the spark.

(d) The cat is positively charged (before the spark), and by the reasoning given in part (b) the flow of charge (electrons) is from the faucet to the cat.

(e) If we think of the nose as a conducting sphere, then the side of the sphere closest to the fur is of one sign (of charge) and the side furthest from the fur is of the opposite sign (which, additionally, is oppositely charged from your bare hand, which had stroked the cat's fur). The charges in your hand and those of the furthest side of the "sphere" therefore attract each other, and when close enough, manage to neutralize (due to the "jump" made by the electrons) in a painful spark.

Problem 21-63

The magnitude of the net force on the $q = 42 \times 10^{-6}$ C charge is

$$k\frac{q_1\,q}{0.28^2} + k\frac{|q_2|q}{0.44^2}$$

where $q_1 = 30 \times 10^{-9}$ C and $|q_2| = 40 \times 10^{-9}$ C. This yields 0.22 N. Using Newton's second law, we obtain

$$m = \frac{F}{a} = \frac{0.22\,\text{N}}{100 \times 10^3\ \text{m/s}^2} = 2.2 \times 10^{-6}\,\text{kg}.$$

Problem 21-64

Let the two charges be q_1 and q_2. Then $q_1 + q_2 = Q = 5.0 \times 10^{-5}$ C. We use Eq. 21-1:

$$1.0\ \text{N} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)q_1q_2}{(2.0\ \text{m})^2}.$$

We substitute $q_2 = Q - q_1$ and solve for q_1 using the quadratic formula. The two roots obtained are the values of q_1 and q_2, since it does not matter which is which. We get $1.2\times10^{-5}\ \text{C}$ and 3.8×10^{-5} C. Thus, the charge on the sphere with the smaller charge is $1.2\times10^{-5}\ \text{C}$.