Physics 4B

Solutions to Chapter 22 HW

Chapter 22: Questions: 4, 8, 10
Problems: 7, 21, 28, 31, 37, 45, 48, 54, 77

Question 22-4
2, 4, 3, 1 (zero)

Question 22-8
(a) positive; (b) same

Question 22-10
(a) rightward;(b) $+q_1$ and $-q_3$, increase; $+q_2$, decrease; *n*, same

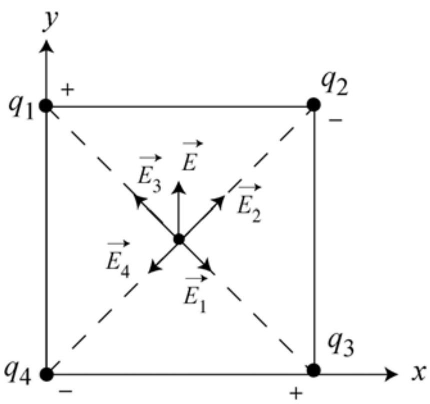
Problem 22-7
The *x* component of the electric field at the center of the square is given by

$$\begin{aligned}E_x &= \frac{1}{4\pi\varepsilon_0}\left[\frac{|q_1|}{(a/\sqrt{2})^2}+\frac{|q_2|}{(a/\sqrt{2})^2}-\frac{|q_3|}{(a/\sqrt{2})^2}-\frac{|q_4|}{(a/\sqrt{2})^2}\right]\cos 45^\circ \\ &= \frac{1}{4\pi\varepsilon_0}\frac{1}{a^2/2}\left(|q_1|+|q_2|-|q_3|-|q_4|\right)\frac{1}{\sqrt{2}} \\ &= 0.\end{aligned}$$

Similarly, the *y* component of the electric field is

$$\begin{aligned}E_y &= \frac{1}{4\pi\varepsilon_0}\left[-\frac{|q_1|}{(a/\sqrt{2})^2}+\frac{|q_2|}{(a/\sqrt{2})^2}+\frac{|q_3|}{(a/\sqrt{2})^2}-\frac{|q_4|}{(a/\sqrt{2})^2}\right]\cos 45^\circ \\ &= \frac{1}{4\pi\varepsilon_0}\frac{1}{a^2/2}\left(-|q_1|+|q_2|+|q_3|-|q_4|\right)\frac{1}{\sqrt{2}} \\ &= \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2.0\times10^{-8}\ \text{C})}{(0.050\ \text{m})^2/2}\frac{1}{\sqrt{2}} = 1.02\times10^5\ \text{N/C}.\end{aligned}$$

Thus, the electric field at the center of the square is $\vec{E} = E_y\hat{\text{j}} = (1.02\times10^5\ \text{N/C})\hat{\text{j}}$. The net electric field is depicted in the figure below (not to scale). The field, pointing to the +*y* direction, is the vector sum of the electric fields of individual charges.



Problem 22-21

Think of the quadrupole as composed of two dipoles, each with dipole moment of magnitude $p = qd$. The moments point in opposite directions and produce fields in opposite directions at points on the quadrupole axis. Consider the point P on the axis, a distance z to the right of the quadrupole center and take a rightward pointing field to be positive. Then, the field produced by the right dipole of the pair is $qd/2\pi\varepsilon_0(z - d/2)^3$ and the field produced by the left dipole is $-qd/2\pi\varepsilon_0(z + d/2)^3$. Use the binomial expansions

$$(z - d/2)^{-3} \approx z^{-3} - 3z^{-4}(-d/2)$$

$$(z + d/2)^{-3} \approx z^{-3} - 3z^{-4}(d/2)$$

to obtain

$$E = \frac{qd}{2\pi\varepsilon_0}\left[\frac{1}{z^3} + \frac{3d}{2z^4} - \frac{1}{z^3} + \frac{3d}{2z^4}\right] = \frac{6qd^2}{4\pi\varepsilon_0 z^4}.$$

Let $Q = 2qd^2$. We have $E = \dfrac{3Q}{4\pi\varepsilon_0 z^4}$.

Problem 22-28

We find the maximum by differentiating Eq. 22-16 and setting the result equal to zero.

$$\frac{d}{dz}\left(\frac{qz}{4\pi\varepsilon_0\left(z^2 + R^2\right)^{3/2}}\right) = \frac{q}{4\pi\varepsilon_0}\frac{R^2 - 2z^2}{\left(z^2 + R^2\right)^{5/2}} = 0$$

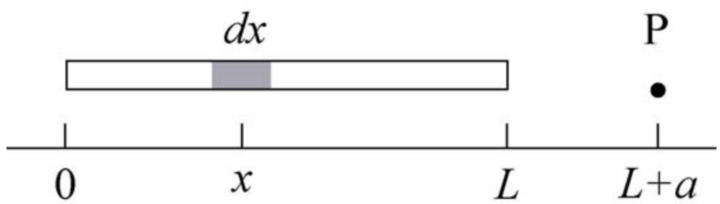
which leads to $z = R/\sqrt{2}$. With $R = 2.40$ cm, we have $z = 1.70$ cm.

Problem 22-31

(a) The linear charge density is the charge per unit length of rod. Since the charge is uniformly distributed on the rod,

$$\lambda = \frac{-q}{L} = \frac{-4.23\times10^{-15}\ \text{C}}{0.0815\ \text{m}} = -5.19\times10^{-14}\ \text{C/m}.$$

(b) We position the *x* axis along the rod with the origin at the left end of the rod, as shown in the diagram.



Let *dx* be an infinitesimal length of rod at *x*. The charge in this segment is $dq = \lambda\, dx$. The charge *dq* may be considered to be a point charge. The electric field it produces at point *P* has only an *x* component, and this component is given by

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{(L+a-x)^2}.$$

The total electric field produced at *P* by the whole rod is the integral

$$E_x = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(L+a-x)^2} = \frac{\lambda}{4\pi\varepsilon_0}\frac{1}{L+a-x}\bigg|_0^L = \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{L+a}\right)$$
$$= \frac{\lambda}{4\pi\varepsilon_0}\frac{L}{a(L+a)} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{a(L+a)},$$

upon substituting $-q = \lambda L$. With q = 4.23 × 10^{-15} C, L =0.0815 m and a = 0.120 m, we obtain $E_x = -1.57\times10^{-3}$ N/C, or $|E_x| = 1.57\times10^{-3}$ N/C.

(c) The negative sign in E_x indicates that the field points in the –*x* direction, or −180° counterclockwise from the +*x* axis.

(d) If *a* is much larger than *L*, the quantity *L* + *a* in the denominator can be approximated by *a*, and the expression for the electric field becomes

$$E_x = -\frac{q}{4\pi\varepsilon_0 a^2}.$$

Since $a = 50$ m $\gg L = 0.0815$ m, the above approximation applies, and we have $E_x = -1.52\times10^{-8}$ N/C, or $|E_x| = 1.52\times10^{-8}$ N/C.

(e) For a particle of charge $-q = -4.23\times10^{-15}$ C, the electric field at a distance a = 50 m away has a magnitude $|E_x| = 1.52\times10^{-8}$ N/C.

Problem 22-37

We use Eq. 22-26, noting that the disk in figure (b) is effectively equivalent to the disk in figure (a) plus a concentric smaller disk (of radius R/2) with the opposite value of σ. That is,

$$E_{(b)} = E_{(a)} - \frac{\sigma}{2\varepsilon_o}\left(1 - \frac{2R}{\sqrt{(2R)^2 + (R/2)^2}}\right)$$

where

$$E_{(a)} = \frac{\sigma}{2\varepsilon_o}\left(1 - \frac{2R}{\sqrt{(2R)^2 + R^2}}\right) .$$

We find the relative difference and simplify:

$$\frac{E_{(a)} - E_{(b)}}{E_{(a)}} = \frac{1-2/\sqrt{4+1/4}}{1-2/\sqrt{4+1}} = \frac{1-2/\sqrt{17/4}}{1-2/\sqrt{5}} = \frac{0.0299}{0.1056} = 0.283$$

or approximately 28%.

Problem 22-45

We combine Eq. 22-9 and Eq. 22-28 (in absolute values).

$$F = |q|E = |q|\left(\frac{p}{2\pi\varepsilon_0 z^3}\right) = \frac{2kep}{z^3}$$

where we have used Eq. 21-5 for the constant k in the last step. Thus, we obtain

$$F = \frac{2\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)\left(3.6\times10^{-29}\text{C}\cdot\text{m}\right)}{\left(25\times10^{-9}\text{m}\right)^3} = 6.6\times10^{-15}\ \text{N} .$$

If the dipole is oriented such that $\vec{p}$ is in the +z direction, then $\vec{F}$ points in the –z direction.

Problem 22-48

We are given $\sigma = 4.00\times10^{-6}$ C/m^2 and various values of z (in the notation of Eq. 22-26, which specifies the field E of the charged disk). Using this with $F = eE$ (the magnitude of Eq. 22-28 applied to the electron) and $F = ma$, we obtain $a = F/m = eE/m$.

(a) The magnitude of the acceleration at a distance R is

$$a = \frac{e\,\sigma\,(2-\sqrt{2}\,)}{4\,m\,\varepsilon_o} = 1.16\times10^{16}\ \text{m/s}^2 .$$

(b) At a distance $R/100$, $a = \dfrac{e\,\sigma\,(10001 - \sqrt{10001}\,)}{20002\,m\,\varepsilon_o} = 3.94 \times 10^{16}\ \text{m/s}^2$.

(c) At a distance $R/1000$, $a = \dfrac{e\,\sigma\,(1000001 - \sqrt{1000001}\,)}{2000002\,m\,\varepsilon_o} = 3.97 \times 10^{16}\ \text{m/s}^2$.

(d) The field due to the disk becomes more uniform as the electron nears the center point. One way to view this is to consider the forces exerted on the electron by the charges near the edge of the disk; the net force on the electron caused by those charges will decrease due to the fact that their contributions come closer to canceling out as the electron approaches the middle of the disk.

Problem 22-54

Due to the fact that the electron is negatively charged, then (as a consequence of Eq. 22-28 and Newton's second law) the field $\vec{E}$ pointing in the $+y$ direction (which we will call "upward") leads to a downward acceleration. This is exactly like a projectile motion problem as treated in Chapter 4 (but with g replaced with $a = eE/m = 8.78 \times 10^{11}\ \text{m/s}^2$). Thus, Eq. 4-21 gives

$$t = \frac{x}{v_0 \cos\theta_0} = \frac{3.00\ \text{m}}{(2.00\times10^6\ \text{m/s})\cos 40.0°} = 1.96\times10^{-6}\ \text{s}\,.$$

This leads (using Eq. 4-23) to

$$\begin{aligned} v_y &= v_0 \sin\theta_0 - at = (2.00\times10^6\ \text{m/s})\sin 40.0° - (8.78\times10^{11}\ \text{m/s}^2)(1.96\times10^{-6}\ \text{s}) \\ &= -4.34\times10^5\ \text{m/s}\,. \end{aligned}$$

Since the x component of velocity does not change, then the final velocity is

$\vec{v} = (1.53 \times 10^6\ \text{m/s})\,\hat{\text{i}} - (4.34 \times 10^5\ \text{m/s})\,\hat{\text{j}}$.

Problem 22-77

(a) Since the two charges in question are of the same sign, the point $x = 2.0$ mm should be located in between them (so that the field vectors point in the opposite direction). Let the coordinate of the second particle be x' ($x' > 0$). Then, the magnitude of the field due to the charge $-q_1$ evaluated at x is given by $E = q_1/4\pi\varepsilon_0 x^2$, while that due to the second charge $-4q_1$ is $E' = 4q_1/4\pi\varepsilon_0(x' - x)^2$. We set the net field equal to zero:

$$\vec{E}_{\text{net}} = 0 \;\Rightarrow\; E = E'$$

so that

$$\frac{q_1}{4\pi\varepsilon_0 x^2} = \frac{4q_1}{4\pi\varepsilon_0 (x' - x)^2}\,.$$

Thus, we obtain $x' = 3x = 3(2.0\ \text{mm}) = 6.0$ mm.

(b) In this case, with the second charge now positive, the electric field vectors produced by both charges are in the negative x direction, when evaluated at x = 2.0 mm. Therefore, the net field points in the negative x direction, or 180°, measured counterclockwise from the $+x$ axis.