Physics 4B

Solutions to Chapter 23 HW

Chapter 23: Questions: 4, 6, 8
Problems: 2, 12, 19, 26, 31, 39, 41, 49, 76

Question 23-4
(a) all tie;(b) *a* uniform, *b* variable, *c* uniform, *d* variable

Question 23-6
either σ, 2σ, 3σ or 3σ, 2σ, σ

Question 23-8
(a) *a*, *b*, *c*, *d*; (b) *a* and *b* tie, then *c*, *d*

Problem 23-2
We use $\Phi = \int \vec{E}\cdot d\vec{A}$ and note that the side length of the cube is (3.0 m–1.0 m) = 2.0 m.

(a) On the top face of the cube *y* = 2.0 m and $d\vec{A} = (dA)\hat{\mathrm{j}}$. Therefore, we have $\vec{E} = 4\hat{\mathrm{i}} - 3\left((2.0)^2 + 2\right)\hat{\mathrm{j}} = 4\hat{\mathrm{i}} - 18\hat{\mathrm{j}}$. Thus the flux is

$$\Phi = \int_{\text{top}} \vec{E}\cdot d\vec{A} = \int_{\text{top}} \left(4\hat{\mathrm{i}} - 18\hat{\mathrm{j}}\right)\cdot(dA)\hat{\mathrm{j}} = -18\int_{\text{top}} dA = (-18)(2.0)^2\ \mathrm{N\cdot m^2/C} = -72\ \mathrm{N\cdot m^2/C}.$$

(b) On the bottom face of the cube *y* = 0 and $d\vec{A} = (dA)\left(-\hat{\mathrm{j}}\right)$. Therefore, we have $E = 4\hat{\mathrm{i}} - 3\left(0^2 + 2\right)\hat{\mathrm{j}} = 4\hat{\mathrm{i}} - 6\hat{\mathrm{j}}$. Thus, the flux is

$$\Phi = \int_{\text{bottom}} \vec{E}\cdot d\vec{A} = \int_{\text{bottom}} \left(4\hat{\mathrm{i}} - 6\hat{\mathrm{j}}\right)\cdot(dA)\left(-\hat{\mathrm{j}}\right) = 6\int_{\text{bottom}} dA = 6(2.0)^2\ \mathrm{N\cdot m^2/C} = +24\ \mathrm{N\cdot m^2/C}.$$

(c) On the left face of the cube $d\vec{A} = (dA)\left(-\hat{\mathrm{i}}\right)$. So

$$\Phi = \int_{\text{left}} \hat{E}\cdot d\vec{A} = \int_{\text{left}} \left(4\hat{\mathrm{i}} + E_y\hat{\mathrm{j}}\right)\cdot(dA)\left(-\hat{\mathrm{i}}\right) = -4\int_{\text{bottom}} dA = -4(2.0)^2\ \mathrm{N\cdot m^2/C} = -16\ \mathrm{N\cdot m^2/C}.$$

(d) On the back face of the cube $d\vec{A} = (dA)\left(-\hat{\mathrm{k}}\right)$. But since $\vec{E}$ has no *z* component $\vec{E}\cdot d\vec{A} = 0$.
Thus, $\Phi = 0$.

(e) We now have to add the flux through all six faces. One can easily verify that the flux through the front face is zero, while that through the right face is the opposite of that through the left one, or +16 $\mathrm{N\cdot m^2/C}$. Thus the net flux through the cube is

$\Phi = (-72 + 24 - 16 + 0 + 0 + 16)\ \text{N}\cdot\text{m}^2/\text{C} = -48\ \text{N}\cdot\text{m}^2/\text{C}$.

Problem 23-12

We note that only the smaller shell contributes a (nonzero) field at the designated point, since the point is inside the radius of the large sphere (and $E = 0$ inside of a spherical charge), and the field points toward the $-x$ direction. Thus, with $R = 0.020$ m (the radius of the smaller shell), $L = 0.10$ m and $x = 0.020$ m, we obtain

$$\vec{E} = E(-\hat{\text{j}}) = -\frac{q}{4\pi\varepsilon_0 r^2}\hat{\text{j}} = -\frac{4\pi R^2\sigma_2}{4\pi\varepsilon_0 (L-x)^2}\hat{\text{j}} = -\frac{R^2\sigma_2}{\varepsilon_0 (L-x)^2}\hat{\text{j}}$$

$$= -\frac{(0.020\ \text{m})^2(4.0\times10^{-6}\,\text{C/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.10\ \text{m} - 0.020\ \text{m})^2}\hat{\text{j}} = \left(-2.8\times10^4\ \text{N/C}\right)\hat{\text{j}}.$$

Problem 23-19

(a) The area of a sphere may be written $4\pi R^2 = \pi D^2$. Thus,

$$\sigma = \frac{q}{\pi D^2} = \frac{2.4\times10^{-6}\ \text{C}}{\pi\left(1.3\ \text{m}\right)^2} = 4.5\times10^{-7}\ \text{C/m}^2.$$

(b) Equation 23-11 gives

$$E = \frac{\sigma}{\varepsilon_0} = \frac{4.5\times10^{-7}\ \text{C/m}^2}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 5.1\times10^4\ \text{N/C}.$$

Problem 23-26

As we approach $r = 3.5$ cm from the inside, we have

$$E_{\text{internal}} = \frac{2\lambda}{4\pi\varepsilon_0 r} = 1000\ \text{N/C}.$$

And as we approach $r = 3.5$ cm from the outside, we have

$$E_{\text{external}} = \frac{2\lambda}{4\pi\varepsilon_0 r} + \frac{2\lambda'}{4\pi\varepsilon_0 r} = -3000\ \text{N/C}\ .$$

Considering the difference ($E_{\text{external}} - E_{\text{internal}}$) allows us to find λ' (the charge per unit length on the larger cylinder). Using $r = 0.035$ m, we obtain $\lambda' = -5.8\times10^{-9}$ C/m.

Problem 23-31

We denote the inner and outer cylinders with subscripts *i* and *o*, respectively.

(a) Since $r_i < r = 4.0$ cm $< r_o$,

$$E(r) = \frac{\lambda_i}{2\pi\varepsilon_0 r} = \frac{5.0\times10^{-6}\ \text{C/m}}{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4.0\times10^{-2}\ \text{m})} = 2.3\times10^{6}\ \text{N/C}.$$

(b) The electric field $\vec{E}(r)$ points radially outward.

(c) Since $r > r_o$,

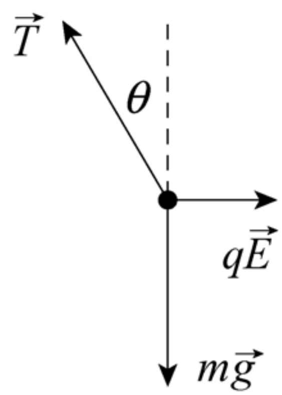
$$E(r = 8.0\ \text{cm}) = \frac{\lambda_i + \lambda_o}{2\pi\varepsilon_0 r} = \frac{5.0\times10^{-6}\ \text{C/m} - 7.0\times10^{-6}\ \text{C/m}}{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(8.0\times10^{-2}\ \text{m})} = -4.5\times10^{5}\ \text{N/C},$$

or $|E(r = 8.0\ \text{cm})| = 4.5\times10^{5}$ N/C.

(d) The minus sign indicates that $\vec{E}(r)$ points radially inward.

Problem 23-39

The forces acting on the ball are shown in the diagram below. The gravitational force has magnitude *mg*, where *m* is the mass of the ball; the electrical force has magnitude *qE*, where *q* is the charge on the ball and *E* is the magnitude of the electric field at the position of the ball; and the tension in the thread is denoted by *T*.



The electric field produced by the plate is normal to the plate and points to the right. Since the ball is positively charged, the electric force on it also points to the right. The tension in the thread makes the angle θ (= 30°) with the vertical.

Since the ball is in equilibrium the net force on it vanishes. The sum of the horizontal components yields

$$qE - T\sin\theta = 0$$

and the sum of the vertical components yields

$$T\cos\theta - mg = 0\,.$$

The expression $T = qE/\sin\theta$, from the first equation, is substituted into the second to obtain $qE = mg\tan\theta$. The electric field produced by a large uniform plane of charge is given by $E = \sigma/2\varepsilon_0$, where σ is the surface charge density. Thus,

$$\frac{q\sigma}{2\varepsilon_0} = mg\tan\theta$$

and

$$\sigma = \frac{2\varepsilon_0 mg\tan\theta}{q} = \frac{2\left(8.85\times10^{-12}\ \mathrm{C^2/N.m^2}\right)\left(1.0\times10^{-6}\ \mathrm{kg}\right)\left(9.8\ \mathrm{m/s^2}\right)\tan 30^\circ}{2.0\times10^{-8}\ \mathrm{C}}$$
$$= 5.0\times10^{-9}\ \mathrm{C/m^2}.$$

Problem 23-41

The charge on the metal plate, which is negative, exerts a force of repulsion on the electron and stops it. First find an expression for the acceleration of the electron, then use kinematics to find the stopping distance. We take the initial direction of motion of the electron to be positive. Then, the electric field is given by $E = \sigma/\varepsilon_0$, where σ is the surface charge density on the plate. The force on the electron is $F = -eE = -e\sigma/\varepsilon_0$ and the acceleration is

$$a = \frac{F}{m} = -\frac{e\sigma}{\varepsilon_0 m}$$

where m is the mass of the electron. The force is constant, so we use constant acceleration kinematics. If v_0 is the initial velocity of the electron, v is the final velocity, and x is the distance traveled between the initial and final positions, then $v^2 - v_0^2 = 2ax.$ Set $v = 0$ and replace a with $-e\sigma/\varepsilon_0 m$, then solve for x. We find

$$x = -\frac{v_0^2}{2a} = \frac{\varepsilon_0 m v_0^2}{2e\sigma}.$$

Now $\frac{1}{2}mv_0^2$ is the initial kinetic energy K_0, so

$$x = \frac{\varepsilon_0 K_0}{e\sigma} = \frac{\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(1.60\times10^{-17}\ \mathrm{J}\right)}{\left(1.60\times10^{-19}\ \mathrm{C}\right)\left(2.0\times10^{-6}\ \mathrm{C/m^2}\right)} = 4.4\times10^{-4}\ \mathrm{m}.$$

Problem 23-49

At all points where there is an electric field, it is radially outward. For each part of the problem, use a Gaussian surface in the form of a sphere that is concentric with the sphere of charge and passes through the point where the electric field is to be found. The field is uniform on the surface, so $\oint \vec{E}\cdot d\vec{A} = 4\pi r^2 E$, where r is the radius of the Gaussian surface.

For $r < a$, the charge enclosed by the Gaussian surface is $q_1(r/a)^3$. Gauss' law yields

$$4\pi r^2 E = \left(\frac{q_1}{\varepsilon_0}\right)\left(\frac{r}{a}\right)^3 \Rightarrow E = \frac{q_1 r}{4\pi\varepsilon_0 a^3}.$$

(a) For $r = 0$, the above equation implies $E = 0$.

(b) For $r = a/2$, we have

$$E = \frac{q_1(a/2)}{4\pi\varepsilon_0 a^3} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(5.00\times10^{-15}\ \mathrm{C})}{2(2.00\times10^{-2}\ \mathrm{m})^2} = 5.62\times10^{-2}\ \mathrm{N/C}.$$

(c) For $r = a$, we have

$$E = \frac{q_1}{4\pi\varepsilon_0 a^2} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(5.00\times10^{-15}\ \mathrm{C})}{(2.00\times10^{-2}\ \mathrm{m})^2} = 0.112\ \mathrm{N/C}.$$

In the case where $a < r < b$, the charge enclosed by the Gaussian surface is q_1, so Gauss' law leads to

$$4\pi r^2 E = \frac{q_1}{\varepsilon_0} \Rightarrow E = \frac{q_1}{4\pi\varepsilon_0 r^2}.$$

(d) For $r = 1.50a$, we have

$$E = \frac{q_1}{4\pi\varepsilon_0 r^2} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(5.00\times10^{-15}\ \mathrm{C})}{(1.50\times2.00\times10^{-2}\ \mathrm{m})^2} = 0.0499\ \mathrm{N/C}.$$

(e) In the region $b < r < c$, since the shell is conducting, the electric field is zero. Thus, for $r = 2.30a$, we have $E = 0$.

(f) For $r > c$, the charge enclosed by the Gaussian surface is zero. Gauss' law yields $4\pi r^2 E = 0 \Rightarrow E = 0.$ Thus, $E = 0$ at $r = 3.50a$.

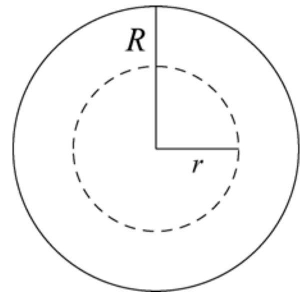
(g) Consider a Gaussian surface that lies completely within the conducting shell. Since the electric field is everywhere zero on the surface, $\oint \vec{E}\cdot d\vec{A} = 0$ and, according to Gauss' law, the net charge enclosed by the surface is zero. If Q_i is the charge on the inner surface of the shell, then $q_1 + Q_i = 0$ and $Q_i = -q_1 = -5.00$ fC.

(h) Let Q_o be the charge on the outer surface of the shell. Since the net charge on the shell is $-q$, $Q_i + Q_o = -q_1$. This means

$Q_o = -q_1 - Q_i = -q_1 - (-q_1) = 0$.

Problem 23-76

(a) The diagram shows a cross section (or, perhaps more appropriately, “end view”) of the charged cylinder (solid circle).



Consider a Gaussian surface in the form of a cylinder with radius r and length ℓ, coaxial with the charged cylinder. An “end view” of the Gaussian surface is shown as a dashed circle. The charge enclosed by it is $q = \rho V = \pi r^2 \ell \rho$, where $V = \pi r^2 \ell$ is the volume of the cylinder.

If ρ is positive, the electric field lines are radially outward, normal to the Gaussian surface and distributed uniformly along it. Thus, the total flux through the Gaussian cylinder is $\Phi = EA_{\text{cylinder}} = E(2\pi r \ell)$. Now, Gauss’ law leads to

$$2\pi\varepsilon_0 r\ell E = \pi r^2 \ell \rho \quad \Rightarrow \quad E = \frac{\rho r}{2\varepsilon_0}.$$

(b) Next, we consider a cylindrical Gaussian surface of radius $r > R$. If the external field E_{ext} then the flux is $\Phi = 2\pi r \ell E_{\text{ext}}$. The charge enclosed is the total charge in a section of the charged cylinder with length ℓ. That is, $q = \pi R^2 \ell \rho$. In this case, Gauss’ law yields

$$2\pi\varepsilon_0 r\ell E_{\text{ext}} = \pi R^2 \ell \rho \quad \Rightarrow \quad E_{\text{ext}} = \frac{R^2 \rho}{2\varepsilon_0 r}.$$