Physics 4B

Solutions to Chapter 24 HW

Chapter 24: Questions: 4, 6, 10
Problems: 11, 19, 25, 33, 35, 53, 54, 67, 99

Question 24-4
(a) 2, 4, and then a tie of 1, 3, and 5 (where $E = 0$); (b) negative x direction; (c) positive x direction

Question 24-6
b, then a, c, and d tie

Question 24-10
(a) $Q/4\pi\varepsilon_0 R$; (b) $Q/4\pi\varepsilon_0 R$; (c) $Q/4\pi\varepsilon_0 R$;(d) a, b, c

Problem 24-11
(a) The potential as a function of r is

$$V(r) = V(0) - \int_0^r E(r)dr = 0 - \int_0^r \frac{qr}{4\pi\varepsilon_0 R^3}dr = -\frac{qr^2}{8\pi\varepsilon_0 R^3}$$

$$= -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.50\times10^{-15}\,\text{C})(0.0145\ \text{m})^2}{2(0.0231\ \text{m})^3} = -2.68\times10^{-4}\ \text{V}.$$

(b) Since $\Delta V = V(0) - V(R) = q/8\pi\varepsilon_0 R$, we have

$$V(R) = -\frac{q}{8\pi\varepsilon_0 R} = -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.50\times10^{-15}\,\text{C})}{2(0.0231\ \text{m})} = -6.81\times10^{-4}\ \text{V}.$$

Problem 24-19
First, we observe that $V(x)$ cannot be equal to zero for $x > d$. In fact $V(x)$ is always negative for $x > d$. Now we consider the two remaining regions on the x axis: $x < 0$ and $0 < x < d$.

(a) For $0 < x < d$ we have $d_1 = x$ and $d_2 = d - x$. Let

$$V(x) = k\left(\frac{q_1}{d_1} + \frac{q_2}{d_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{x} + \frac{-3}{d-x}\right) = 0$$

and solve: $x = d/4$. With $d = 24.0$ cm, we have $x = 6.00$ cm.

(b) Similarly, for $x < 0$ the separation between q_1 and a point on the x axis whose coordinate is x is given by $d_1 = -x$; while the corresponding separation for q_2 is $d_2 = d - x$. We set

$$V(x) = k\left(\frac{q_1}{d_1} + \frac{q_2}{d_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{-x} + \frac{-3}{d-x}\right) = 0$$

to obtain $x = -d/2$. With $d = 24.0$ cm, we have $x = -12.0$ cm.

Problem 24-25

(a) All the charge is the same distance R from C, so the electric potential at C is

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{R} - \frac{6Q_1}{R}\right) = -\frac{5Q_1}{4\pi\varepsilon_0 R} = -\frac{5(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.20\times10^{-12}\,\text{C})}{8.20\times10^{-2}\ \text{m}} = -2.30\ \text{V},$$

where the zero was taken to be at infinity.

(b) All the charge is the same distance from P. That distance is $\sqrt{R^2+D^2}$, so the electric potential at P is

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{\sqrt{R^2+D^2}} - \frac{6Q_1}{\sqrt{R^2+D^2}}\right] = -\frac{5Q_1}{4\pi\varepsilon_0\sqrt{R^2+D^2}}$$

$$= -\frac{5(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.20\times10^{-12}\,\text{C})}{\sqrt{(8.20\times10^{-2}\ \text{m})^2 + (6.71\times10^{-2}\ \text{m})^2}}$$

$$= -1.78\ \text{V}.$$

Problem 24-33

Consider an infinitesimal segment of the rod, located between x and $x + dx$. It has length dx and contains charge $dq = \lambda\, dx = cx\, dx$. Its distance from P_1 is $d + x$ and the potential it creates at P_1 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{d+x} = \frac{1}{4\pi\varepsilon_0}\frac{cx\,dx}{d+x}.$$

To find the total potential at P_1, we integrate over the length of the rod and obtain

$$V = \frac{c}{4\pi\varepsilon_0}\int_0^L \frac{xdx}{d+x} = \frac{c}{4\pi\varepsilon_0}[x - d\ln(x+d)]\Big|_0^L = \frac{c}{4\pi\varepsilon_0}\left[L - d\ln\left(1+\frac{L}{d}\right)\right]$$

$$= (8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(28.9\times10^{-12}\,\text{C/m}^2)\left[0.120\ \text{m} - (0.030\ \text{m})\ln\left(1+\frac{0.120\ \text{m}}{0.030\ \text{m}}\right)\right]$$

$$= 1.86\times10^{-2}\ \text{V}.$$

Problem 24-35

We use Eq. 24-41:

$$E_x(x,y) = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left((2.0\text{V}/\text{m}^2)x^2 - 3.0\text{V}/\text{m}^2)y^2\right) = -2(2.0\text{V}/\text{m}^2)x;$$
$$E_y(x,y) = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left((2.0\text{V}/\text{m}^2)x^2 - 3.0\text{V}/\text{m}^2)y^2\right) = 2(3.0\text{V}/\text{m}^2)y.$$

We evaluate at x = 3.0 m and y = 2.0 m to obtain

$\vec{E} = (-12\text{ V/m})\hat{\text{i}} + (12\text{ V/m})\hat{\text{j}}$.

Problem 24-53

(a) The potential energy is

$$U = \frac{q^2}{4\pi\varepsilon_0 d} = \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(5.0\times10^{-6}\text{ C}\right)^2}{1.00\text{ m}} = 0.225\text{ J}$$

relative to the potential energy at infinite separation.

(b) Each sphere repels the other with a force that has magnitude

$$F = \frac{q^2}{4\pi\varepsilon_0 d^2} = \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(5.0\times10^{-6}\text{ C}\right)^2}{\left(1.00\text{ m}\right)^2} = 0.225\text{ N}.$$

According to Newton's second law the acceleration of each sphere is the force divided by the mass of the sphere. Let m_A and m_B be the masses of the spheres. The acceleration of sphere A is

$$a_A = \frac{F}{m_A} = \frac{0.225\text{ N}}{5.0\times10^{-3}\text{ kg}} = 45.0\text{ m/s}^2$$

and the acceleration of sphere B is

$$a_B = \frac{F}{m_B} = \frac{0.225\text{ N}}{10\times10^{-3}\text{ kg}} = 22.5\text{ m/s}^2.$$

(c) Energy is conserved. The initial potential energy is U = 0.225 J, as calculated in part (a). The initial kinetic energy is zero since the spheres start from rest. The final potential energy is zero since the spheres are then far apart. The final kinetic energy is $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$, where v_A and v_B are the final velocities. Thus,

$$U = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2.$$

Momentum is also conserved, so

$$0 = m_A v_A + m_B v_B.$$

These equations may be solved simultaneously for v_A and v_B. Substituting $v_B = -(m_A / m_B)v_A$, from the momentum equation into the energy equation, and collecting terms, we obtain

$$U = \tfrac{1}{2}(m_A / m_B)(m_A + m_B)v_A^2.$$

Thus,

$$v_A = \sqrt{\frac{2Um_B}{m_A(m_A + m_B)}} = \sqrt{\frac{2(0.225 \text{ J})(10\times10^{-3} \text{ kg})}{(5.0\times10^{-3} \text{ kg})(5.0\times10^{-3} \text{ kg} + 10\times10^{-3} \text{ kg})}} = 7.75 \text{ m/s}.$$

We thus obtain

$$v_B = -\frac{m_A}{m_B}v_A = -\left(\frac{5.0\times10^{-3} \text{ kg}}{10\times10^{-3} \text{ kg}}\right)(7.75 \text{ m/s}) = -3.87 \text{ m/s},$$

or $|v_B| = 3.87$ m/s.

Problem 24-54

(a) Using $U = qV$ we can "translate" the graph of voltage into a potential energy graph (in eV units). From the information in the problem, we can calculate its kinetic energy (which is its total energy at $x = 0$) in those units: $K_i = 284$ eV. This is less than the "height" of the potential energy "barrier" (500 eV high once we've translated the graph as indicated above). Thus, it must reach a turning point and then reverse its motion.

(b) Its final velocity, then, is in the negative x direction with a magnitude equal to that of its initial velocity. That is, its speed (upon leaving this region) is 1.0×10^7 m/s.

Problem 24-67

(a) The magnitude of the electric field is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{\left(3.0\times10^{-8}\text{C}\right)\left(8.99\times10^{9} \text{ N}\cdot\text{m}^2/\text{C}^2\right)}{\left(0.15\,\text{m}\right)^2} = 1.2\times10^{4} \text{ N/C}.$$

(b) $V = RE = (0.15 \text{ m})(1.2 \times 10^4 \text{ N/C}) = 1.8 \times 10^3$ V.

(c) Let the distance be x. Then

$$\Delta V = V(x) - V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R+x} - \frac{1}{R}\right) = -500\,\text{V},$$

which gives

$$x = \frac{R\Delta V}{-V - \Delta V} = \frac{(0.15\,\text{m})(-500\,\text{V})}{-1800\,\text{V} + 500\,\text{V}} = 5.8 \times 10^{-2}\,\text{m}.$$

Problem 24-99

(a) The charge on every part of the ring is the same distance from any point P on the axis. This distance is $r = \sqrt{z^2 + R^2}$, where R is the radius of the ring and z is the distance from the center of the ring to P. The electric potential at P is

$$V = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{\sqrt{z^2 + R^2}} = \frac{1}{4\pi\varepsilon_0} \frac{1}{\sqrt{z^2 + R^2}} \int dq = \frac{1}{4\pi\varepsilon_0} \frac{q}{\sqrt{z^2 + R^2}}.$$

(b) The electric field is along the axis and its component is given by

$$E = -\frac{\partial V}{\partial z} = -\frac{q}{4\pi\varepsilon_0} \frac{\partial}{\partial z} (z^2 + R^2)^{-1/2} = \frac{q}{4\pi\varepsilon_0} \left(\frac{1}{2}\right) (z^2 + R^2)^{-3/2} (2z) = \frac{q}{4\pi\varepsilon_0} \frac{z}{(z^2 + R^2)^{3/2}}.$$

This agrees with Eq. 23-16.