Physics 4B

Solutions to Chapter 25 HW

Chapter 25: Questions: 4, 6, 10
Problems: 4, 15, 19, 27, 32, 45, 48, 51, 57

Question 25-4
(a) 2; (b) 3; (c) 1

Question 25-6
(a) less; (b) less; (c) less; (d) less

Question 25-10
(a) increase; (b) increase; (c) decrease; (d) decrease; (e) same, increase, increase, increase

Problem 25-4
(a) We use Eq. 25-17:

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} = \frac{(40.0\,\text{mm})(38.0\,\text{mm})}{\left(8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(40.0\,\text{mm}-38.0\,\text{mm})} = 84.5\ \text{pF}.$$

(b) Let the area required be A. Then $C = \varepsilon_0 A/(b-a)$, or

$$A = \frac{C(b-a)}{\varepsilon_0} = \frac{(84.5\,\text{pF})(40.0\,\text{mm}-38.0\,\text{mm})}{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 191\,\text{cm}^2.$$

Problem 25-15
(a) First, the equivalent capacitance of the two 4.00 μF capacitors connected in series is given by 4.00 μF/2 = 2.00 μF. This combination is then connected in parallel with two other 2.00-μF capacitors (one on each side), resulting in an equivalent capacitance C = 3(2.00 μF) = 6.00 μF. This is now seen to be in series with another combination, which consists of the two 3.0-μF capacitors connected in parallel (which are themselves equivalent to C' = 2(3.00 μF) = 6.00 μF). Thus, the equivalent capacitance of the circuit is

$$C_{\text{eq}} = \frac{CC'}{C+C'} = \frac{(6.00\,\mu\text{F})\,(6.00\,\mu\text{F})}{6.00\,\mu\text{F}+6.00\,\mu\text{F}} = 3.00\,\mu\text{F}.$$

(b) Let V = 20.0 V be the potential difference supplied by the battery. Then

$$q = C_{\text{eq}}V = (3.00\ \mu\text{F})(20.0\ \text{V}) = 6.00\times10^{-5}\ \text{C}.$$

(c) The potential difference across C_1 is given by

$$V_1 = \frac{CV}{C+C'} = \frac{(6.00\,\mu\text{F})\,(20.0\,\text{V})}{6.00\,\mu\text{F}+6.00\,\mu\text{F}} = 10.0\,\text{V}.$$

(d) The charge carried by C_1 is $q_1 = C_1V_1$= (3.00 μF)(10.0 V) = 3.00×10^{-5} C.

(e) The potential difference across C_2 is given by $V_2 = V - V_1$ = 20.0 V – 10.0 V = 10.0 V.

(f) The charge carried by C_2 is $q_2 = C_2V_2$ = (2.00 μF)(10.0 V) = 2.00×10^{-5} C.

(g) Since this voltage difference V_2 is divided equally between C_3 and the other 4.00-μF capacitors connected in series with it, the voltage difference across C_3 is given by $V_3 = V_2/2$ = 10.0 V/2 = 5.00 V.

(h) Thus, $q_3 = C_3V_3$ = (4.00 μF)(5.00 V) = 2.00×10^{-5} C.

Problem 25-19

(a) and (b) We note that the charge on C_3 is q_3 = 12 μC – 8.0 μC = 4.0 μC. Since the charge on C_4 is q_4 = 8.0 μC, then the voltage across it is q_4/C_4 = 2.0 V. Consequently, the voltage V_3 across C_3 is 2.0 V $\Rightarrow$ $C_3 = q_3/V_3$ = 2.0 μF.

Now C_3 and C_4 are in parallel and are thus equivalent to 6 μF capacitor which would then be in series with C_2; thus, Eq 25-20 leads to an equivalence of 2.0 μF which is to be thought of as being in series with the unknown C_1. We know that the total effective capacitance of the circuit (in the sense of what the battery "sees" when it is hooked up) is (12 μC)/V_{battery} = 4 μF/3. Using Eq 25-20 again, we find

$$\frac{1}{2\ \mu\text{F}} + \frac{1}{C_1} = \frac{3}{4\ \mu\text{F}} \quad \Rightarrow \quad C_1 = 4.0\ \mu\text{F}\ .$$

Problem 25-27

(a) In this situation, capacitors 1 and 3 are in series, which means their charges are necessarily the same:

$$q_1 = q_3 = \frac{C_1C_3V}{C_1+C_3} = \frac{(1.00\,\mu\text{F})\,(3.00\,\mu\text{F})\,(12.0\,\text{V})}{1.00\,\mu\text{F}+3.00\,\mu\text{F}} = 9.00\,\mu\text{C}.$$

(b) Capacitors 2 and 4 are also in series:

$$q_2 = q_4 = \frac{C_2C_4V}{C_2+C_4} = \frac{(2.00\,\mu\text{F})\,(4.00\,\mu\text{F})\,(12.0\,\text{V})}{2.00\,\mu\text{F}+4.00\,\mu\text{F}} = 16.0\,\mu\text{C}.$$

(c) $q_3 = q_1 = 9.00\,\mu\text{C}$.

(d) $q_4 = q_2 = 16.0\,\mu\text{C}$.

(e) With switch 2 also closed, the potential difference V_1 across C_1 must equal the potential difference across C_2 and is

$$V_1 = \frac{C_3 + C_4}{C_1 + C_2 + C_3 + C_4} V = \frac{(3.00\,\mu\text{F} + 4.00\,\mu\text{F})(12.0\,\text{V})}{1.00\,\mu\text{F} + 2.00\,\mu\text{F} + 3.00\,\mu\text{F} + 4.00\,\mu\text{F}} = 8.40\ \text{V}.$$

Thus, $q_1 = C_1V_1$ = (1.00 μF)(8.40 V) = 8.40 μC.

(f) Similarly, $q_2 = C_2V_1$ = (2.00 μF)(8.40 V) = 16.8 μC.

(g) $q_3 = C_3(V - V_1)$ = (3.00 μF)(12.0 V – 8.40 V) = 10.8 μC.

(h) $q_4 = C_4(V - V_1)$ = (4.00 μF)(12.0 V – 8.40 V) = 14.4 μC.

Problem 25-32

(a) The capacitance is

$$C = \frac{\varepsilon_0 A}{d} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(40\times10^{-4}\ \text{m}^2)}{1.0\times10^{-3}\,\text{m}} = 3.5\times10^{-11}\ \text{F} = 35\,\text{pF}.$$

(b) $q = CV$ = (35 pF)(600 V) = 2.1×10^{-8} C = 21 nC.

(c) $U = \frac{1}{2}CV^2 = \frac{1}{2}(35\,\text{pF})(21\,\text{nC})^2 = 6.3\times10^{-6}\,\text{J} = 6.3\,\mu\text{J}$.

(d) $E = V/d$ = 600 V/1.0 × 10^{-3} m = 6.0 × 10^{5} V/m.

(e) The energy density (energy per unit volume) is

$$u = \frac{U}{Ad} = \frac{6.3\times10^{-6}\,\text{J}}{(40\times10^{-4}\text{m}^2)(1.0\times10^{-3}\text{m})} = 1.6\ \text{J/m}^3.$$

Problem 25-45

Using Eq. 25-29, with $\sigma = q/A$, we have

$$\left|\vec{E}\right| = \frac{q}{\kappa\varepsilon_0 A} = 200\times 10^3 \text{ N/C}$$

which yields $q = 3.3 \times 10^{-7}$ C. Eq. 25-21 and Eq. 25-27 therefore lead to

$$U = \frac{q^2}{2C} = \frac{q^2 d}{2\kappa\varepsilon_0 A} = 6.6\times 10^{-5}\text{ J} .$$

Problem 25-48

The capacitor can be viewed as two capacitors C_1 and C_2 in parallel, each with surface area $A/2$ and plate separation d, filled with dielectric materials with dielectric constants κ_1 and κ_2, respectively. Thus, (in SI units),

$$C = C_1 + C_2 = \frac{\varepsilon_0 (A/2)\kappa_1}{d} + \frac{\varepsilon_0 (A/2)\kappa_2}{d} = \frac{\varepsilon_0 A}{d}\left(\frac{\kappa_1+\kappa_2}{2}\right)$$

$$= \frac{(8.85\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(5.56\times 10^{-4}\text{ m}^2)}{5.56\times 10^{-3}\text{ m}}\left(\frac{7.00+12.00}{2}\right) = 8.41\times 10^{-12}\text{ F.}$$

Problem 25-51

(a) The electric field in the region between the plates is given by $E = V/d$, where V is the potential difference between the plates and d is the plate separation. The capacitance is given by $C = \kappa\varepsilon_0 A/d$, where A is the plate area and κ is the dielectric constant, so $d = \kappa\varepsilon_0 A/C$ and

$$E = \frac{VC}{\kappa\varepsilon_0 A} = \frac{(50\text{ V})(100\times 10^{-12}\text{ F})}{5.4(8.85\times 10^{-12}\text{ F/m})(100\times 10^{-4}\text{ m}^2)} = 1.0\times 10^4\text{ V/m}.$$

(b) The free charge on the plates is $q_f = CV = (100 \times 10^{-12}\text{ F})(50\text{ V}) = 5.0 \times 10^{-9}$ C.

(c) The electric field is produced by both the free and induced charge. Since the field of a large uniform layer of charge is $q/2\varepsilon_0 A$, the field between the plates is

$$E = \frac{q_f}{2\varepsilon_0 A} + \frac{q_f}{2\varepsilon_0 A} - \frac{q_i}{2\varepsilon_0 A} - \frac{q_i}{2\varepsilon_0 A},$$

where the first term is due to the positive free charge on one plate, the second is due to the negative free charge on the other plate, the third is due to the positive induced charge on one dielectric surface, and the fourth is due to the negative induced charge on the other dielectric surface. Note that the field due to the induced charge is opposite the field due to the free charge, so they tend to cancel. The induced charge is therefore

$$q_i = q_f - \varepsilon_0 AE = 5.0\times10^{-9}\ \text{C} - \left(8.85\times10^{-12}\ \text{F/m}\right)\left(100\times10^{-4}\ \text{m}^2\right)\left(1.0\times10^{4}\ \text{V/m}\right)$$
$$= 4.1\times10^{-9}\ \text{C} = 4.1\,\text{nC}.$$

Problem 25-57

The pair C_3 and C_4 are in parallel and consequently equivalent to 30 μF. Since this numerical value is identical to that of the others (with which it is in series, with the battery), we observe that each has one-third the battery voltage across it. Hence, 3.0 V is across C_4, producing a charge

$$q_4 = C_4V_4 = (15\ \mu\text{F})(3.0\ \text{V}) = 45\ \mu\text{C}.$$