Physics 4B

Solutions to Chapter 26 HW

Chapter 26: Questions: 2, 8, 10
Problems: 2, 11, 13, 18, 33, 41, 46, 56, 71

Question 26-2
b, *a*, *c*

Question 26-8
(a) 1 and 2 tie, then 3;(b) 1 and 2 tie, then 3;(c) 1 and 2 tie, then 3

Question 26-10
C, *A*, *B*

Problem 26-2
Suppose the charge on the sphere increases by Δq in time Δt. Then, in that time its potential increases by

$$\Delta V = \frac{\Delta q}{4\pi\varepsilon_0 r},$$

where r is the radius of the sphere. This means $\Delta q = 4\pi\varepsilon_0 r\,\Delta V$. Now, $\Delta q = (i_{in} - i_{out})\,\Delta t$, where i_{in} is the current entering the sphere and i_{out} is the current leaving. Thus,

$$\Delta t = \frac{\Delta q}{i_{in} - i_{out}} = \frac{4\pi\varepsilon_0 r\,\Delta V}{i_{in} - i_{out}} = \frac{(0.10\text{ m})(1000\text{ V})}{(8.99\times10^9\text{ F/m})(1.0000020\text{ A} - 1.0000000\text{ A})}$$
$$= 5.6\times10^{-3}\text{ s}.$$

Problem 26-11
(a) The current resulting from this nonuniform current density is

$$i = \int_{\text{cylinder}} J_a\,dA = \frac{J_0}{R}\int_0^R r\cdot 2\pi r dr = \frac{2}{3}\pi R^2 J_0 = \frac{2}{3}\pi(3.40\times10^{-3}\text{m})^2(5.50\times10^4\text{ A/m}^2)$$
$$= 1.33\text{ A}.$$

(b) In this case,

$$i = \int_{\text{cylinder}} J_b\,dA = \int_0^R J_0\left(1 - \frac{r}{R}\right)2\pi r dr = \frac{1}{3}\pi R^2 J_0 = \frac{1}{3}\pi(3.40\times10^{-3}\text{m})^2(5.50\times10^4\text{ A/m}^2)$$
$$= 0.666\text{ A}.$$

(c) The result is different from that in part (a) because J_b is higher near the center of the cylinder (where the area is smaller for the same radial interval) and lower outward, resulting in a lower average current density over the cross section and consequently a lower current than that in part (a). So, J_a has its maximum value near the surface of the wire.

Problem 26-13

We use $v_d = J/ne = i/Ane$. Thus,

$$t = \frac{L}{v_d} = \frac{L}{i/Ane} = \frac{LAne}{i} = \frac{(0.85\,\text{m})\left(0.21\times10^{-14}\,\text{m}^2\right)\left(8.47\times10^{28}/\text{m}^3\right)\left(1.60\times10^{-19}\,\text{C}\right)}{300\,\text{A}}$$

$$= 8.1\times10^2\,\text{s} = 13\,\text{min}.$$

Problem 26-18

(a) $i = V/R = 23.0\text{ V}/15.0\times10^{-3}\,\Omega = 1.53\times10^3$ A.

(b) The cross-sectional area is $A = \pi r^2 = \frac{1}{4}\pi D^2$. Thus, the magnitude of the current density vector is

$$J = \frac{i}{A} = \frac{4i}{\pi D^2} = \frac{4\left(1.53\times10^{-3}\text{ A}\right)}{\pi\left(6.00\times10^{-3}\text{ m}\right)^2} = 5.41\times10^7\text{ A/m}^2.$$

(c) The resistivity is

$$\rho = \frac{RA}{L} = \frac{(15.0\times10^{-3}\Omega)\pi(6.00\times10^{-3}\text{ m})^2}{4(4.00\text{ m})} = 10.6\times10^{-8}\ \Omega\cdot\text{m}.$$

(d) The material is platinum.

Problem 26-33

(a) The current in the block is $i = V/R = 35.8\text{ V}/935\ \Omega = 3.83\times10^{-2}$ A.

(b) The magnitude of current density is

$$J = i/A = (3.83\times10^{-2}\text{ A})/(3.50\times10^{-4}\text{ m}^2) = 109\text{ A/m}^2.$$

(c) $v_d = J/ne = (109\text{ A/m}^2)/[(5.33\times10^{22}/\text{m}^3)(1.60\times10^{-19}\text{ C})] = 1.28\times10^{-2}$ m/s.

(d) $E = V/L = 35.8\text{ V}/0.158\text{ m} = 227$ V/m.

Problem 26-41

(a) Electrical energy is converted to heat at a rate given by $P = V^2/R,$ where V is the potential difference across the heater and R is the resistance of the heater. Thus,

$$P = \frac{(120\ \text{V})^2}{14\ \Omega} = 1.0 \times 10^3\ \text{W} = 1.0\ \text{kW}.$$

(b) The cost is given by $(1.0\,\text{kW})(5.0\,\text{h})(5.0\,\text{cents/kW}\cdot\text{h}) = \text{US\$}0.25$.

Problem 26-46

(a) Using Table 26-1 and Eq. 26-10 (or Eq. 26-11), we have

$$|\vec{E}| = \rho|\vec{J}| = \left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right)\left(\frac{2.00\,\text{A}}{2.00\times10^{-6}\,\text{m}^2}\right) = 1.69\times10^{-2}\ \text{V/m}.$$

(b) Using L = 4.0 m, the resistance is found from Eq. 26-16:

$$R = \rho L/A = 0.0338\ \Omega.$$

The rate of thermal energy generation is found from Eq. 26-27:

$$P = i^2 R = (2.00\ \text{A})^2(0.0338\ \Omega) = 0.135\ \text{W}.$$

Assuming a steady rate, the amount of thermal energy generated in 30 minutes is found to be (0.135 J/s)(30 × 60 s) = 2.43×10^2 J.

Problem 26-56

(a) The current is

$$i = \frac{V}{R} = \frac{V}{\rho L/A} = \frac{\pi V d^2}{4\rho L} = \frac{\pi(1.20\ \text{V})[(0.0400\,\text{in.})(2.54\times10^{-2}\text{m/in.})]^2}{4(1.69\times10^{-8}\,\Omega\cdot\text{m})(33.0\ \text{m})} = 1.74\ \text{A}.$$

(b) The magnitude of the current density vector is

$$|\vec{J}| = \frac{i}{A} = \frac{4i}{\pi d^2} = \frac{4(1.74\ \text{A})}{\pi[(0.0400\,\text{in.})(2.54\times10^{-2}\ \text{m/in.})]^2} = 2.15\times10^6\ \text{A/m}^2.$$

(c) $E = V/L$ = 1.20 V/33.0 m = 3.63×10^{-2} V/m.

(d) $P = Vi$ = (1.20 V)(1.74 A) = 2.09 W.

Problem 26-71

(a) In Eq. 26-17, we let $\rho = 2\rho_0$ where ρ_0 is the resistivity at T_0 = 20°C:

$$\rho - \rho_0 = 2\rho_0 - \rho_0 = \rho_0\alpha\left(T - T_0\right),$$

and solve for the temperature T: $T = T_0 + \dfrac{1}{\alpha} = 20°\,\text{C} + \dfrac{1}{4.3\times 10^{-3}\,/\,\text{K}} \approx 250°\,\text{C}.$

(b) Since a change in Celsius is equivalent to a change on the Kelvin temperature scale, the value of α used in this calculation is not inconsistent with the other units involved. It is worth noting that this agrees well with Fig. 26-10.